10.213 Recitation
April 8, 2002

A heat pump is used to heat a house in the winter and to cool it in the summer. During the winter, the outside air serves as a low-temperature heat source; during the summer, it acts as a high-temperature heat sink. The heat-transfer rate through the walls and roof of the house is 0.75 kJ/s for each ºC difference between the inside and outside of the house, summer and winter. The heat-pump motor is rated at 1.5 kW. Determine the minimum outside temperature for which the house can be maintained at 20 ºC during the winter and the maximum outside temperature for which the house can be maintained at 25 ºC during the summer.

Consider Carnot analyses where $\frac{|\dot{Q}_C|}{|\dot{Q}_H|} = \frac{T_C}{T_H}$ *and* $|\dot{Q}_H| = |\dot{Q}_C| + |\dot{W}_S|$*:*

In winter, $T_H = 293.15$ *K,* T_C *is unknown,* $|\dot{W}_S| = 1.5$ *kW, and* $|\dot{Q}_H| = 0.75\ (T_H - T_C)$ *kW/ºC.*

We can relate $|\dot{W}_S|$ *and* $|\dot{Q}_H|$ *to give* $\frac{\dot{W}_S}{|\dot{Q}_H|} = \frac{|\dot{Q}_H| - |\dot{Q}_C|}{|\dot{Q}_H|} = 1 - \frac{T_C}{T_H}$

Thus, $\frac{1.5\text{kW}}{0.75\,(293.15\,\text{K} - T_C)\,\text{kW/}^o\text{C}} = 1 - \frac{T_C}{293.15\text{K}}$ *which gives* $T_C = -4.2$ *ºC.*

In summer, $T_C = 298.15$ *K,* T_H *is unknown,* $|\dot{W}_S| = 1.5$ *kW, and* $|\dot{Q}_C| = 0.75\ (T_H - T_C)$ *kW/ºC.*

We can relate $|\dot{W}_S|$ *and* $|\dot{Q}_C|$ *to give* $\frac{\dot{W}_S}{|\dot{Q}_C|} = \frac{|\dot{Q}_H| - |\dot{Q}_C|}{|\dot{Q}_C|} = \frac{|\dot{Q}_H|}{|\dot{Q}_C|} - 1$

Thus, $\frac{1.5\text{kW}}{0.75\,(T_H - 298.15\text{K})\,\text{kW/}^o\text{C}} = \frac{T_H}{298.15\text{K}} - 1$ *which gives* $T_H = 49.4$ *ºC.*

An inventor has devised a complicated non-flow process in which 1 mole of air is the working fluid. The net effects of the process are claimed to be:

- A change in state of air from 250 °C and 3 bar to 80 °C and 1 bar.
- The production of 1.8 kJ of work
- The transfer of an undisclosed amount of heat to a heat reservoir at 30 °C.

Determine whether the claimed performance of the process is consistent with the second law. Assume that air is an ideal gas for which $C_p = 3.5$ R.

First, consider a first-law balance: $\Delta U^t = Q + W$ *and realize that* $C_V = C_P - R = 2.5R$

(1 mole)($C_V = 2.5R$*)(353 K – 523 K) = Q + (-1.8 kJ) gives Q = -1.73 kJ*

For the second law balance, we have two effects:

Change in state: $n\Delta S = nC_P \ln(T_2/T_1) - R\ln(P_2/P_1) = -2.3$ *J/K*

Heat transfer: $n\Delta S = -Q/T_{res} = -1.73$ *kJ/303K = 5.7 J/K*

Total entropy changes = -2.3 J/K + 5.7 J/K = 3.4 J/K. As $\Delta S^t > 0$*, process possible!*