10.213 Problem #18 Solution

$$\rho = A + Bx_1 + Cx_1^2$$
$$= 0.02 - 0.01x_1 + 0.005x_1^2$$

a) ◇ Pure ①, $x_1 = 1$, so

$$\rho_1 = (0.02) - (0.01)(1) + (0.005)(1)^2$$
$$= 0.015 \text{ mol/cm}^3$$

$$\boxed{V_1 = 1/\rho_1 = 66.67 \text{ cm}^3/\text{mol}}$$

◇ Pure ②, $x_1 = 0$, so

$$\rho_2 = (0.02) - (0.01)(0) + (0.005)(0)^2$$
$$= 0.02 \text{ mol/cm}^3$$

$$\boxed{V_2 = 1/\rho_2 = 50 \text{ cm}^3/\text{mol}}$$

b) ◇ from S+VN, p. 323

$$\overline{V_1} = V + x_2\left(\frac{dV}{dx_1}\right)_{T,P} \qquad (10.15)$$
$$\overline{V_2} = V - x_1\left(\frac{dV}{dx_1}\right)_{T,P} \qquad (10.16)$$

◇ at $x_1 = 0.8$,

$$V = 1/\rho = \frac{1}{(0.02)-(0.01)(0.8)+(0.005)(0.8)^2} = \frac{1}{(0.0152)}$$
$$= \underline{65.79 \text{ cm}^3/\text{mol}}$$

◊ Also,

$$\frac{dV}{dx_1} = \frac{d(1/\rho)}{dx_1} = -\frac{1}{\rho^2}\frac{d\rho}{dx_1}$$

$$= -\frac{1}{\rho^2}\,\frac{d\left(0.02 - 0.01x_1 + 0.005x_1^{\,2}\right)}{dx_1}$$

$$= \frac{0.01 - 0.005(2)x_1}{\rho^2}$$

$$= \frac{0.01 - 0.01x_1}{\left(0.02 - 0.01x_1 + 0.005x_1^{\,2}\right)^2}$$

◊ at $x_1 = 0.8$,

$$\frac{dV}{dx_1} = \frac{0.01 - 0.01(0.8)}{(0.0152)^2}$$

$$= \underline{8.6565 \text{ cm}^3/\text{mol}}$$

◊ Thus,

$$\bar{V}_1 = (65.79) + (1-0.8)(8.6565)$$

$$= 67.5 \text{ cm}^3/\text{mol}$$

$$\bar{V}_2 = (65.79) - (0.8)(8.6565)$$

$$= 58.9 \text{ cm}^3/\text{mol}$$

$$\boxed{\begin{aligned}\bar{V}_1 &= 67.5 \text{ cm}^3/\text{mol}\\ \bar{V}_2 &= 58.9 \text{ cm}^3/\text{mol}\end{aligned}}$$

c).

$$V^{E} = V^{REAL} - V^{ID}$$

◇ and

$$V^{REAL} = x_1\overline{V_1} + x_2\overline{V_2}$$

$$V^{ID} = x_1 V_1 + x_2 V_2$$

◇ where:

$V \equiv$ volume of total solution [cm³/mole soln.]

$V_1 \equiv$ volume of pure component ① (NOT in solution)

$\overline{V_1} \equiv$ partial molar volume of ①/ "apparent volume" (IN solution)

◇ thus,

$$V^{REAL} = 65.79 \text{ cm}^3/\text{mol} \quad (\text{part b)})$$

$$V^{ID} = (0.8)(66.67) + (1-0.8)(50)$$

$$= 63.33 \text{ cm}^3/\text{mol}$$

$$\boxed{\begin{aligned} V^{E} &= 65.79 - 63.33 \\ &= 2.5 \text{ cm}^3/\text{mol} \end{aligned}}$$

d) See over.

◇ We need to figure out how many moles of ① and ② are in the final mixture C. Thus we need to convert weight % to mole %:

– Consider 100g of soln. C:

in 100g soln. $\Rightarrow$ 20g ① = $^{20g}/_{40 g/mole}$ = 0.5 mole ①
$\Rightarrow$ 80g ② = $^{80g}/_{60 g/mole}$ = 1.33 mole ②

– Thus,

$$x_1 = \frac{0.5 \text{ mole}}{0.5 \text{ mole} + 1.33 \text{ mole}}$$

← moles of ①

← total moles in soln = moles ① + moles ②

$$= \underline{0.273}$$

$$x_2 = 1 - 0.273$$

$$= \underline{0.727}$$

◇ What is the molar density of solution C? From our empirical relationship:

$$\rho_c = 0.02 - 0.01(0.273) + 0.005(0.273)^2$$
$$= 0.01764 \text{ mol/cm}^3$$

◇ We want 1,000 cm³ of solution. We thus need:

$$N_c = V_c^{total} \cdot \rho_c$$
$$= (1{,}000 \text{ cm}^3)(0.01764 \text{ mol/cm}^3)$$
$$= \underline{17.64 \text{ moles total in solution C}}$$

◇ How much ② will soln. A give us?

moles of ② from soln A = $X_{2A} N_A$

= (0.2)(6.01)

= $\underline{1.203 \text{ moles}}$ ②

⇒ but we need 12.82 moles of ②

— thus we do need to add pure ② (soln. B)

◇ So,

=1 since B is pure ②

$\#$ moles of ② req'd = $X_{2A} N_A + \cancel{X_{2B}} N_B$

$(12.82) = (1.203) + N_B$

thus $N_B = (12.82) - (1.203)$

= $\underline{11.62 \text{ moles of soln. B}}$

◇ Finally, we want to know what total volumes of A and B we need to add:

total # moles of soln A [mole]

intensive volume for soln A ($x_1 = 0.8$) [cm³/mol]

total volume [cm³]

$$V_A^{total} = N_A V_A$$

$$= (6.01)(65.79)$$

$$\boxed{V_A^t = 395.4 \text{ cm}^3}$$

B is pure ②

$$V_B^{total} = N_B V_B = N_B V_2$$

$$= (11.62)(50)$$

$$\boxed{V_B^t = 581.0 \text{ cm}^3}$$

* NOTE: $V_A^t + V_B^t = 976.4 \text{ cm}^3 < 1{,}000 \text{ cm}^3$ because of volume increase w/ mixing