1.041/1.200 Spring 2024: Recitation 5

Date: Mar 11, 2:00 PM

1 Problem 1: Queueing Models

Consider a city block with on-street parking available. There are four parking spots available. Inter-arrival times between vehicles which try to park along the block are independent and distributed exponentially with rate λ. If a vehicle arrives and wants to park but finds all four spots full, it leaves and finds somewhere else to park. The time from a vehicle parking in a space to it leaving is distributed exponentially with rate μ, independent and identical for each of the four spaces on the block and independent across vehicles.

1. Use Kendall's notation to specify the type of queueing system that represents this on-street parking system.
2. Define the state space of the queueing system and draw its state transition diagram.
3. What condition(s), if any, must the system satisfy for it to be able to reach its steady state?
4. Assuming the condition(s) from part c are met, write the system of global balance equations that can be used to compute the steady state probability distribution.
5. Solve the global balance equations analytically, i.e., provide an analytical expression for the stationary distribution.

   For parts f and G=g, suppose that the arrival rate of vehicles into the system is 6 vehicles per hour, and that the expected duration that a vehicle occupies a parking space is 30 minutes.

6. Write an analytical expression and provide a numerical solution for the expected number of vehicles per hour that arrive to the system but are turned away because it is full.

2 Quiz 1 Review Highlights

1. The quiz will not focus on those analytical calculations, such as numerical integration or IDM models.

2. The main focus will be the fundamental diagrams, queueing system, and Little's Law.
3. Make sure you understand the components of Little's Law. $\bar{N}_{car} = \lambda' \bar{T}_{car}$. Also, see the L3 slides examples and its application combined with queueing system.
4. Understand the cumulative plot, like how to find some features (e.g. longest queue or total time in the system) based on the arrival and departure curves.

3 M/D/1 System

Note this part is not required by the lecture nor the quiz, but is related to some formulas in CL2.

1. Due to Pollaczek-Khinchine formula, Expected Waiting Time in the Buffer $W_q = \frac{\lambda\sigma^2 + \rho^2}{2\mu(1-\rho)}$ with $= 0$.
2. Expected Number of Vehicles in the Buffer $L_q = \lambda W_q$
3. Expected Time in the System $W = W_q + \frac{1}{\mu}$
4. Expected Number of Vehicles in the System $L = \lambda W$