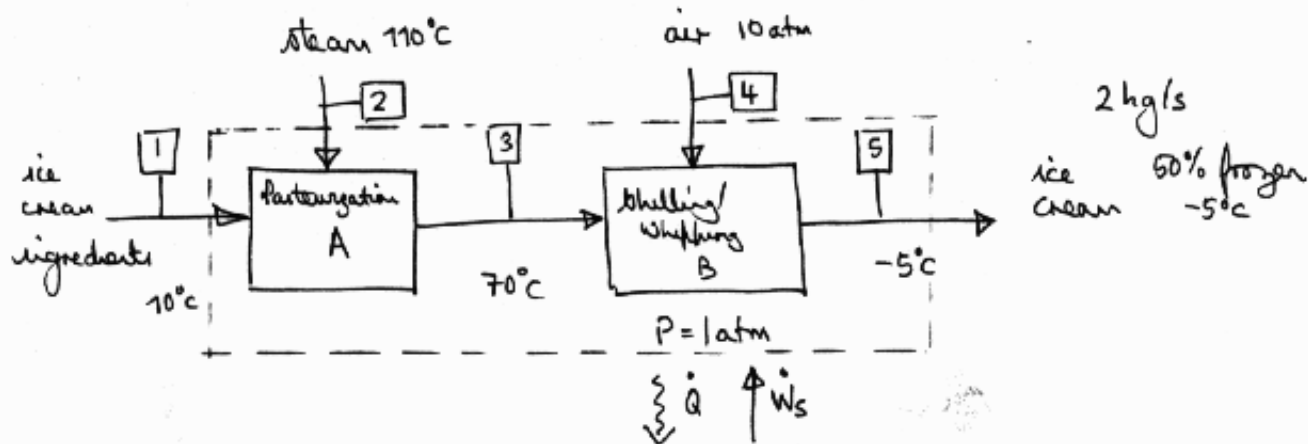


10.213

PROBLEM 11 SOLUTION

a)
- Perform overall mass balance:

$$\text{IN} - \text{OUT} + \underset{\substack{\text{no} \\ \text{reactions}}}{\text{GEN}} = \underset{\text{ACC}}{\text{ACC}} \quad \text{assume steady state}$$

$$\dot{m}_1 + \dot{m}_2 + \dot{m}_4 = \boxed{\dot{m}_5} \quad \text{known}$$

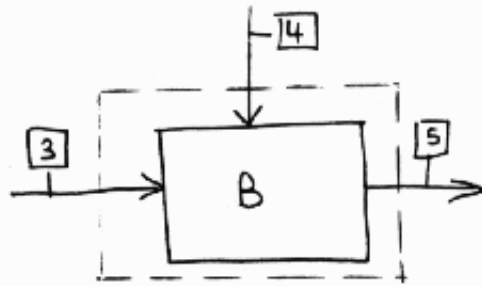
- What is (4)? Air used to whip the ice-cream product. Since the density of gas is typically $\sim 1000\times$ less than that of liquids, we might assume that the mass flow rate of the air is negligible compared to that of the liquid flows. We can check this assumption in part (c) but for now let's go with it.

$$\text{assuming } \dot{m}_4 < \dot{m}_2 \ll \dot{m}_1 \text{ \& } \dot{m}_5,$$

$$\dot{m}_1 + \dot{m}_2 \approx \dot{m}_5$$

$$\text{thus } \dot{m}_1 + \dot{m}_2 = 2 \text{ kg/s}$$

- Looking at unit B, the chilling/whipping unit:

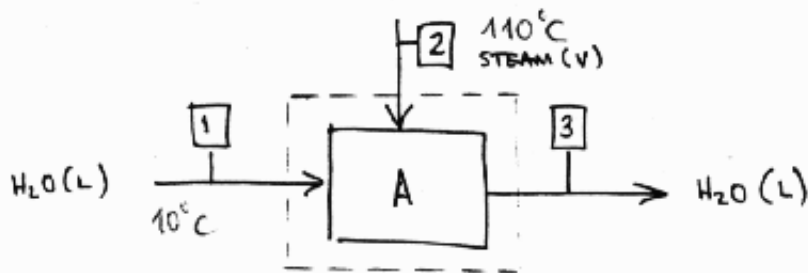


mass balance on B: $\dot{m}_3 + \dot{m}_4 = \dot{m}_5$

using our assumption that $\dot{m}_4 \ll \dot{m}_3$ & \dot{m}_5

$$\dot{m}_3 = \dot{m}_5 = 2 \text{ kg/s} \rightarrow$$

- Looking at unit A, the pasteurization unit:



* mass balance on A: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2 \text{ kg/s}$

thus,
$$\dot{m}_1 = \underbrace{\dot{m}_3}_{\text{known}} - \dot{m}_2 \quad (1)$$

* energy balance on A: $\sum_i \dot{m}_i E_i + \dot{Q} + \dot{W}_s = 0$

$$\dot{m}_1 \left(H_1 + \frac{u_1^2}{2} + g z_1 \right) + \dot{m}_2 \left(H_2 + \frac{u_2^2}{2} + g z_2 \right) - \dot{m}_3 \left(H_3 + \frac{u_3^2}{2} + g z_3 \right) + \dot{Q} + \dot{W}_s = 0$$

but,
$$\begin{aligned} \dot{Q} &= 0 && \text{(well insulated)} \\ \dot{W}_s &= 0 && \text{(no shaft work)} \end{aligned}$$

also, assume that $\frac{u_i^2}{2}, gz_i \ll H_i$ for each stream, thus:

$$\dot{m}_1 H_1 + \dot{m}_2 H_2 - \overset{\text{known}}{\dot{m}_3} H_3 = 0 \quad \text{--- (2)}$$

- Using mass balance (1) in energy balance (2),

$$(\dot{m}_3 - \dot{m}_2) H_1 + \dot{m}_2 H_2 - \dot{m}_3 H_3 = 0$$

$$\therefore \dot{m}_2 (H_2 - H_1) = \dot{m}_3 (H_3 - H_1)$$

$$\dot{m}_2 = \dot{m}_3 \left(\frac{H_3 - H_1}{H_2 - H_1} \right) \quad \text{--- (3)}$$

- so we can solve for \dot{m}_2 using enthalpies of streams ①, ② and ③.

- From steam tables:

$$\left. \begin{aligned} H_1 &= H_{H_2O}^L (10^\circ\text{C}) = 42.0 \text{ kJ/kg} \\ H_2 &= H_{H_2O}^V (110^\circ\text{C}) = 2691.3 \text{ kJ/kg} \\ H_3 &= H_{H_2O}^L (70^\circ\text{C}) = 293 \text{ kJ/kg} \end{aligned} \right\}$$

- Using in (3):

$$\dot{m}_2 = (2 \text{ kg/s}) \left\{ \frac{(293) - (42)}{(2691.3) - (42)} \right\}$$

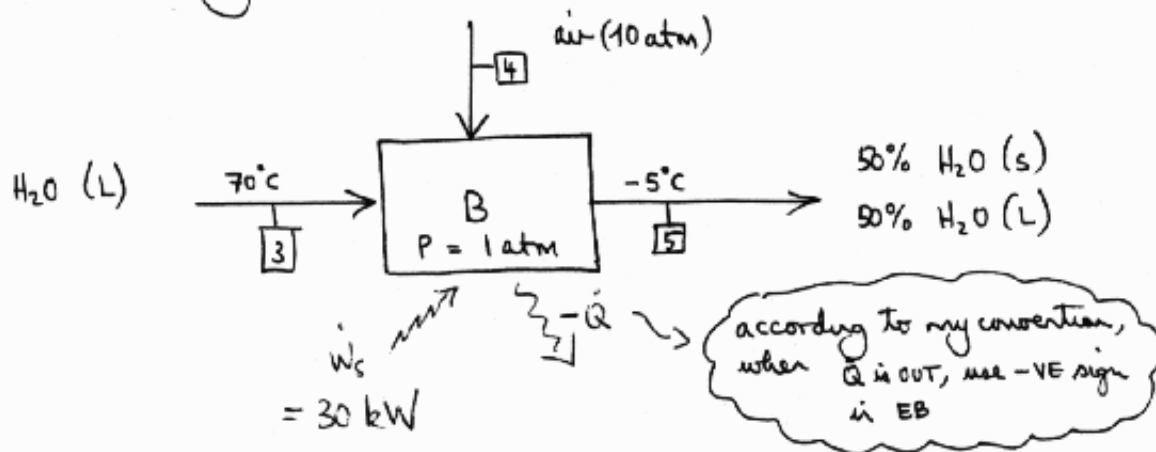
$$= \underline{\underline{0.19 \text{ kg/s}}} \rightarrow \text{steam}$$

- From (1):

$$\dot{m}_1 = \dot{m}_3 - \dot{m}_2 = (2) - (0.19)$$

$$= \underline{\underline{1.81 \text{ kg/s}}} \rightarrow \text{ingredients}$$

b) consider chilling unit, B:



* From our mass balance in part a), we know that

$$\dot{m}_3 \approx \dot{m}_5 = 2 \text{ kg/s}$$

* Energy balance on B:

$$\dot{m}_3 H_3 + \dot{m}_4 H_4 - \dot{m}_5 H_5 + \dot{Q} + \dot{w}_s = 0$$

What about this term? We know that $\dot{m}_4 \ll \dot{m}_3$ & \dot{m}_5 ; also $C_p(\text{gas})$ typically less than $C_p(\text{liq})$ and thus we expect that $H_4 \ll H_3$ & H_5 . So, we can probably neglect this term.

thus,

$$\dot{m}_3 H_3 - \dot{m}_5 H_5 + \dot{Q} + \dot{w}_s = 0 \quad (4)$$

Now,

$$\dot{m}_3 = \text{known} = 2 \text{ kg/s} = \dot{m}_5$$

$$H_3 = \text{known from (a)}$$

$$= H_{40}^L(70^\circ\text{C})$$

$$= 293 \text{ kJ/kg}$$

$$\dot{w}_s = 30 \text{ kW}$$

$$\dot{Q} = \text{unknown} \quad \dots \text{ what about } H_5?$$

- Stream ⑤ (product stream) is a mixture of liquid and solid. Hence

$$H_5 = \overbrace{0.5 H_{H_2O}^L(-5^\circ\text{C})}^{50\% \text{ LIQ.}} + \overbrace{0.5 H_{H_2O}^S(-5^\circ\text{C})}^{50\% \text{ SOLID}}$$

- Why -5°C ? Well, we have a mixture of water and some sugars and thus the freezing point of the water will be depressed. We thus have to cool down the solution to -5°C before parts of it will freeze.

- For the above expression, we can rewrite it to include the heat of fusion as follows:

$$H_5 = 0.5 H_{H_2O}^L(-5^\circ\text{C}) + 0.5 \left[H_{H_2O}^L(-5^\circ\text{C}) - \Delta H_{H_2O}^{\text{fus}}(-5^\circ\text{C}) \right]$$

$$= \textcircled{\text{I}} H_{H_2O}^L(-5^\circ\text{C}) - (0.5) \Delta H_{H_2O}^{\text{fus}}(-5^\circ\text{C})$$

so,

$$H_5 = \underbrace{H_{H_2O}^L(-5^\circ\text{C})}_{\textcircled{\text{I}}} - (0.5) \underbrace{\Delta H_{H_2O}^{\text{fus}}(-5^\circ\text{C})}_{\textcircled{\text{II}}} \quad \text{--- (5)}$$

- to find term $\textcircled{\text{I}}$ in (5):

$$H_{H_2O}^L(-5^\circ\text{C}) = H_{H_2O}^L(0^\circ\text{C}) + \Delta H_{H_2O}^L(0^\circ\text{C} \rightarrow -5^\circ\text{C})$$

FROM STEAM TABLES,
 $\Delta H_{H_2O}^L$ is ~ 0

$$= \bar{c}_p^L \Delta T$$

$$= \bar{c}_p^L(-5-0)$$

$$= -5\bar{c}_p^L$$

and we can estimate \bar{c}_p^L by $\bar{c}_p^L(0^\circ\text{C}) = 4.18 \text{ kJ/kg}\cdot\text{K}$

thus, (I) $H_{H_2O}^L = -(5K)(4.18 \text{ kJ/kg}\cdot\text{K})$
 $= \underline{-20.9 \text{ kJ/kg}}$ at -5°C

- what about term (II)?

$$\Delta H_{H_2O}^{fus}(-5^\circ\text{C}) = ?$$

we could correct this from $\Delta H_{H_2O}^{fus}(0^\circ\text{C})$, but it doesn't seem worth it. Thus,

$$\begin{aligned} \text{(II)} \quad \Delta H_{H_2O}^{fus}(-5^\circ\text{C}) &\approx \Delta H_{H_2O}^{fus}(0^\circ\text{C}) = 6.01 \text{ kJ/mol} \\ &= \frac{6.01 \text{ kJ/mol}}{18 \text{ g/mol}} \cdot \frac{1000 \text{ g}}{\text{kg}} \\ &= \underline{333.9 \text{ kJ/kg}} \end{aligned} \left. \begin{array}{l} \text{CRC} \\ \text{Handbook} \\ \text{p 6-126} \end{array} \right\}$$

- Hence, using these in (5):

$$\begin{aligned} H_5 &= \text{(I)} - 0.5 \text{(II)} \\ &= (-20.9 \text{ kJ/kg}) - 0.5(333.9 \text{ kJ/kg}) \\ &= \underline{-187.8 \text{ kJ/kg}} \end{aligned}$$

- Physically, this is equivalent to freezing 50% of a solution at -5°C .

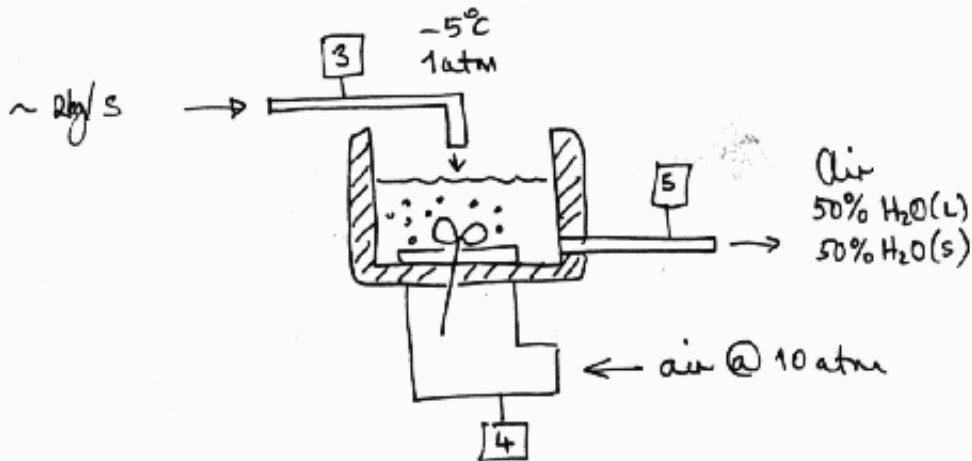
Finally, going back to equation (4):

$$\dot{m}_3 H_3 - \dot{m}_5 H_5 - \dot{Q} + \dot{W}_s = 0$$

$$\therefore \dot{Q} = \dot{m}_3 H_3 - \dot{m}_5 H_5 + \dot{W}_s$$

$$\begin{aligned}\dot{Q} &= (2 \text{ kg/s})(293 \text{ kJ/kg}) - (2 \text{ kg/s})(-187.8 \text{ kJ/kg}) \\ &\quad + 30 \text{ kW} \\ &= (586 \text{ kW}) + (376 \text{ kW}) + (30 \text{ kW}) \\ &= \underline{\underline{992 \text{ kW}}}\end{aligned}$$

c)



- So, volume doubles after addition of air:

$$\dot{V}_5 = 2\dot{V}_3$$

- and volume flow rate of (3), $\dot{V}_3 = \frac{\dot{m}_3}{\rho_{\text{H}_2\text{O}}}$

$$= \frac{2 \text{ kg/s}}{1000 \text{ kg/m}^3} \cdot \frac{10^6 \text{ cm}^3}{\text{m}^3}$$

$$= \underline{\underline{2,000 \text{ cm}^3/\text{s}}}$$

- Assuming little volume change upon freezing, we need $2,000 \text{ cm}^3$ of air to double the volume in (5)

- Hence air flow = $\underline{\underline{2,000 \text{ cm}^3/\text{s}}} = \dot{V}_4$

- To check our previous assumption, let's calculate \dot{M}_4

- Assuming ideal gas (air is at 1 atm in the chiller),

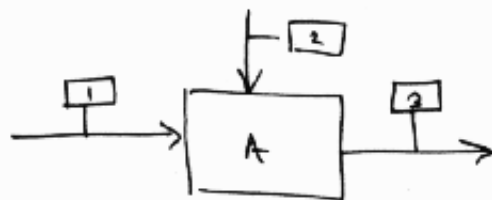
$$P\dot{V}_4 = \dot{n}_4 RT$$

$$\dot{n}_4 = \frac{P\dot{V}_4}{RT} = \frac{(1 \text{ atm})(2,000 \text{ cm}^3/\text{s})}{(82.06 \frac{\text{cm}^3 \text{ atm}}{\text{mol} \cdot \text{K}})(268.15 \text{ K})}$$

$$\begin{aligned} \dot{m}_4 &= 0.0909 \text{ mole/s} \cdot 28.8 \text{ g/mole} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \\ &= \underline{0.0026 \text{ kg/s}} \rightarrow \end{aligned}$$

- Which is 0.1% of the liquid mass flows \Rightarrow assumption is fine.

d) If we do the energy balances we did in a) and b) using internal energy, u , rather than enthalpy, H , we do not include work required to push the gas feeds into the units and pull them out (i.e. the PV work):



$$\dot{m}_1 - \dot{m}_2 - \dot{m}_3 = 0$$

$$\begin{aligned} \dot{m}_1 u_1 + \dot{m}_2 u_2 - \dot{m}_3 u_3 \\ + P_1 V_1 + P_2 V_2 - P_3 V_3 = 0 \end{aligned}$$

$\underbrace{\hspace{10em}}_{\dot{W}_{\text{process, say}}}$

$$\therefore \dot{W}_{\text{process}} = \dot{m}_3 u_3 - \dot{m}_2 u_2 - \dot{m}_1 u_1$$

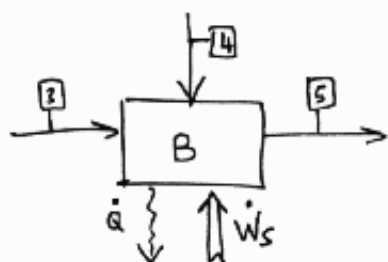
- We can get u 's from steam tables:

$$\left. \begin{aligned} u_1 &= u^L(10^\circ\text{C}) = 42.0 \text{ kJ/kg} \\ u_2 &= u^V(110^\circ\text{C}) = 2518.0 \text{ kJ/kg} \\ u_3 &= u^L(70^\circ\text{C}) = 292.9 \text{ kJ/kg} \end{aligned} \right\}$$

- Thus,

$$\begin{aligned} \dot{W}_{\text{process}} &= (2)(293) - (0.19)(2518) - (1.81)(42) \\ &= \underline{\underline{31.56 \text{ kW}}} \quad (+ve): \text{ need to put work into} \\ &\quad \text{process to move feeds} \\ &\quad \text{and products} \end{aligned}$$

e) Similarly for B:



$$\text{so,} \quad \underbrace{\dot{m}_3 u_3 + \dot{m}_4 u_4 + P_3 V_3 + P_4 V_4 + \dot{W}_s}_{\text{IN}} - \underbrace{\dot{m}_5 u_5 + P_5 V_5}_{\text{OUT}} - \dot{Q} = 0$$

$$\dot{m}_3 u_3 + \dot{m}_4 u_4 - \dot{m}_5 u_5 + \dot{W}_s - \dot{Q} + \dot{W}_{\text{process}} = 0 \quad (6)$$

(P₃V₃ + P₄V₄ - P₅V₅)

- as before,

$$u_3 = 292.7 \text{ kJ/kg}$$

$$u_4 = u_{\text{AIR}}^g(T=?)$$

* To get an estimate, I found u_{AIR}^g at 298.15 K (25°C) and 10 atm,
 $u_{\text{AIR}} \approx 213 \text{ kJ/kg}$

- again

$$u_5 = 0.5 u_{\text{H}_2\text{O}}^L(-5^\circ\text{C}) + 0.5 u_{\text{H}_2\text{O}}^S(-5^\circ\text{C})$$

$$= 0.5 u_{\text{H}_2\text{O}}^L(-5^\circ\text{C}) +$$

$$0.5 [u_{\text{H}_2\text{O}}^L(-5^\circ\text{C}) - \Delta u_{\text{H}_2\text{O}}^{\text{fus}}(-5^\circ\text{C})]$$

- What about $\Delta U_{H_2O}^{fus}$?

- Well,

$$\begin{aligned}\Delta H_{H_2O}^{fus} &= \Delta U_{H_2O}^{fus} + \Delta(PV)^{fus} \\ &= \Delta U_{H_2O}^{fus} + P\Delta V^{fus}\end{aligned}$$

and since $\Delta V^{fus} \sim 0$,

$$\Delta H_{H_2O}^{fus} \cong \Delta U_{H_2O}^{fus}$$

\nearrow
P const during phase change

- Thus,

$$\begin{aligned}U_5 &= U_{H_2O}^L(-5^\circ\text{C}) - (0.5)\Delta H_{H_2O}^{fus}(-5^\circ\text{C}) \\ &= \underbrace{U_{H_2O}^L(0^\circ\text{C})}_{\sim 0} + \Delta U_{H_2O}^L(0^\circ\text{C} \rightarrow -5^\circ\text{C}) - (0.5)\Delta H_{H_2O}^{fus} \\ &= \bar{C}_V(-5-0) - 0.5\Delta H_{H_2O}^{fus}\end{aligned}$$

- For liquids, especially water, $C_p \cong C_v$ so use C_p :

$$\begin{aligned}U_5 &= (-5)(4.18) - (0.5)(333.9 \text{ kJ/kg}) \\ &= \underline{-187.8 \text{ kJ/kg}} = H_5\end{aligned}$$

- Using the above in (6), we get:

$$(2)(293) + \underbrace{(0.0026)(213)}_{\text{negligible compared to other terms!}} - (2)(-187.8) + (30) - (992) + \dot{W}_{\text{process}} = 0$$

$$\therefore \dot{W}_{\text{process}} = 0!$$

- Hence, there is no extra work associated with the process in moving feeds and products in and out at the level of our assumptions.

- This is not true in real life, of course. If we included the contribution from the air we would get, in (6),

$$\dot{m}_4 u_4 - \dot{m}_5^{AIR} u_5^{AIR} + \underbrace{\dot{W}_{process}^{AIR}}_{P_4 V_4} = 0$$

where, \dot{m}_5^{AIR} = flow of air in (5) = $\dot{m}_4 = 0.0026 \text{ kg/s}$
 u_5^{AIR} = internal energy of air in (5)

which gives

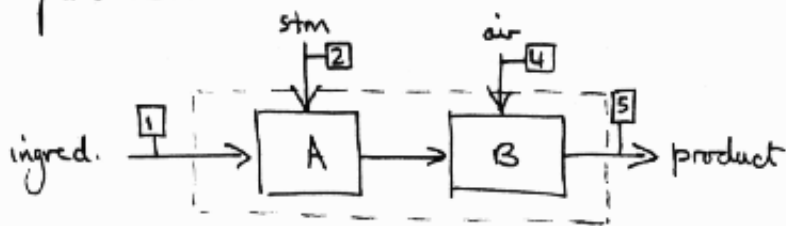
$$\dot{m}_4 (u_5 - u_4) = \dot{W}_{process}^{AIR}$$

$$\dot{m}_4 C_V (T_5 - T_4) = \dot{W}_{process}^{AIR}$$

$$\begin{aligned} \dot{W}_{process}^{AIR} &= \dot{m}_4 C_P / \gamma \Delta T \\ &= (0.0026 \text{ kg/s}) \left(\frac{1 \text{ kJ/kg}\cdot\text{K}}{1.4} \right) (-30 \text{ K}) \\ &= 0.0557 \text{ kW} \end{aligned}$$

$\underline{\underline{\hspace{2cm}}}$

f) Overall process:



- so, for each feed stream and output:

$$\dot{E}_{k_i} = \frac{1}{2} \dot{m}_i u_i^2 \quad \text{units } [\text{kg/s}] [\text{m}^2/\text{s}^2] = [\text{W}]$$

- assuming pipe diameters of say 4",

$$A = \frac{\pi (4 \cdot 0.0254)^2}{4} = 8.107 \times 10^{-3} \text{ m}^2$$

- Velocity, u , is then

$$u_i = \frac{\dot{V}_i}{A} = \frac{\dot{m}_i}{\rho_i A}$$

◇ for liquid streams, $\rho_i \approx \rho_{H_2O} = 1,000 \text{ kg/m}^3$

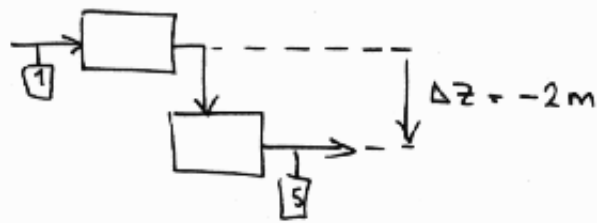
◇ for gas streams, $\rho_i = \left(\frac{PM}{RT}\right)_i$ at pipe entrance

- Thus

| Stream | ρ_i [kg/m ³] | \dot{m}_i [kg/s] | u_i [m/s] | \dot{E}_{k_i} [W] |
|--------|--|--------------------|-------------|---------------------|
| 1 | 1,000 | 1.81 | 0.223 | 0.05 |
| 2 | $\frac{(1 \text{ atm})(18 \text{ g/mol})}{(82.06 \frac{\text{cm}^3 \cdot \text{atm}}{\text{mol} \cdot \text{K}})(343.15 \text{ K})}$ = 0.64 | 0.19 | 36.6 | 127.4 |
| 4 | $\frac{(1 \text{ atm})(28.8 \text{ g/mol})}{(82.06 \frac{\text{cm}^3 \cdot \text{atm}}{\text{mol} \cdot \text{K}})(268 \text{ K})}$ = 1.31 | 0.0026 | 0.245 | < 0.0001 |
| 5 | 1,000 | 2 | 0.245 | 0.06 |

- We can see that the major contribution comes from steam, but the kinetic energy associated with this steam is still orders of magnitude below that of enthalpies -- W compared to kW!

g) Finally then, consider the magnitude of a potential energy term:



$$\begin{aligned}\therefore \Delta \dot{E}_p &= \dot{m}_5 g z_5 - \dot{m}_1 g z_1 \\ &= (2 \text{ kg/s})(10 \text{ m/s}^2)(-2 \text{ m}) \\ &= \underline{\underline{-40 \text{ W}}}\end{aligned}$$

ref. height = 0

- Again, orders of magnitude less than enthalpy flows.

\Rightarrow It is OK to neglect kinetic and potential energy terms in this problem!

10.213 ENERGY BALANCE

ADDENDUM

The sign convention for the energy balance is sometimes confusing. I use the following convention to keep things consistent:

◇ For a MULTI-INPUT, MULTI-OUTPUT flow process with MULTIPLE HEAT INFLOWS + OUTFLOWS & SHAFT WORK IN AND OUT:

$$\underbrace{\text{IN}}_{\substack{\text{ALL FLOWS,} \\ \text{HEAT TRANSFER} + \\ \text{SHAFT WORK IN}}} - \underbrace{\text{OUT}}_{\substack{\text{ALL FLOWS} \\ \text{HEAT TRANSFER} + \\ \text{SHAFT WORK OUT}}} + \underbrace{\text{GEN}}_{\text{ALL GEN. TERMS}} = 0 \quad \begin{array}{l} \text{STEADY} \\ \text{STATE} \\ \text{ACC} = 0 \end{array}$$

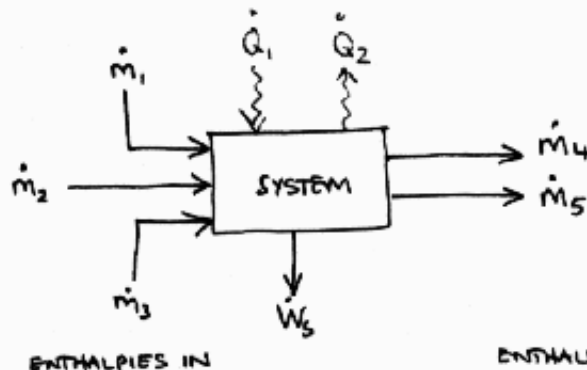
ENERGY BALANCE

$$\sum_{\text{IN}} \dot{m}_i H_i - \sum_{\text{OUT}} \dot{m}_j H_j + \sum_k \dot{Q}_k + \sum_p (\dot{W}_s)_p = 0$$

◇ For heat and shaft work:

- * FOR \dot{Q}_{IN} , USE +VE SIGN ~~(-VE SIGN)~~
- * FOR \dot{Q}_{OUT} , USE -VE SIGN
- * FOR $\dot{W}_s \text{ IN}$, USE +VE SIGN
- * FOR $\dot{W}_s \text{ OUT}$, USE -VE SIGN

◇ Example:



$$\begin{aligned} \text{EB: } & \underbrace{(\dot{m}_1 H_1 + \dot{m}_2 H_2 + \dot{m}_3 H_3)}_{\text{ENTHALPIES IN}} - \underbrace{(\dot{m}_4 H_4 + \dot{m}_5 H_5)}_{\text{ENTHALPIES OUT}} \\ & + (\dot{Q}_1 - \dot{Q}_2) + (-\dot{W}_s) = 0 \\ & \quad \quad \quad \text{HEAT IN} \quad \text{HEAT OUT} \quad \quad \text{SHAFT WORK OUT} \end{aligned}$$

◇ \dot{Q} & \dot{W}_s are then always positive and are the quantities of heat and shaft work, without any sign, i.e. $|\dot{Q}|$ & $|\dot{W}_s|$.