

Notes on
1.63 Advanced Environmental Fluid Mechanics
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 3-8a-pert.tex

Appendix to §3.8: Details of perturbation analysis

Consider

$$1 \gg \frac{\nu}{\omega L^2} \gg \epsilon = \frac{U_o T}{L} \sim \frac{U_o}{\omega L}$$

Dimensionless equations

$$u_x + v_y = 0, \quad (3.8.a.1)$$

$$u_t + \epsilon(uu_x + vv_y) = U_t + \epsilon U U_x + u_{yy} \quad (3.8.a.2)$$

Introduce

$$u = u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots, \quad v = v_1 + \epsilon v_2 + \epsilon^2 v_3 + \dots, \quad (3.8.a.3)$$

Plugging into (3.8.a.1),

$$\frac{\partial}{\partial x} (u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots) + \frac{\partial}{\partial y} (v_1 + \epsilon v_2 + \epsilon^2 v_3 + \dots) = 0$$

Plugging into (3.8.a.2)

$$\begin{aligned} & \frac{\partial}{\partial t} (u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots) \\ & + \epsilon (u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots) \frac{\partial}{\partial x} (u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots) \\ & + \epsilon (v_1 + \epsilon v_2 + \epsilon^2 v_3 + \dots) \frac{\partial}{\partial y} (u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots) \\ & = \frac{\partial U}{\partial t} + \epsilon U \frac{\partial U}{\partial x} + \frac{\partial^2}{\partial y^2} (u_1 + \epsilon u_2 + \epsilon^2 u_3 + \dots) \end{aligned}$$

Order $O(\epsilon^0)$:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \quad (3.8.a.4)$$

$$\frac{\partial u_1}{\partial t} = \frac{\partial U}{\partial t} + \frac{\partial^2 u_1}{\partial y^2} \quad (3.8.a.5)$$

Order $O(\epsilon)$:

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \quad (3.8.a.6)$$

$$\frac{\partial u_2}{\partial t} + \left(u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} \right) = U \frac{\partial U}{\partial x} + \frac{\partial^2 u_2}{\partial y^2} \quad (3.8.a.7)$$

Boundary conditions: $O(\epsilon^0)$:

$$u_1 = v_1 = 0, \quad y = 0 \quad (3.8.a.8)$$

$$u_1 \rightarrow U, \quad y \rightarrow \infty \quad (3.8.a.9)$$

$O(\epsilon)$:

$$u_2 = v_2 = 0, \quad y = 0 \quad (3.8.a.10)$$