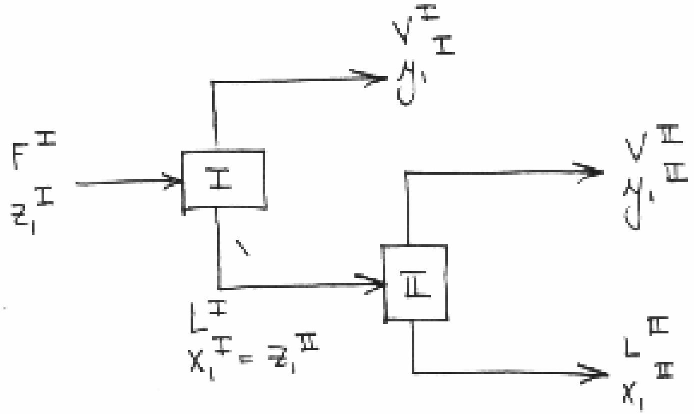
Problem 22 Solution

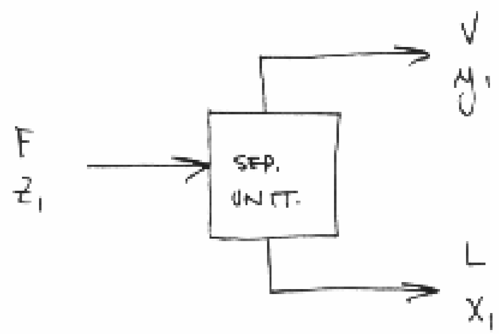


① – Component 1
② – Component 2

◇ For each of the separation units we can write a <u>mass balance</u> and an <u>equilibrium relationship</u>. (eg Raoult's Law)

◇ Let's write the equations for the general case, assuming both <u>IDEAL SOLUTIONS</u> (liquid phase) and <u>IDEAL GAS</u> (vapour phase)

◇ <u>GENERAL CASE</u>:



* <u>MB ON COMP. 1</u>

$$F z_1 = V y_1 + L x_1 \qquad (1)$$

* <u>RAOULT'S LAW</u>

$$y_1 P = x_1 P_1^{sat} \qquad (2)$$

$$(1 - y_1) P = (1 - x_1) P_2^{sat} \qquad (3)$$

◇ Solving for y_1 in ①:

$$y_1 = \left(\frac{F z_1}{V}\right) - \left(\frac{L}{V}\right) x_1 \qquad (4)$$

◇ Using in ②:

$$\left[\left(\frac{F z_1}{V}\right) - \left(\frac{L}{V}\right) x_1 \right] P = x_1 P_1^{sat}$$

◇ Solving for P,

$$P = \frac{x_1 P_1^{sat}}{\left[\left(\frac{F z_1}{V}\right) - \left(\frac{L}{V}\right)x_1\right]} \qquad \text{——— (5)}$$

◇ Using both (4) and (5) in (3):

$$\left[1 - \left(\left(\frac{F z_1}{V}\right) + \left(\frac{L}{V}\right)x_1\right)\right]\frac{x_1 P_1^{sat}}{\left[\left(\frac{F z_1}{V}\right) - \left(\frac{L}{V}\right)x_1\right]} = (1 - x_1)P_2^{sat}$$

◇ Collecting like terms, we can generate a quadratic equation in x_1 :

$$x_1^2\left[\left(\frac{L}{V}\right)\left(\frac{P_1^{sat}}{P_2^{sat}} - 1\right)\right]$$
$$+ x\left[\left(\frac{P_1^{sat}}{P_2^{sat}}\right)\left(1 - \frac{F z_1}{V}\right) + \left(\frac{L}{V}\right) + \left(\frac{F z_1}{V}\right)\right]$$
$$+ \left[-\frac{F z_1}{V}\right] = 0 \qquad \text{——— (6)}$$

◇ Rewriting in more familiar form; we need to solve for x_1 in:

$$\boxed{\alpha x_1^2 + \beta x_1 + \gamma = 0}$$

where:

$$\begin{cases} \alpha = \left(\frac{L}{V}\right)\left(\frac{P_1^{sat}}{P_2^{sat}} - 1\right) \\ \beta = \left(\frac{P_1^{sat}}{P_2^{sat}}\right)\left(1 - \frac{F z_1}{V}\right) + \left(\frac{L}{V}\right) + \left(\frac{F z_1}{V}\right) \\ \gamma = -\frac{F z_1}{V} \end{cases}$$

◇ The solution is given by:

$$\boxed{x_1 = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}}$$

• For example,

Case a)

$\boxed{\text{UNIT I:}}$

$$z_1^I = 0.5$$
$$F^I = 100 \text{ moles}$$
$$L^I = 50 \text{ moles}$$
$$V^I = 50 \text{ moles}$$

thus,

$$\alpha^I = \left(\frac{L}{V}\right)\left(\frac{P_1^{sat}}{P_2^{sat}} - 1\right)$$
$$= \left(\frac{50}{50}\right)\left(\frac{10}{20} - 1\right)$$
$$= -0.5$$

$$\beta^I = \left(\frac{10}{20}\right)\left(1 - \frac{50}{50}\right) + \left(\frac{50}{50}\right) + \left(\frac{50}{50}\right)$$
$$= 2$$

$$\gamma^I = -\frac{(50)}{(50)}$$
$$= -1$$

so,

$$x_1^I = \frac{-(2) \pm \sqrt{(2)^2 - 4(-0.5)(-1)}}{2(-0.5)}$$
$$= \frac{-2 + \sqrt{2}}{-1.0} \quad \text{or} \quad \frac{-2 - \sqrt{2}}{-1.0}$$
$$= \underline{\underline{0.5858}} \qquad \cancel{= 3.414} \; > 1.0$$

so,

$$x_2^I = 1 - x_1^I = 1 - 0.5858 = \underline{\underline{0.4142}}$$

$$y_1^I = x_1^I P_1^{sat}/P \quad \text{so need } P$$

and

$$P^I = x_1^I P_1^{sat} + x_2^I P_2^{sat} = (0.5858)(10) + (0.4142)(20)$$
$$= \underline{\underline{14.14 \text{ kPa}}}$$

◇ Thus,

$$y_1^{I} = \frac{x_1^{I} P_1^{sat}}{P} = \frac{(0.5858)(10)}{(14.14)}$$

$$= \underline{\underline{0.4142}}$$

$$y_2^{I} = 1 - y_1^{I} = 1 - 0.4142$$

$$= \underline{\underline{0.5858}}$$

◇ We then use this data with: $\left\{ \begin{array}{ll} z_1^{II} = x_1^{I} & F^{II} = 50 \text{ mol/s} \\ z_2^{II} = x_2^{I} & V^{II} = L^{II} = 25 \text{ mol/s} \end{array} \right\}$

and solve $\boxed{\text{UNIT II}}$ similarly for each of parts b) and c).

◇ I did this on a spreadsheet and the results and Pxy plots follow.

NOTE:

◇ You could also solve this iteratively as follows:

* EQUILIBRIUM:

$$y_1 P = x_1 P_1^{sat}$$

$$(+) \quad y_2 P = x_2 P_2^{sat}$$

$$\boxed{P = x_1 (P_1^{sat} - P_2^{sat}) + P_2^{sat}} \quad \text{—— ①}$$

* MB ON A:

$$F z_1 = V y_1 + L x_1$$

$$= V(x_1 P_1^{sat}/P) + L x_1$$

$$\boxed{x_1 = \frac{(F z_1)}{(V P_1^{sat}/P) + L}} \quad \text{—— ②}$$

⟹ two equations in 2 unknowns, P & x_1:

Ⓐ guess P
Ⓑ calculate x_1 from ②
Ⓒ use Ⓑ's calculated x_1 to get new, corrected P from ①
Ⓓ repeat until convergence

Part a)

$P_1^{(sat)}$ =	10 kPa
$P_2^{(sat)}$ =	20 kPa

UNIT I

Solving: $\alpha x^2 + \beta x + \gamma = 0$

F^{I} =	100 mol/s
L^{I} =	50 mol/s
V^{I} =	50 mol/s
z_1^{I} =	0.5
α =	-0.5
β =	2
γ =	-1

x_1^{I} =	**0.58579**	= z_1^{II}
x_2^{I} =	**0.41421**	= z_2^{II}
y_1^{I} =	**0.41421**	
y_2^{I} =	**0.58579**	
P_{total}^{I} =	**14.14 kPa**	

$P_1^{(sat)}$ =	10 kPa
$P_2^{(sat)}$ =	20 kPa

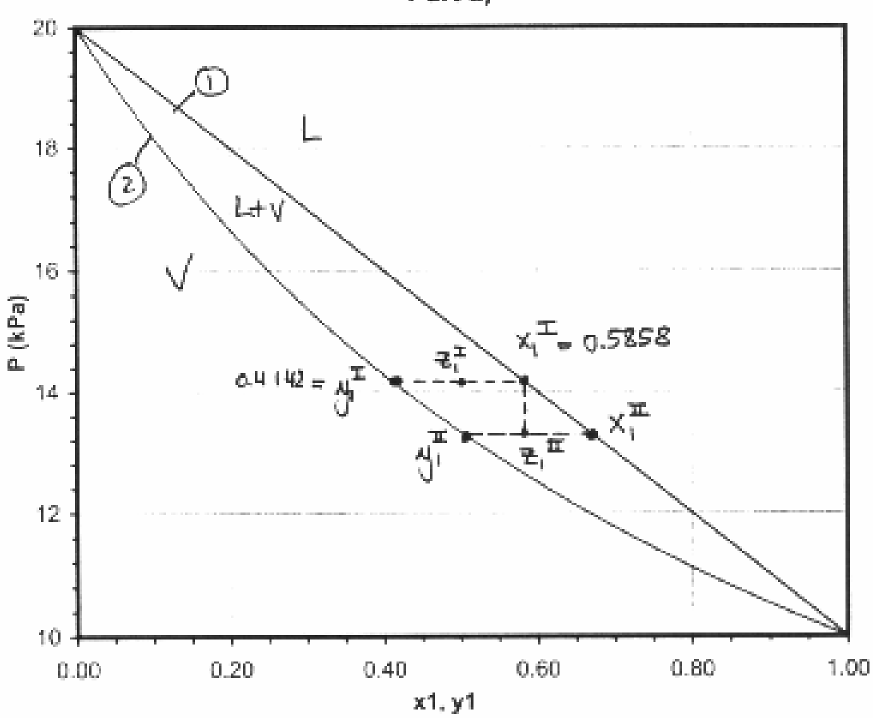
UNIT II

Solving: $\alpha x^2 + \beta x + \gamma = 0$

F^{II} =	50 mol/s
L^{II} =	25 mol/s
V^{II} =	25 mol/s
z_1^{II} =	0.58579
α =	-0.5
β =	2.0857864
γ =	-1.171573

x_1^{II} =	**0.66897**
x_2^{II} =	**0.33103**
y_1^{II} =	**0.50260**
y_2^{II} =	**0.49740**
P_{total}^{II} =	**13.31 kPa**

Part a)



Pxy diagram generated from:

① $P = x_1(P_1^{sat} - P_2^{sat}) + P_2^{sat}$ (P-x)

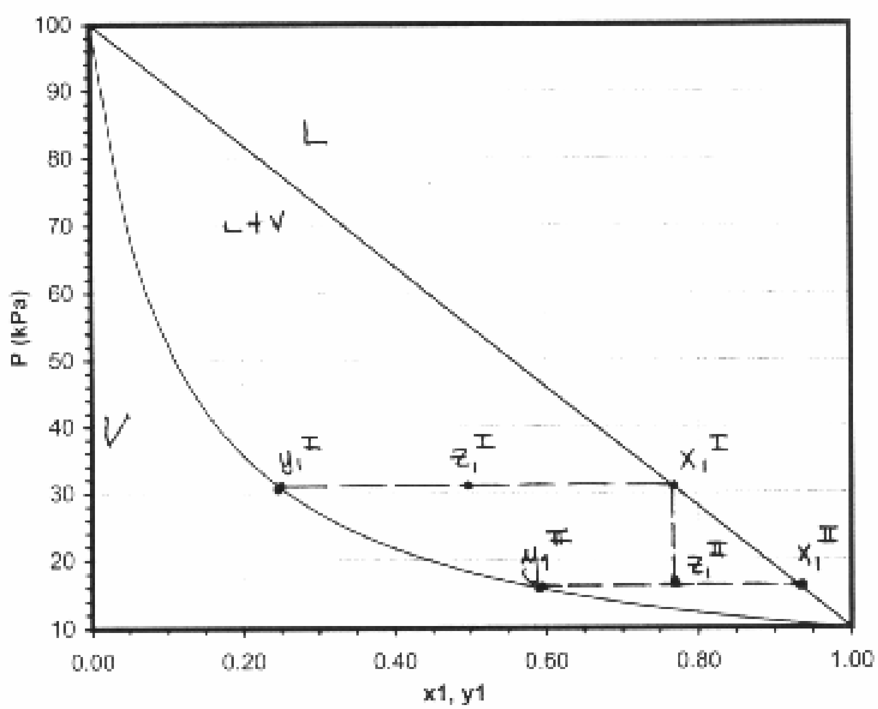
② $P = \dfrac{P_2^{sat}}{1 - y_1(1 - P_2^{sat}/P_1^{sat})}$

Part b)

$P_1^{(sat)}$ =	10 kPa	$P_1^{(sat)}$ =	10 kPa
$P_2^{(sat)}$ =	100 kPa	$P_2^{(sat)}$ =	100 kPa

UNIT I		**UNIT II**	
Solving:	$\alpha x^2 + \beta x + \gamma = 0$	**Solving:**	$\alpha x^2 + \beta x + \gamma = 0$
F^I =	100 mol/s	F^{II} =	50 mol/s
L^I =	50 mol/s	L^{II} =	25 mol/s
V^I =	50 mol/s	V^{II} =	25 mol/s
z_1^I =	0.5	z_1^{II} =	0.75975
α =	-0.9	α =	-0.9
β =	2	β =	2.467544
γ =	-1	γ =	-1.519494

x_1^I =	**0.75975** = z_1^{II}	x_1^{II} =	**0.93391**
x_2^I =	**0.24025** = z_2^{II}	x_2^{II} =	**0.06609**
y_1^I =	**0.24025**	y_1^{II} =	**0.58559**
y_2^I =	**0.75975**	y_2^{II} =	**0.41441**
P_{total}^I =	**31.62 kPa**	P_{total}^{II} =	**15.95 kPa**



Part c)

$P_1^{(sat)}$ =	10 kPa
$P_2^{(sat)}$ =	5000 kPa

$P_1^{(sat)}$ =	10 kPa
$P_2^{(sat)}$ =	5000 kPa

UNIT I

Solving: $\alpha x^2 + \beta x + \gamma = 0$

F^I =	100 mol/s
L^I =	50 mol/s
V^I =	50 mol/s
z_1^I =	0.5
α =	-0.998
β =	2
γ =	-1

x_1^I =	**0.95719**	= z_1^{II}
x_2^I =	**0.04281**	= z_2^{II}
y_1^I =	**0.04281**	
y_2^I =	**0.95719**	
P_{total}^I =	**223.61 kPa**	

UNIT II

Solving: $\alpha x^2 + \beta x + \gamma = 0$

F^{II} =	50 mol/s
L^{II} =	25 mol/s
V^{II} =	25 mol/s
z_1^{II} =	0.95719
α =	-0.998
β =	2.912557
γ =	-1.914386

x_1^{II} =	**0.99981**
x_2^{II} =	**0.00019**
y_1^{II} =	**0.91457**
y_2^{II} =	**0.08543**
P_{total}^{II} =	**10.93 kPa**

Notice:
"3 9's pure" requires extremes of pressure:
2 atm for I
0.1 atm for II
⇒ costs energy!