Solution method for Practice Problems in Problem Sets E, F, and G

Let me know (thio@mit.edu) if you disagree with the methods or if you spot a typo.

Practice Problem P7

50 kg/s of steam at 700 kPa and 280°C is mixed with some liquid water at 40°C.
Exits at 700 kPa and 200°C. What is the flow rate of liquid water?
Operation is adibatic (Q = 0). Just a mixing process, we can also assume no shaft work (Ws = 0).

Material balance: $m_{steam,in} + m_{water,in} = m_{steam,out}$ (1)
We only know $m_{water,in} = 50$ kg/s.

1st law open system: $\Delta(mH) = Q + W = 0 + 0 = 0$.
Remember that the "Δ" here means "out minus in".
So this basically says that the inlet and outlet enthalpy rates (kJ/s, not kJ/kg) have to be the same. Otherwise, we have a build-up of enthalpy in the exchanger. So we need to multiply the enthalpies with the corresponding flow rates to get:
$m_{steam,out} H_{steam,out} - (m_{steam,in} H_{steam,in} + m_{water,in} H_{water,in}) = 0$ (2)

We can get $H_{steam,in}$ (700 kPa, 280°C) and $H_{steam,out}$ (700 kPa, 200°C) from the steam tables.
What about $H_{water,in}$? Steam tables tell us the enthalpy of *saturated* liquid at 40°C. But this is for pressure = P^{sat}at 40°C. Does it matter? Remember that for liquid we usually assume that the enthalpy and volume do not depend too much on the pressure. Especially since the pressure is not specified here, we shall assume $H_{water,in} \approx H_{sat\ liq\ water}$ at 40°C.
For a more rigorous way to get ΔH for liquid, see p. 200 of book (for which we need more data).

Therefore we have 2 equations, (1) and (2) above, and 2 unknowns: $m_{steam,out}$ and $m_{water,in}$.
We can solve this linear equation quite easily.

ΔS can be calculated as: $m_{steam,out} S_{steam,out} - (m_{steam,in} S_{steam,in} + m_{water,in} S_{water,in}) = 0$
There is no $\Delta S_{surrounding}$ in this case because the process is adiabatic. ($\Delta S_{surr} = - Q_{sys}/T_{surr}$).
We can get the S values from the steam tables at the conditions just like the H above.
We should find ΔS > 0 (irreversible) → the steam is not going to "unmix" back to steam and water.

Practice Problem P8

a)
Water at 24°C is mixed with saturated steam at 0.4 MPa to form 5 kg/s of water at 85°C.
What are the unknown flow rates assuming adiabatic operation?
The solution is very similar to Problem P7 above. We use the same reasoning to get 2 equations:
$m_{steam,in} + m_{water,in} = m_{water,out}$ (1)
$m_{water,out} H_{water,out} - (m_{steam,in} H_{steam,in} + m_{water,in} H_{water,in}) = 0$ (2)

$H_{steam,in} = H_{sat\ vap}$ at 0.4 MPa; $H_{water,in} \approx H_{sat\ liq}$ at 24°C; $H_{water,out} \approx H_{sat\ liq}$ at 85°C.

Two equations, two unknowns ($m_{steam,in}$ and $m_{water,in}$). Can solve.

Practice Problem P8 (cont'd)

b)
Choose as our system 2 m^3 tank (total volume constant).
Initial picture: 10 mass% vapor, 90 mass% liquid. $P_1 = 3$ MPa. $T_1 = T^{sat}$ at 3MPa (can look up).
Final picture: 40% of initial mass remains. $T_2 = T_1$.

Open or closed system? Closed, because we have this "before and after" picture of the tank. For open system, we usually have an inlet stream and an outlet stream (at some flow rates).
1^{st} law for closed system: $\Delta U^t = Q + W$ (where everything now is in terms of kJ)
There is no work (constant total volume) → $Q = \Delta U^t = U^t_2 - U^t_1$.
So we need to figure out the total internal energy for before and after.

Initially: What is the total internal energy? It would be a combination of U^t_{liq} and U^t_{vap}. Mind that total. We want kJ, not kJ/kg. This is especially important because the initial and final mass are different. So we need to figure out the masses of the liquid and the vapor.

$m_{liq,1} = 0.9\ m_1$ and $m_{vap,1} = 0.1\ m_1 \rightarrow m_{liq,1} = (1/9)\ m_{vap,1}$ (1)
$V^t_{liq} + V^t_{vap} = V^t_1 \rightarrow m_{liq}\ V_{liq} + m_{vap}\ V_{vap} = 2\ m^3$ (2)

Combining the two equations: $(m_{vap,1}/9)\ V_{liq} + m_{vap,1}\ V_{vap} = 2\ m^3$
We can get V_{liq} and V_{vap} from steam tables (saturated condition at 3 MPa).
We can solve for m_{vap}. Then we can get m_{liq}. (And also $m_1 = m_{vap,1} + m_{liq,1}$)

$U^t_1 = m_{liq,1}\ U_{liq} + m_{vap,1}\ U_{vap}$
Again, we can get U_{liq} and U_{vap} from steam tables (saturated condition at 3 MPa). Can get $\mathbf{U^t_1}$.

What about the final condition? We know that the mass now is 40% of initial mass: $m_2 = 0.4m_1$.
What is the condition now? Is it liquid, vapor, or a mixture? We should check first:

The molar volume $V_2 = V^t_2/m_2 = (2\ m^3) / (m_2)$
Compare this V_2 with V_{vap} and V_{liq} at the same temperature (hence same pressure = 3 MPa).
If $V_2 > V_{vap}$ then we have superheated steam. If we have V_2 in between V_{vap} and V_{liq}, we have a mixture. (I haven't done the calculation, so I don't know where it will be).

If superheated: We can find U_2 in the superheated steam tables. Then $U^t_2 = m_2\ U^t_2$

If a mixture: We can write similar equations as (1) and (2) for the new condition, except that the quality (vapor fraction) is not known. We'll designate this as X:
$m_{liq,2} = (1\text{-}X)\ m_2$ and $m_{vap,2} = X\ m_2$ (3)
$V^t_{liq} + V^t_{vap} = V^t_2 \rightarrow m_{liq,2}\ V_{liq} + m_{vap,2}\ V_{vap} = 2\ m^3$ (4)

Combining the two: $(1\text{-}X)m_2\ V_{liq,2} + X\ m_2\ V_{vap,2} = 2\ m^3$
We can solve for X. And therefore we can get $m_{liq,2}$ and $m_{vap,2}$. Then
$\mathbf{U^t_2} = m_{liq,2}\ U_{liq} + m_{vap,2}\ U_{vap}$

Finally we can calculate $Q = U^t_2 - U^t_1$ (in kJ, not kJ/kg). Done.

Practice Problem P9

Propane is compressed isothermally at 100°C from 1 bar to 10 bar. Estimate ΔH and ΔS.

This is just an exercise in using …. ideal gas? No. Since ideal gas is not specified, you can't assume that. Do we have data for propane? No. Then we must use [ominous music] generalized correlation. But it's not so bad, really. Try it out. It's not bad because we have both temperature and pressure for both initial and final condition.

$$(\Delta H) = \Delta H^{ig} + \Delta H^R = \int_{T_1}^{T_2} Cp^{ig} dT + H_2^R - H_1^R \qquad \Delta S = \Delta S^{ig} + \Delta S^R = \int_{T_1}^{T_2} \frac{Cp^{ig}}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dP + S_2^R - S_1^R$$

$$(\Delta H) = H_2^R - H_1^R \quad \text{(because T is constant)} \qquad \Delta S = -\int_{P_1}^{P_2} \frac{R}{P} dP + S_2^R - S_1^R \quad \text{(because T is constant)}$$

I'll assume you know how to calculate H^R and S^R at the two conditions.
(If you don't, do let me know (thio@mit.edu). It's very important to know how to do this).

Note: Can we use eqns (6.78) and (6.79) ("that thingy with the B stuff", applicable for low to moderate P's)? Probably yes. $P_{r,2}$ is only about 0.2: P is low enough, it seems. But why use that when using the charts is so much quicker? We want to use those eqns only when we don't know T or P; in these cases, using equations is easier than guessing-and-looking-up-the-charts over and over. But in our case here, since we know the T and P (for both conditions), it's much easier to use chart.

Quick way to check your numbers: Use the NIST webbook. You should get the same ΔH and ΔS.

Practice Problem P10

Ah, the generic compressor problem… I hope you can do this problem without help at this point. Do try it out before reading the solution method.

Since the compressor is only 0.78 efficient, we need to construct the imaginary reversible process.
The reversible process goes from 3,500 kPa and 35°C (same initial point as the real one) and goes to 5,500 kPa. Since it's reversible $\Delta S = 0 \rightarrow$ we follow a constant S line from 3,500 kPa and 35°C up to 5,500 kPa. **Remember to convert the units; the diagram is in English units**.
The outlet enthalpy for the reversible process can be read (top axis). Call this $H_{2,rev}$.
The inlet enthalpy can be read also (at 3,500 kPa and 35°C). Call this H_1.

$(\Delta H)_S = W_{s,rev} = H_{2,rev} - H_1$. (Negative or positive? Compression, system works or gets worked?)
Then the real W_s can be calculated from efficiency:
$\eta_{comp} = W_s / W_{s,rev}$. Get W_s.
Power requirement would be W_s (Btu/lbm) multiplied by flow rate (kg-mol/s)
(Convert the units first of course. We want Btu/s or kJ/s)

Then the outlet of the real compressor is identified by 5,500 kPa (outlet pressure) and the enthalpy $H_2 = H_1 + W_s$. Find in the chart: 5,500 kPa and H_2. Read the temperature.

Read this: Alright… At this point have you gone "Wait a minute! He did it wrong!" ?
If not, check the efficiency expression again. The way I wrote it, $|W_s|$ will be smaller than $|W_{s,rev}|$.
Does it make sense for a real compressor to have smaller work requirement than the reversible?
You need to make checks like this if you're not sure. No. ("The universe is always against us").
The correct expression: $\eta_{comp} = W_{s,rev} / W_s$. For turbine: $\eta_{turbine} = W_s / W_{s,rev}$.

Practice Problem P11

This is really similar to Problem 24, but simpler: we only have one cycle. Let me just copy the method from that problem.

Strategy: (see Figure 9.1 in text)
We want to find the heat and shaft work terms, for which we need values of enthalpy for the points 1,2,3, and 4. Point 2 and 4 are usually defined in the problem ("anchor points") because they are saturated vapor and saturated liquid, respectively. We can start with point 4 and identify point 1. Then we can start with point 2 and identify point 3.

Throttle valve (4 → 1)
Inlet (point 4): Saturated liquid at 80°F. (temperature specified in problem statement)
Table 9.1 (or Figure G.2) gives us for 80°F: $P_4 = P^{sat}$ and $H_4 = H_{sat\ liq}$.
Outlet (point 1): Mixture of vapor and liquid.
For throttle valve $\Delta H = 0$. Then $H_1 = H_4$. There is no heat or work produced.

Evaporator (1 → 2)
Outlet (point 2): Saturated vapor at 0°F. (temperature specified in problem statement)
Table 9.1 (or Figure G.2) at 0°F: $P_2 = P^{sat}$; the enthalpy = $H_2 = H_{sat\ vap}$.
Since there is no work done, $Q = \mathbf{Q_C} = \Delta H = H_2 - H_1$.

Compressor (2 → 3)
We calculate the work of a reversible compressor first (to use the 76% efficiency information).

For a reversible compressor, $\Delta S = 0$, or $S_{3*} = S_2$. We compress from P_2 to P_3.
On Figure G.2: We follow constant S line from point 2 (sat. vap. at 0°F) to $P_3 = P_4$.
This gives us point 3* with some H_{3*}.
For this reversible compressor, $W_{c,2,rev} = (\Delta H)_S = H_{3*} - H_2$
For our *real* compressor, $\eta = 76\%$. Then $\mathbf{W_c} = W_{c,rev} / 0.76$.

Putting it all together
We can calculate $Q_{H,2}$ (heat released in the condenser).
Since for a cycle ΔH must be 0, $W_c + Q_H + Q_C = 0$. Then $Q_H = -(W_c + Q_C)$

Coefficient of performance = COP = $|Q_c| / |W_{net}| = |Q_C| / |W_c|$ (since the valve produces no work)

The circulation rate = (refrigeration rate)/($|Q_c|$)
(Why? Qc is the cooling in Btu/lbm of refrigerant. Refrigeration rate is in Btu/s. So we need to multiply Qc by the flow rate [lbm/s] to get the cooling rate)
Then to find the power, we multiply the |Ws| by this circulation rate to get some Btu/s.

COP of Carnot refrigerator = $T_H/(T_H-T_c)$