Problem Set E

Problem 17

Hot water tank 472K

House 293K

Q_1

1000 kJ

E

W

C

Q_2

Q_3

Surrounding 305K
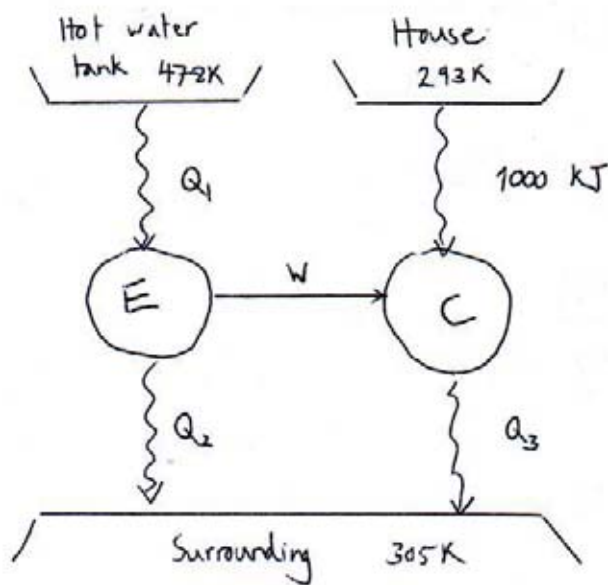
Both are Carnot engines

For system operating at maximum efficiency

$$\frac{Q_C}{T_C} + \frac{Q_H}{T_H} = 0 \qquad \text{or} \qquad \frac{|Q_C|}{T_C} = \frac{|Q_H|}{T_H}$$

For refrigerator C, we have

$$\frac{|Q_3|}{305} = \frac{1000}{293}$$

$$\therefore |Q_3| = 10\,41 \text{ kJ}$$

$|W| = |Q_3| - 1000$ From energy balance around C

$\quad = 41 \text{ kJ}$

For heat engine E, $\qquad \frac{|Q_1|}{478} = -\frac{|Q_2|}{305}$

$\therefore \quad |Q_1| = \dfrac{478}{305} |Q_2|$

From energy balance around E,

$$|Q_1| = |W| + |Q_2|$$

$$\therefore \quad \dfrac{478}{305} |Q_2| = |W| + |Q_2|$$

$$|Q_2| = \dfrac{|W|}{\left(\dfrac{478}{305} - 1\right)}$$

$$\therefore \quad |Q_2| = 72.3 \text{ kJ}$$

$$\boxed{\therefore \quad |Q_1| = 113 \text{ kJ}}$$

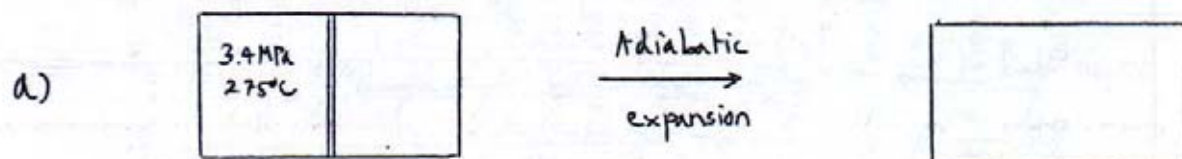Another way of solving this problem is to consider overall energy balance

$$Q_{tank} + Q_{surr} + Q_{house} = 0$$

and for a process to be reversible, $\Delta S = 0$

ie. $\quad \dfrac{Q_{tank}}{T_{tank}} + \dfrac{Q_{surr}}{T_{surr}} + \dfrac{Q_{house}}{T_{house}} = 0$

this should leads to the same answer

Problem 18

a)

3.4 MPa
275°C

Adiabatic
————————→
expansion

Adiabatic $\therefore$ $Q = 0$

Irreversible expansion against vacuum $\therefore$ the gas do no wo

$$\Rightarrow \quad W = 0$$

From 1st law $\qquad \Delta u = Q + W$

$$\therefore \quad \Delta u = 0$$

From steam table $\qquad P = 3.4 \, M \, Pa$

$$T = 275°C$$

(page 711) $\Rightarrow$ $\qquad u = 2687.7 \, kJ \, kg^{-1}$

$$V = 65.982 \, cm^3 g^{-1}$$

Since the final total volume of the gas is doubled

$$V_{final} = 131.964 \, cm^3 g^{-1}$$

$\Delta u = 0$ $\Rightarrow$ $u_{final} = 2687.7 \, kJ/kg^{-1}$

From steam table
we know V and u

$$\boxed{\begin{array}{l} P \approx 1.7 \, MPa \\ T \approx 250°C \end{array}}$$

$\therefore$ ⌐

* Remember phase rule , we only need to define 2 variables, u and
in this case, then P, T etc. are automatically fix
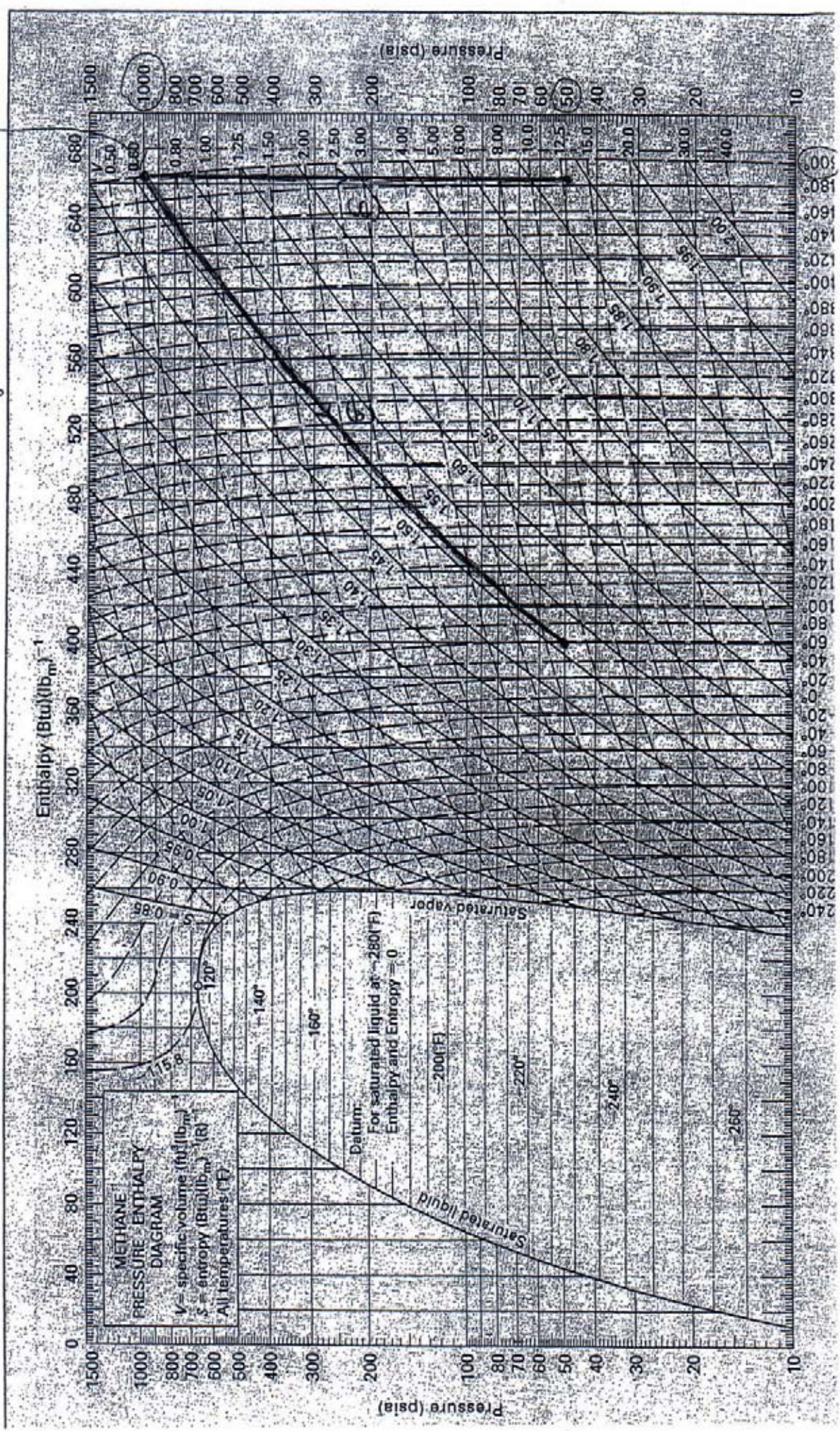
(b)

starting point

Isentropic $\Delta S = 0$

$\therefore$ $T = 60°F$
$\Delta T = -440°F$
and $\Delta H = 400 - 664$
$= -264 \ Btu/lb$

(c)

Isenthalpic $\Delta H = 0$

$T = 484°F$
$\Delta T = -16°F$

$\Delta S \approx 188 - 151$
$= 37 \ Btu/lb.R$

This process is
irreversible as
$\Delta S > 0$

METHANE
PRESSURE-ENTHALPY
DIAGRAM

V = specific volume (ft³ lb$_m$)⁻¹
S = entropy (Btu lb$_m$)⁻¹(R)⁻¹
All temperatures (°F)

Datum:
For saturated liquid at −280(°F)
Enthalpy and Entropy = 0

Pressure (psia)

Enthalpy (Btu)(lb$_m$)⁻¹

Saturated vapor

Saturated liquid

Problem 19

Ethane

$T_1 = 200°C$

$P_0 = 25\ bar$

$\xrightarrow[\text{expansion}]{\text{Isentropic}}$

$T_2 = ?$

$P = 2\ bar$

Isentropic $=$ reversible and adiabatic

$\therefore\quad \Delta S = 0$

a) For an ideal gas

$$dS = \frac{C_p}{T}\,dT - \frac{R}{P}\,dP$$

$$\Delta S = \int \frac{C_p}{T}\,dT - R\,\ln\frac{P}{P_0}$$

in this case

$$\frac{C_p^{ig}}{R} = 1.131 + 19.225\times10^{-3}\,T - 5.561\times10^{-6}\,T^2$$

$$\therefore\quad \frac{C_p}{T} = R\left(\frac{1.131}{T} + 19.225\times10^{-3} - 5.561\times10^{-6}\,T\right)$$

$$\therefore\quad \int_{T_1}^{T_2} \frac{C_p}{T}\,dT = R\int_{T_1}^{T_2} \frac{1.131}{T} + 19.225\times10^{-3} - 5.561\times10^{-6}\,T \quad dT$$

$$= R\left[1.131\,\ln T + 19.225\times10^{-3}\,T - 2.7805\times10^{-6}\,T^2\right]_{T_1}^{T_2}$$

$$= R\left[1.131\,\ln T_2 + 19.225\times10^{-3}\,T_2 - 2.780\times10^{-6}\,T_2{}^2 - 15.44\right]$$

with $\Delta S = 0$

$$R[\ 1.131 \ln T_2 + 19.225 \times 10^{-3} T_2 - 2.7805 \times 10^{-6} T_2{}^2 - 15.44\ ] = R \ln \frac{2}{25}$$

$$1.131 \ln T_2 + 19.225 \times 10^{-3} T_2 - 2.7805 \times 10^{-6} T_2{}^2 = 12.91$$

$$\therefore \quad \boxed{T_2 = 345\ K}$$

From 1st law $\quad du = dQ + dW$

as $\quad dQ = 0 \qquad dW = du$

For ideal gas $\qquad du = C_v\ dT$

with $\qquad C_p = C_v + R$

$$\therefore \quad du = (C_p - R)\ dT$$
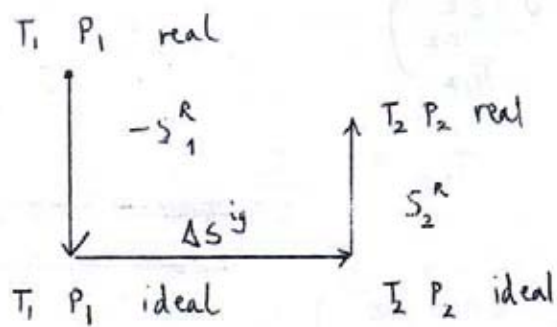
$$\therefore \quad W = \int (C_p - R)\ dT$$

$$= \int R[\ 1.131 + 19.225 \times 10^{-3} T - 5.561 \times 10^{-6} T^2\ ] - R \quad dT$$

$$= \int_{T_1}^{T_2} R(\ 0.131 + 19.225 \times 10^{-3} T - 5.561 \times 10^{-6} T^2\ )\ dT$$

$$= R\left[\ 0.131 T + 9.6125 \times 10^{-3} T^2 - 1.854 \times 10^{-6} T^3\ \right]_{T_1}^{T_2}$$

$$\boxed{W = -7510\ J/mol}$$

b)

T₁ P₁ real

$-S_1^R$

ΔS^{iy}

T₁ P₁ ideal

T₂ P₂ real

S_2^R

T₂ P₂ ideal

$$\therefore \Delta S = \Delta S^{iy} + S_2^R - S_1^R$$

For ethane $T_c = 305.3$ K $P_c = 48.72$ bar

$\omega = 0.100$

using generalized correlation for gases with residual property relationship (eqn. 6.79) ; this however only valid for low to moderate pressure

ie. $P_r < 0.8$

$$\frac{S^R}{R} = -P_r \left(\frac{dB^\circ}{dT_r} + \omega \frac{dB'}{dT_r} \right)$$

$$\frac{dB^\circ}{dT_r} = \frac{0.675}{T_r^{2.6}}$$

$$\frac{dB'}{dT_r} = \frac{0.722}{T_r^{5.2}}$$

$$\therefore \quad S^R = -RP_r \left(\frac{0.675}{T_r^{2.6}} + \omega \cdot \frac{0.722}{T_r^{5.2}} \right)$$

$$S_1^R = -RP_{r,1} \left(\frac{0.675}{T_{r,1}^{2.6}} + \omega \cdot \frac{0.722}{T_{r,1}^{5.2}} \right)$$

with $P_{r,1} = \dfrac{25}{48.72} = 0.513$ $T_{r,1} = \dfrac{473}{305.3} = 1.550$

$$\therefore \quad S_1^R = -0.953 \quad J/mol\,K$$

$$S_2^R = -R \, P_{r,2} \left(\frac{0.675}{T_{r,2}^{2.6}} + \omega \cdot \frac{0.722}{T_{r,2}^{5.2}} \right)$$

$$P_{r,2} = \frac{2}{48.72} = 0.0411 \qquad T_{r,2} = \frac{T_2}{305.3}$$

$$\therefore \; S_2^R = -0.3413 \left(\frac{1.948 \times 10^6}{T_2^{2.6}} + \frac{6.013 \times 10^{11}}{T_2^{5.2}} \right)$$

with $\quad \Delta S^{ig} = R \left[1.131 \ln T_2 + 19.225 \times 10^{-3} T_2 - 2.780 \times 10^{-6} T_2^2 - 15.44 \right] - R \ln 0.08$

we have $\quad \Delta S = 0$

$$0 = R \left[1.131 \ln T_2 + 19.225 \times 10^{-3} T_2 - 2.780 \times 10^{-6} T_2^2 - 15.44 \right] + 21$$

$$- 0.3413 \left(\frac{1.948 \times 10^6}{T_2^{2.6}} + \frac{6.013 \times 10^{11}}{T_2^{5.2}} \right) + 0.953$$

$$\therefore \quad \boxed{T_2 = 341 \text{ K}}$$

To calculate work, we use $\quad dW = dU$

with $\quad U = U^{ig} + U_2^R - U_1^R$

with $\quad U^{ig} = R \left[0.131 T + 9.6125 \times 10^{-3} T^2 - 1.854 \times 10^{-6} T^3 \right]_{T_1}^{T_2}$

$$= -7726 \text{ J/mol}$$

now we need to calculate U^R

$$H = U + PV$$

$$\therefore \; U = H - PV$$

$$\Rightarrow \quad U^R = H^R - PV^R$$

using eqn. 6.78 for H^R

$$\frac{H^R}{RT_c} = P_r\left[B^0 - T_r\frac{dB^0}{dT_r} + \omega\left(B^1 - T_r\frac{dB^1}{dT_r}\right)\right]$$

with $\quad B^0 = 0.083 - \dfrac{0.422}{T_r^{1.6}}$ $\qquad B^1 = 0.139 - \dfrac{0.172}{T_r^{4.2}}$

$$\frac{dB^0}{dT_r} = \frac{0.675}{T_r^{2.6}} \qquad\qquad \frac{dB^1}{dT_r} = \frac{0.722}{T_r^{5.2}}$$

$\therefore \qquad H^R_1 = -600.8 \quad J/mol$

$\qquad\qquad H^R_2 = -91.6 \quad J/mol$

$$V^R = (Z-1)\frac{RT}{P}$$

with $\quad Z = 1 + B^0\dfrac{P_r}{T_r} + \omega B^1 \dfrac{P_r}{T_r}$

$\therefore \quad V^R_1 = -6\times10^{-5} \quad m^3/mol$

$\qquad\quad V^R_2 = -1.395\times10^{-4} \quad m^3/mol$

$\therefore \quad u^R_1 = -450.8 \quad J/mol$

$\qquad\quad u^R_2 = -63.7 \quad J/mol$

$\therefore \quad \Delta u = -7726 - 63.7 + 450.8 \qquad = \qquad -7339 \quad J/mol$

$$\boxed{W = -7340 \quad J/mol}$$

c)   From NIST

$T_1 = 473\ K \qquad P_1 = 25\ bar$

$\Rightarrow \qquad S_1 = 103.53\ J/mol\,K \qquad\qquad U_1 = 26.836\ kJ/mol$

with $\ P_2 = 2\ bar \qquad S_2 = 103.53\ J/mol\,K$

$$\boxed{T_2 = 331.6\ K}$$

with $\ U_2 = 19.510\ kJ/mol$

$\therefore \qquad W = 19.510 - 26.836$

$$\boxed{W = -7.326\ kJ/mol}$$

similar to part b) $\therefore$ we're justified in using those eqns in part b)

e)   For isenthalpic process $\Delta H = 0$

$T_1 = 473\ K \qquad P_1 = 25\ bar$

$\Rightarrow \qquad H_1 = 30.618\ kJ/mol \qquad U_1 = 26.836\ kJ/mol$

with $\quad P_2 = 2\ bar \qquad H_2 = 30.618\ kJ/mol$

$$\boxed{T_2 = 465.35\ K}$$

with $U_2 = 26.763\ kJ/mol$

$$\therefore \qquad \boxed{W = -73\ J/mol}$$