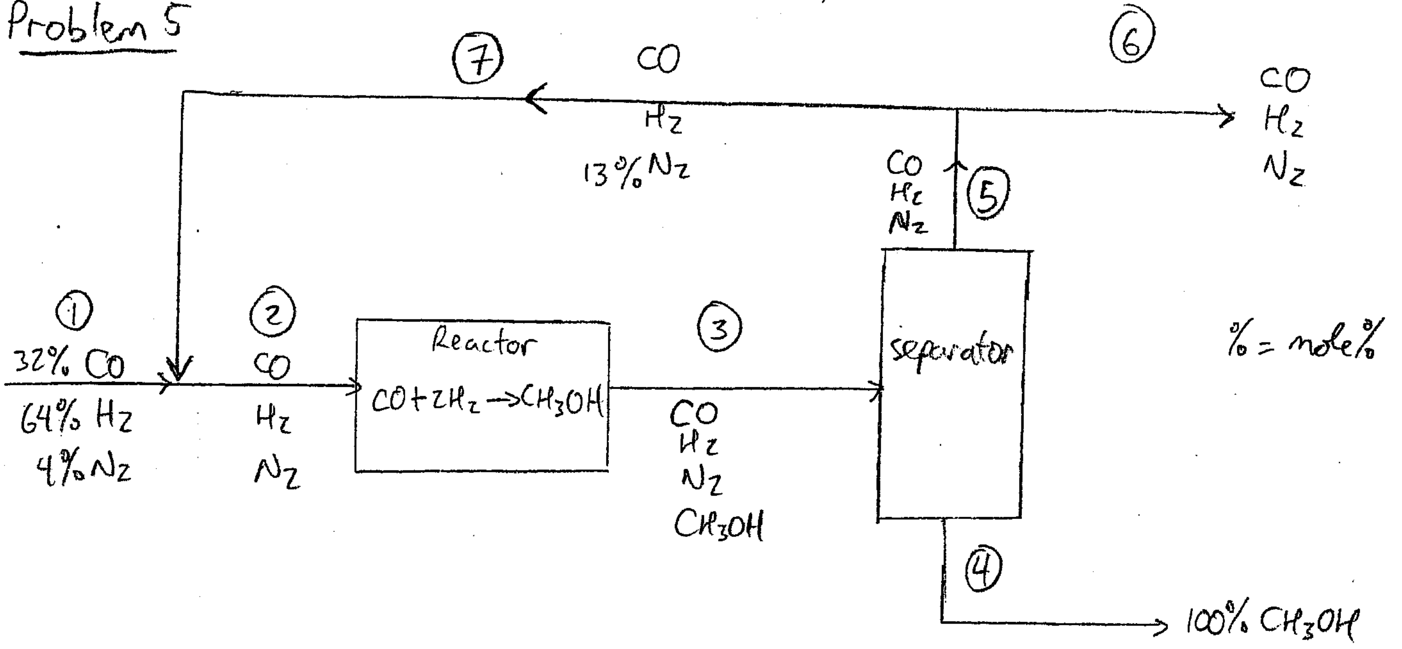
10.213
Spring 2002

Problem Set B

Solution

Problem 5



First, we draw the diagram and label the streams and all known information

Nomenclature:

$F_i \equiv$ flow rate of stream i (mol/hr)

$x_{a,i} \equiv$ mole % of species a in stream i, e.g. $x_{N_2,7} = 0.13$

$F_{a,i} \equiv$ flow rate of species a in stream i $= F_i \cdot x_{a,i}$

We are given that: $\dfrac{F_7}{F_1} = \dfrac{\text{mole recycle}}{\text{mole fresh feed}} = \dfrac{5}{1}$

Since we are not given a basis, let us choose one: let $F_1 = 1$ mol/hr

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a) We want to find out production rate of CH_3OH ($= F_4$).

Performing atomic/element balance over the whole system:

$$\text{Atom} \quad \text{In (stream 1)} + \overset{=0 \text{ for atom bal.}}{\cancel{\text{Gen}}} = \text{Out (streams 4 and 6)} + \overset{\text{steady state}}{\cancel{\text{Acc}}}$$

$H: (2)(0.64)(1 \text{ mol/hr}) = (4)(F_4) + (2)(F_{H_2,6}) \quad$ (i)

$C: (1)(0.32)(1 \text{ mol/hr}) = (1)(F_4) + (1)(F_{CO,6}) \quad$ (ii)

$N: (2)(0.04)(1 \text{ mol/hr}) = (2)(F_{N_2,6}) \quad$ (iii)

$O: (1)(0.32)(1 \text{ mol/hr}) = (1)(F_4) + (1)(F_{CO,6}) \quad$ (iv)

We have 4 equations and 4 unknowns : F_4; $F_{CO,6}$; $F_{H_2,6}$; $F_{N_2,6}$
But eq (ii) is the same as eq (iV) ! This is because C and O always stay together in our process ⇒ when there is C, there is O.
(We would still lack an equation if we do a species balance instead).
We do have one more piece of information:

$$x_{N_2,6} = 0.13 \qquad (v)$$

Why? Because the compositions of streams 5, 6, and 7 must be the same we are told $x_{N_2,7} = 13\%$ ⇒ $x_{N_2,6} = 13\%$ too. This means:

$$x_{N_2,6} = \frac{F_{N_2,6}}{F_6} = \frac{F_{N_2,6}}{F_{CO,6} + F_{H_2,6} + F_{N_2,6}} = 0.13$$

$$\Rightarrow \quad F_{N_2,6} = 0.13\left(F_{CO,6} + F_{H_2,6} + F_{N_2,6}\right) \qquad (vi)$$

So we have 4 unknowns in 4 equations. Rearranging the terms:

$$\left.\begin{array}{llr}
(i): & 4F_4 + 2F_{H_2,6} & = 1.28 \text{ mol/hr} \\
(ii): & F_4 + F_{CO,6} & = 0.32 \text{ mol/hr} \\
(iii): & 2F_{N_2,6} & = 0.08 \text{ mol/hr} \\
(vi): & 0.13F_{H_2,6} + 0.13F_{CO,6} - 0.87F_{N_2,6} & = 0
\end{array}\right] \begin{array}{l}\text{4 unknowns} \\ \text{4 independent eq'ns}\end{array}$$

Solving the 4 equations (by hand or your favorite program):

$F_{H_2,6} = 0.178$ mol/hr $\qquad F_{N_2,6} = 0.040$ mol/hr

$F_{CO,6} = 0.089$ mol/hr $\qquad F_4 = 0.231$ mol/hr

So production rate of $CH_3OH = F_4 = \boxed{0.231 \text{ mol/hr}}$ (ans)
(for basis of 1 mol/hr feed)

Note: There are many ways to solve this problem. All of them, however, will require using the information that $x_{N_2,7} = 13\%$

b) We have solved for the composition of the purge gas in the previous problem:

$F_{H_2,6} = 0.178$ mol/hr
$F_{CO,6} = 0.089$ mol/hr
$F_{N_2,6} = 0.040$ mol/hr

] purge $F_6 = \boxed{0.307 \text{ mol/hr}}$

$\boxed{\begin{array}{l} X_{H_2,6} = 58\% \\ X_{CO,6} = 29\% \\ X_{N_2,6} = 13\% \end{array}}$

c) Overall conversion $= \dfrac{\text{mole reacted over entire process}}{\text{mole fed into entire process}} = \dfrac{\text{mole in} - \text{mole out}}{\text{mole in}}$

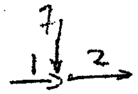
$$= \frac{F_{CO,1} - F_{CO,6}}{F_{CO,1}} = \frac{0.32 \text{ mol/hr} - 0.089 \frac{\text{mol}}{\text{hr}}}{0.32 \text{ mol/hr}} = \underline{\underline{72.1\%}}$$

Single-pass conversion $= \dfrac{\text{mole reacted in reactor}}{\text{mole fed into reactor}} = \dfrac{\text{mole into reactor} - \text{mole out reactor}}{\text{mole into reactor}}$

Single-pass conv. $= \dfrac{F_{CO,2} - F_{CO,3}}{F_{CO,2}}$ Let's find $F_{CO,2}$ and $F_{CO,3}$

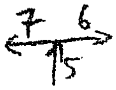
$F_7 = 5 F_1$ (given) $F_7 = 5(1 \text{ mol/hr}) = 5$ mol/hr

$F_{CO,7} = X_{CO,7} \cdot F_7 = X_{CO,6} \cdot F_7 = 1.45$ mol/hr $\leftarrow F_{CO,7}$



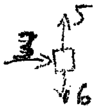
CO balance over "joint": $F_{CO,1} + F_{CO,7} = F_{CO,2}$ (no rxn)

$F_{CO,2} = 0.32$ mol/hr $+ 1.45$ mol/hr $= 1.77 \frac{\text{mol}}{\text{hr}}$



CO balance over "split": $F_{CO,5} = F_{CO,6} + F_{CO,7}$ (no rxn)

$= (0.089 + 1.45)$ mol/hr $= 1.539 \frac{\text{mol}}{\text{hr}}$



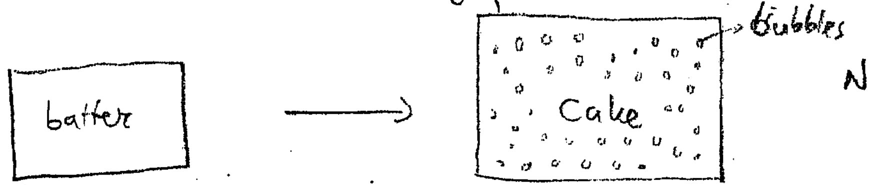
CO balance over separator: $F_{CO,3} = F_{CO,5} = 1.539$ mol/hr

Finally: single pass Conversion $= \dfrac{F_{CO,2} - F_{CO,3}}{F_{CO,2}} = \dfrac{(1.77 - 1.539) \text{ mol/hr}}{1.77 \text{ mol/hr}} = \underline{\underline{13.1\%}}$

Conversion: $\boxed{\begin{array}{l} \text{overall} = 72.1\% \\ \text{single pass} = 13.1\% \end{array}}$ (cons) again many ways to get to the ans.

Problem 6

System: cake (with baking soda inside)



$NaHCO_3 + H^+ = Na^+ + H_2O + CO_{2(g)}$

$V_1^t = 500\ mL$ $\quad$ $V_2^t = 2{,}000\ mL$ → see note at the end about "increasing fourfold"

$T_1 = 25°C$ $\quad$ $T_2 = 350°F$

1) Estimate M_{NaHCO_3} added if bubbles contain 100% CO_2.

Assume: - reaction is complete; all CO_2 goes into bubbles.
- Without $NaHCO_3$ added, cake volume increases 11%
- process is at $P = 1\ atm$ (constant); ideal gas

extrinsic

$\Delta V^t = V_2^t - V_1^t = 1500\ mL$; but $(11\% \times 500\ mL) = 55\ mL$ is not from bubbles.

So $V^t_{bubbles} = 1500\ mL - 55\ mL = 1445\ mL$

Ideal gas: $PV^t = nRT$ $\quad$ $T = 350°F = 176.7°C = 449.8\ K$

$$n_{bubble\ gas} = \frac{PV^t}{RT} = \frac{(1\ atm)(1445\ mL)}{82.06\ \frac{mL\ atm}{mol\ K} \cdot 449.8\ K} = 0.039\ mol$$

$n_{bubble\ gas} = n_{CO_2} = 0.039\ mol \xrightarrow{\text{stoichiometry}} n_{NaHCO_3,\ reacted} = 0.039\ mol$

$M_{NaHCO_3} = 0.039\ mol \times (23+1+12+3\times16)\frac{g}{mol} = \boxed{3.28\ g}$ ans.

2) What if bubbles are only 60% CO_2?

$n_{CO_2} = 0.6\ n_{bubble\ gas} = (0.6)(0.039\ mol) = 0.0234\ mol$

$m_{NaHCO_3} = 0.0234\ mol \times (23+1+12+3\times16)\frac{g}{mol} = \boxed{1.97\ g}$ ans

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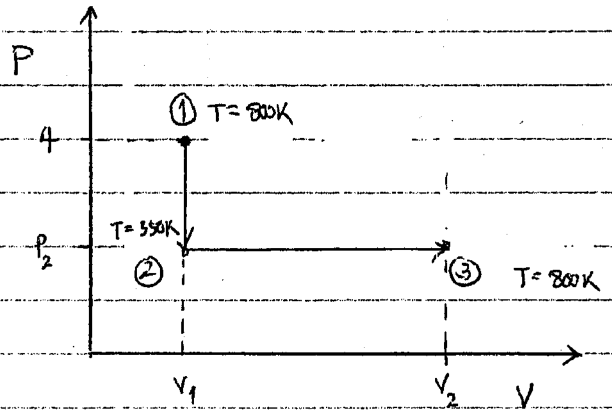
Note: If you chose "increase fourfold" to mean $\Delta V^t = 2000\ mL$, the answers will be 4.41 g and 2.65 g

Problem 7

Two-step process:



Assuming the gas is ideal, we can use $PV^t = nRT$ to calculate P, V, or T of state ①, ②, and ③

* remember, the values of P, V, T are not path dependent

① T= 800K, P= 4 bar $\therefore V^t = \frac{nRT}{P} = \frac{1 \times R \times 800}{4 \times 10^5}$

$V_1 = 2 \times 10^{-3} R \ m^3$

② T= 350 K, $V = V_1 = 2 \times 10^{-3} R \ m^3$

$\therefore P = \frac{nRT}{V^t} = \frac{1 \times R \times 350}{2 \times 10^{-3} R} = 1.75 \times 10^5 \ Pa = 1.75 \ bar$

③ $P = P_2 = 1.75$ bar T= 800 K

$\therefore V^t = \frac{nRT}{P} = \frac{1 \times R \times 800}{1.75 \times 10^5} = 4.57 \times 10^{-3} R \ m^3$

Assuming mechanical reversibility

$$W = -\int P\, dV^t = -n\int_{V_i}^{V_f} P\, dV$$

for a constant volume process from ① → ②

$$dV^t = dV = 0$$

$$\therefore \quad W_{step\,1} = 0$$

For a constant pressure process

$$W = -nP\int_{V_i}^{V_f} dV = -nP(V_f - V_i)$$

$$= -P(V_f^t - V_i^t) \quad \text{as } n \text{ is constant (closed system)}$$

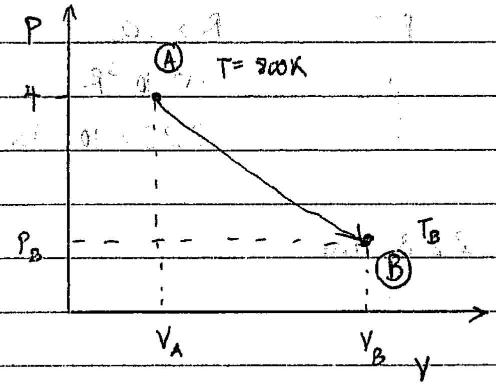
$$W_{step\,2} = -P_2(V_2 - V_1) = -1.75\times10^5(4.57\times10^{-3}R - 2\times10^{-3}R)$$

$$= -450R \quad J$$

-ve sign as the gas expands and do work on the surroundings.

For a single step isothermal expansion:

T is constant in isothermal process



$T_B = T_A = 800$ K

$$W = -n\int P\,dV$$

for ideal gas $PV = RT$ ∴ $P = \frac{RT}{V}$

$$W = -n\int \frac{RT}{V}\,dV$$

T is constant ← remember W is path dependent, here the path we chose is the constant T path

$$W = -nRT\int_{V_A}^{V_B} \frac{1}{V}\,dV$$

$$W = -nRT\ln\left(\frac{V_B}{V_A}\right)$$

we know that $T = 800$ K

$V_A = 2\times10^{-3}R$ m^3

and we need W to be $-450R$ J

$$-450R = -1\times R\times 800\times\ln\left(\frac{V_B}{2\times10^{-3}R}\right)$$

$$\ln\left(\frac{V_B}{2\times10^{-3}R}\right) = 0.5625$$

$V_B = 3.51 \times 10^{-3} R \ m^3/mol$

From ideal gas EOS $PV = RT$

$$P = \frac{RT}{V} = \frac{R \times 800}{3.51 \times 10^{-3} R}$$

$$= 2.28 \times 10^5 \ Pa$$

$\therefore$ Final pressure is 2.28 bar

Problem 8

a) At all times the pressure in the cylinder equals the weight of the elevator

ie. the piston moves by reversible expansion

$$P = \frac{F}{A} = \frac{4000 \times 9.81}{2.5} = 15696 \text{ Pa}$$

b) This is a constant pressure process

Initial condition ① $T = 293$ K $V = 25 \text{ m}^3$

Final condition ② $T = ?$ $V = 25 + (2.5 \times 3)$

$= 32.5 \text{ m}^3$

as the gas is ideal, we can calculate its final temperature from

$$PV^t = nRT$$

$$PV_1^t = nRT_1 \longrightarrow \frac{P}{nR} = \frac{T_1}{V_1^t}$$

$$PV_2^t = nRT_2 \longrightarrow \underset{\uparrow \text{ constant}}{\frac{P}{nR}} = \frac{T_2}{V_2^t}$$

$$\therefore \frac{T_1}{V_1^t} = \frac{T_2}{V_2^t}$$

$$T_2 = \frac{V_2^t}{V_1^t} T_1 = \frac{32.5}{25} \times 293 = 380.9 \text{ K}$$

also $n = \dfrac{PV_2^t}{RT_2} = \dfrac{15696 \times 32.5}{8.314 \times 380.9} = 161$ moles for a constant pressure process $Q = n\Delta H = n\int_{T_1}^{T_2} C_p \, dT$

$Q = n C_p (T_2 - T_1)$ as n is constant

$= 161 \times 30 \times (380.9 - 293)$

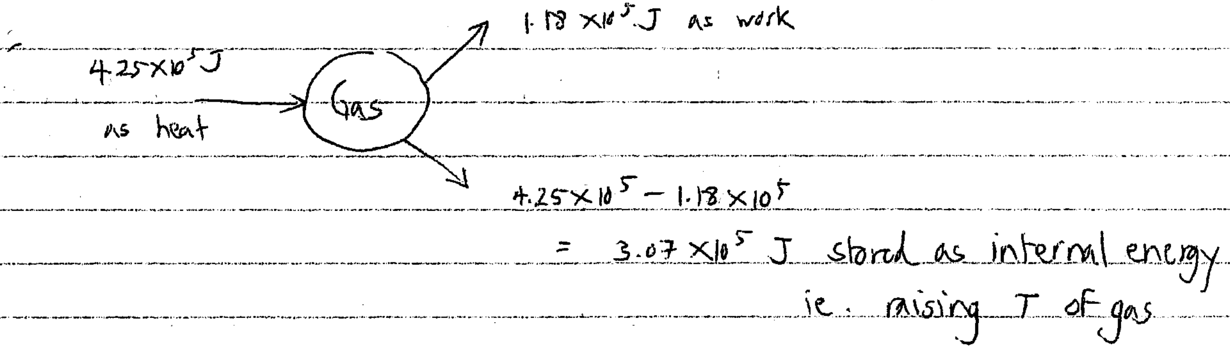
$= 4.25 \times 10^5 \text{ J}$

() for a constant pressure process

$$\dot{W} = -n\int_{V_1}^{V_2} P\,dV = -nP(V_2 - V_1) = -P(V_2^t - V_1^t)$$

$$= -15696 \times (32.5 - 25)$$

$$= -1.18 \times 10 \text{ J}$$



$\therefore \dfrac{1.18\times10^5}{4.25\times10^5} \times 100 = 28\%$ is used to do work

and 72% to raise its temperature

d) During lowering process both P and T remain constant, we simply reduce the gas volume by venting away some of the gas

$$PV^t = nRT$$

$$\frac{P}{RT} = \frac{n}{V^t} = \text{constant}$$

$$\therefore \frac{n_{\text{initial}}}{V^t_{\text{initial}}} = \frac{n_{\text{final}}}{V^t_{\text{final}}}$$

$$\frac{161}{32.5} = \frac{n_{\text{final}}}{25}$$

$$\therefore n_{\text{final}} = 124 \text{ moles}$$

$\therefore$ vent 161 − 124 = 37 moles

e) this is not an efficient process both in terms of energy and time