Problem # 19 Solution

$$T_0 = 300 K$$

pure A, $X_{A1} = 1$ $\dot{m}_1$ A ①——→ [MIX] ③——→ MIX $\dot{m}_3$ X_{A3} = mol. fraction of A in ③

pure B, $X_{B2} = 1$ $\dot{m}_2$ B ②——→ X_{B3} = mol. fraction of B in ③

$$P_1 = 1 \text{ bar} = P_2$$
$$T_1 = 50°C = T_2$$

$\dot{Q}$

$$P_3 = 40 \text{ bar}$$
$$T_3 = 50°C$$

◊ Known:

$$X_{A1} = 1, \quad X_{B1} = 0 \quad \text{is pure A}$$
$$X_{B2} = 1, \quad X_{A1} = 0 \quad \text{is pure B}$$
$$\dot{m}_1 = 50 \text{ mol/s}$$
$$\dot{m}_2 = 23 \text{ mol/s}$$
$$H^E = -X_1 X_2 (10 X_1 + 20 X_2) \quad\Big\}\ [kJ/mol]$$
$$TS^E = -5 X_1 X_2$$

◊ [MB]

◊ Overall:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \qquad\qquad —①$$

◊ Component:

— on [A] $\dot{m}_1 X_{A1}^{=1} + \dot{m}_2 X_{A2}^{=0} = \dot{m}_3 X_{A3}$

$$\dot{m}_1 = \dot{m}_3 X_{A3} \qquad\qquad —②$$

— on [B] $\dot{m}_1 X_{B1}^{=0} + \dot{m}_2 X_{B2}^{=1} = \dot{m}_3 X_{B3}$

$$\dot{m}_2 = \dot{m}_3 X_{B3} \qquad\qquad —③$$

◊ from ①,② and ③:

① : $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$= (50) + (23)$

$= \underline{73 \ mol/s}$

② : $x_{A3} = \dot{m}_1 / \dot{m}_3$

$= (50)(73)$

x_1 $\boxed{x_{A3} = 0.6849}$

similarly $\boxed{x_{B3} = 0.3151}$

◊ Knowing x_{A3}, x_{B3} we can calculate H^E and S^E:

$H^E = - x_1 x_2 (10 x_1 + 20 x_2)$

$= - (0.6849)(0.3151)[10(0.6849) + 20(0.3151)]$

$= \underline{-2.838 \ kJ/mol}$

$S^E = -5 x_1 x_2 / T$

$= -5(0.6849)(0.3151)/(50 + 273.15)$

$= \underline{-3.34 \ J/mol}$

a)

◊ $\boxed{EB}$

$\dot{m}_1 H_1 + \dot{m}_2 H_2 - \dot{m}_3 H_3 + \dot{Q} + \cancel{\dot{W}_s}^0 = 0$ ———④

◊ but $H_1 = H_{A1} @ 1 \ bar, 50°C$ (ie, H of pure A)

$H_2 = H_{B2} @ 1 \ bar, 50°C$

◊ What about H_3?

$$H_3 = H_3^{REAL} = H_3^{ID} + H_3^{E}$$

$$\underbrace{} = \underbrace{\left[X_{A3} H_{A3} + X_{B3} H_{B3} \right]}_{} + \underbrace{\left[H_3^{E} \right]}_{}$$

REAL   IDEAL SOLUTION   EXCESS
SOLUTION

◇ Using the above in ④:

$$\dot{m}_1 H_{A1} + \dot{m}_2 H_{A2} - \dot{m}_3 \left[(X_{A3} H_{A3} + X_{B3} H_{B3}) + H^{E} \right] + \dot{Q} = 0$$

▢ From our component balances, we know that:

②: $\dot{m}_3 X_{A3} = \dot{m}_1$

③: $\dot{m}_3 X_{B3} = \dot{m}_3$

◇ Thus,

$$\dot{m}_1 \underbrace{\left(H_{A1} - H_{A3} \right)}_{} + \dot{m}_2 \underbrace{\left(H_{B2} - H_{B3} \right)}_{} - \dot{m}_3 H^{E} + \dot{Q} = 0$$

(−ΔH) from conditions $= -\Delta H_B$
of stream ① to stream ③
for pure A
$= -\Delta H_A$

◇ Now, for a liquid, both H & S are weak functions of
pressure, thus:

$H_A (1\,bar, 50°c) \simeq H_A (40\,bar, 50°c)$
$H_B (1\,bar, 50°c) \simeq H_B (40\,bar, 50°c)$

◇ So, $\Delta H_A \sim 0$ $\Delta H_B \sim 0$ and we end up with:

$$\dot{Q} = \dot{m}_3 H^{E}$$
$$= (73\,mol/s)(-2.838 \; kJ/mol)$$

$$\boxed{\dot{Q} = -207\,kW}$$

◦ Thus, heat must be <u>REMOVED</u> from system to keep it
  <u>ISOTHERMAL</u>.

(b)

$\boxed{2^{ND} \text{LAW}}$

◦ From S+VN, pg 617 (16.9):

$$\dot{W}_{lost} = T_\sigma \, \Delta(s\dot{m})_{fs} - \dot{Q} \qquad (16.9) \underline{\qquad} ⑤$$

◦ Where:

$$\Delta(s\dot{m})_{fs} = \dot{m}_3 S_3 - \dot{m}_2 S_2 - \dot{m}_1 S_1$$

◦ But,

$$S_1 = S_{A1} \text{ @ 1 bar, } 50°C$$
$$S_2 = S_{B2} \text{ @ 1 bar, } 50°C$$
$$S_3 = \underbrace{S_3^{ID}}_{} + S_3^E$$
$$= \overbrace{\left[X_{A3} S_{A3} + X_{B3} S_{B3} - R X_{A3} \ln X_{A3} - R X_{B3} \ln X_{B3} \right]}^{S^{ID}}$$
$$+ S_3^E$$

◦ Using above:

$$\Delta(s\dot{m})_{fs} = \dot{m}_3 \left(X_{A3} S_{A3} + X_{B3} S_{B3} - R X_{A3} \ln X_{A3} - R X_{B3} \ln X_{B3} + S^E \right)$$
$$- \dot{m}_1 S_{A1} - \dot{m}_2 S_{A2}$$

  ◦ try similar use of ② & ③ as in a):

$$= \dot{m}_1 \left(S_{A3} - S_{A1} \right) + \dot{m}_2 \left(S_{B3} - S_{A2} \right) - \left[R X_{A3} \ln X_{A3} - R \ln X_{B3} X_B \right. $$
$$\left. + S^E \right] \dot{m}_3$$

  ◦ Again, S is a weak function of pressure for
    liquids.

◊ Thus,

$$\Delta(\dot{Sm})_{fs} = \dot{m}_3 S^E - \dot{m}R x_{A3} \ln x_{A3} - \dot{m}R x_{B3} \ln x_{B3}$$

$$= (73)(-0.00334)(1000)$$

$$[\text{mol/s}] \; [\text{kJ/mol·K}] \; [\text{J/kJ}]$$

$$- (73)(8.314)(0.6849) \ln (0.6849)$$

$$[\text{mol/s}][\text{J/mol·K}][-][-]$$

$$- (73)(8.314)(0.3151) \ln (0.3151)$$

$$= -243.8 - (-378.2) \qquad [\text{J/s·K}]$$

$$= \underline{134.4 \; \text{W/K}}$$

◊ Using in (5) with $T_\sigma = 300\text{K}$:

$$\dot{W}_{lost} = (300\text{K})(134.4 \; \text{W/K}) - (207.2 \; \text{kW})(1000 \; \text{W/kW})$$

$$= 247,484 \; \text{W}$$

$$\boxed{\dot{W}_{lost} = 247.5 \; \text{kW}}$$

c) ◊
$$G^E = H^E - TS^E = (-2.838) - (-1.0791)$$
$$= -1.76 \; \text{kJ/mol}$$

◊ Thus,

$$\left.\begin{array}{l} G^E < 0 \\ H^E < 0 \\ S^E < 0 \end{array}\right\}$$

◊ From fig. 10.7, solution will likely be:

☐ NA/NA eg acetone/chloroform

(or ▲ AS/NA eg ethanol/chloroform)

————END————