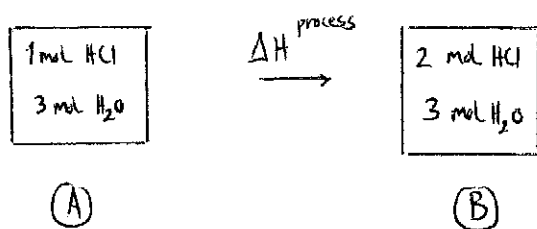


# Problem Set K

## Problem 36

a)



Condition A:  $\tilde{n}_A = 3$

$$\tilde{\Delta H}_A = -56 \text{ kJ/mol solute}$$

$$\begin{aligned} \Delta H_A^{total} &= -56 \times 1 \\ &= -56 \text{ kJ} \end{aligned}$$

$$\Delta H^{process} = (-82) - (-56)$$

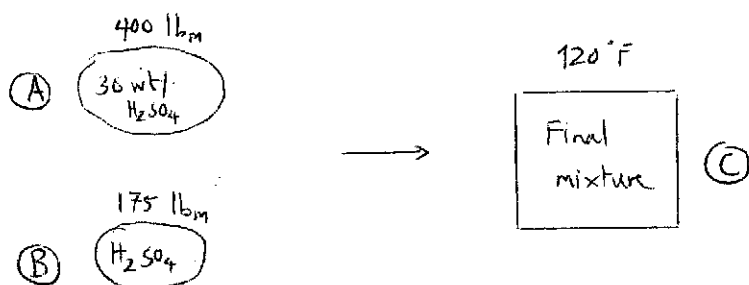
$$\Delta H^{process} = -26 \text{ kJ}$$

Condition B:  $\tilde{n}_B = \frac{3}{2}$

$$\tilde{\Delta H}_B = -41 \text{ kJ/mol solute}$$

$$\begin{aligned} \Delta H_B^{total} &= -41 \times 2 \\ &= -82 \text{ kJ} \end{aligned}$$

b)



(A):  $0.3 \times 400 = 120 \text{ lb}_m \text{ H}_2\text{SO}_4$   
 $0.7 \times 400 = 280 \text{ lb}_m \text{ H}_2\text{O}$   
 i.e. 36 wt% H<sub>2</sub>SO<sub>4</sub>

$$H_A = -24 \text{ Btu/lb}_m \text{ solution}$$

$$H_A^{total} = -24 \times 400 = -9600 \text{ Btu}$$

(B):  $H_{\text{H}_2\text{SO}_4} = 8 \text{ Btu/lb}_m$

$$H_B^{total} = 8 \times 175 = 1400 \text{ Btu}$$

$$\textcircled{C} : \quad 120 + 175 = 295 \text{ lb}_m \text{ H}_2\text{SO}_4$$

$$280 \text{ lb}_m \text{ H}_2\text{O}$$

$$\therefore \text{ we have } \left( \frac{295}{295 + 280} \right) \times 100 = 51.3 \text{ wt\% H}_2\text{SO}_4$$

$$H_B = -82 \text{ Btu/lb}_m \text{ solution}$$

$$\therefore H_B^{\text{total}} = -82 \times (295 + 280) = -47150 \text{ Btu}$$

$$\therefore \Delta H^{\text{process}} = -47150 - (-9600 + 1400)$$

$$\boxed{\Delta H^{\text{process}} = -38950 \text{ Btu}}$$

$$\textcircled{C}) \quad H^E = H - H^{\text{id}}$$

from chart, 60 wt%  $\text{H}_2\text{SO}_4$  at  $100^\circ\text{F}$

$$H = -110 \text{ Btu/lb}_m \text{ solution}$$

$$\text{also from chart } H_{\text{H}_2\text{SO}_4} = 8 \text{ Btu/lb}_m \quad (\text{at } 100 \text{ wt\% H}_2\text{SO}_4)$$

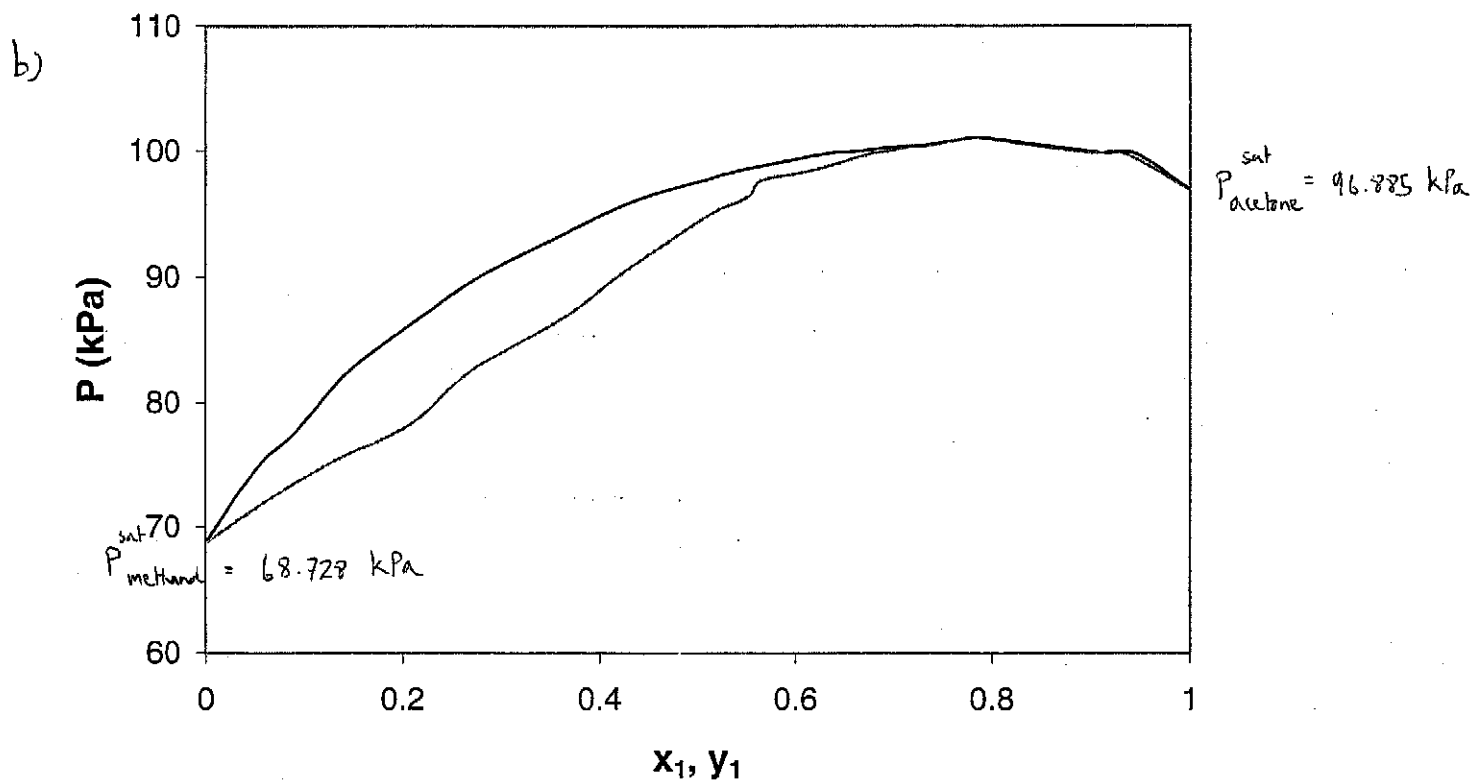
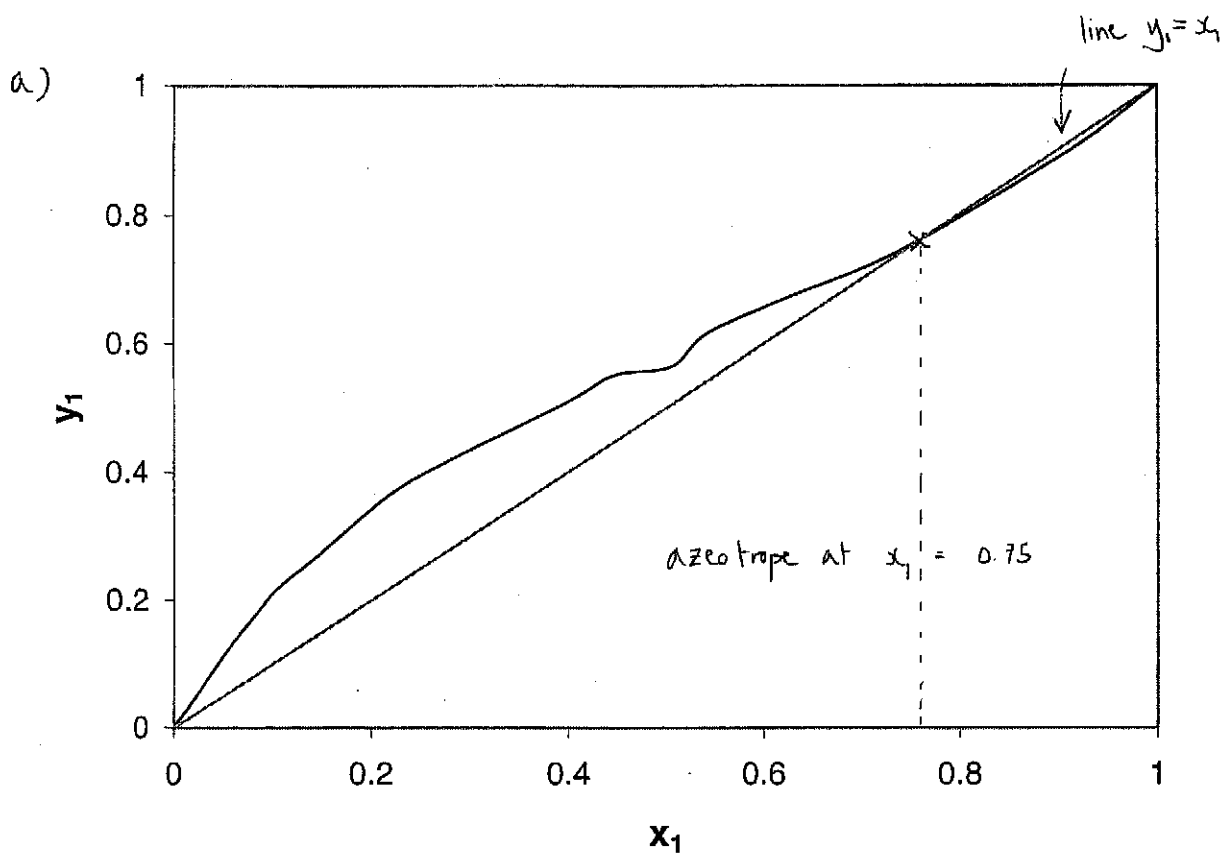
$$H_{\text{H}_2\text{O}} = 68 \text{ Btu/lb}_m \quad (\text{at } 0 \text{ wt\% H}_2\text{SO}_4)$$

$$\begin{aligned} \therefore H^{\text{id}} &= \sum_i x_i H_i = x_{\text{H}_2\text{SO}_4} H_{\text{H}_2\text{SO}_4} + x_{\text{H}_2\text{O}} H_{\text{H}_2\text{O}} \\ &= (0.6 \times 8) + (0.4 \times 68) \\ &= 32 \text{ Btu/lb}_m \end{aligned}$$

$$\therefore H^E = -110 - 32 = -142 \text{ Btu/lb}_m$$

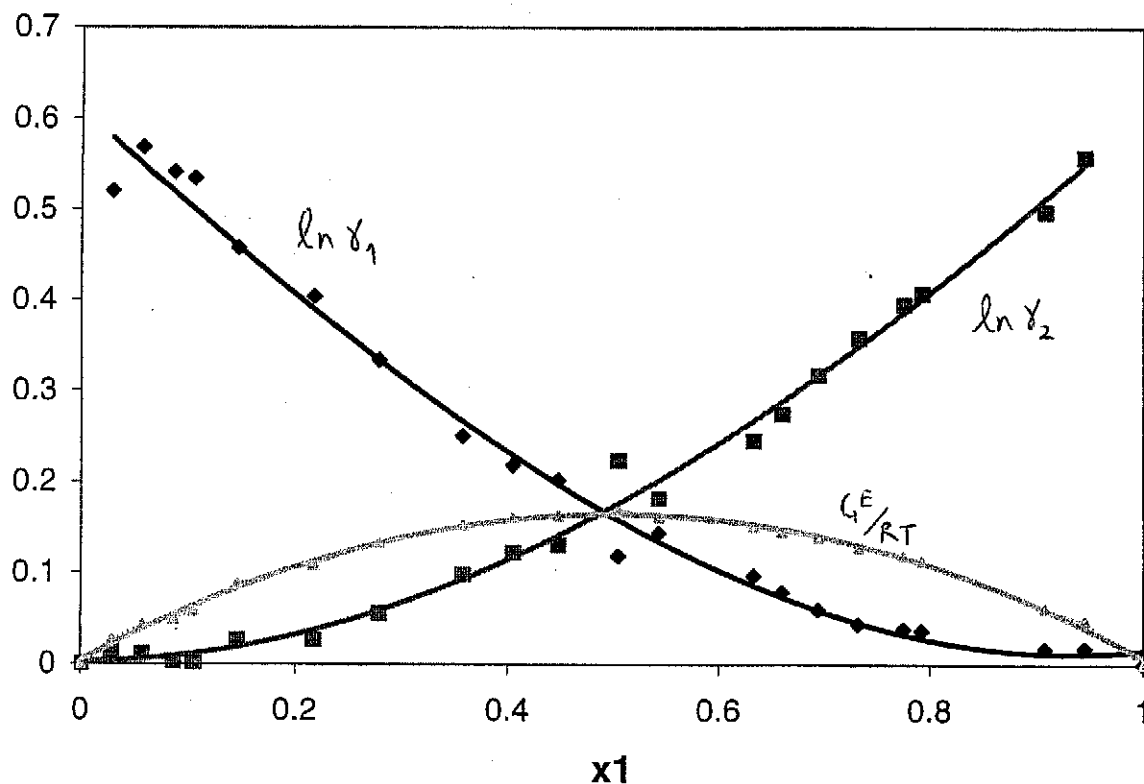
$$\boxed{H^E = -142 \text{ Btu/lb}_m}$$

# Problem 37



C)

| P (kPa) | $x_1$  | $y_1$  | $\gamma_1$ | $\gamma_2$ | $\ln \gamma_1$ | $\ln \gamma_2$ | $G^E/RT$ |
|---------|--------|--------|------------|------------|----------------|----------------|----------|
| 68.728  | 0      | 0      | #DIV/0!    | 1          | #DIV/0!        | 0              | 0        |
| 72.278  | 0.0287 | 0.0647 | 1.681791   | 1.012675   | 0.519859       | 0.012595       | 0.027154 |
| 75.279  | 0.057  | 0.1295 | 1.765274   | 1.011107   | 0.568306       | 0.011046       | 0.04281  |
| 77.254  | 0.0858 | 0.1848 | 1.71743    | 1.002329   | 0.540829       | 0.002326       | 0.04853  |
| 78.951  | 0.1046 | 0.219  | 1.706136   | 1.001977   | 0.534231       | 0.001975       | 0.057649 |
| 82.528  | 0.1452 | 0.2694 | 1.580432   | 1.02632    | 0.457698       | 0.02598        | 0.088665 |
| 86.762  | 0.2173 | 0.3633 | 1.497196   | 1.026917   | 0.403594       | 0.026561       | 0.10849  |
| 90.088  | 0.2787 | 0.4184 | 1.395935   | 1.056919   | 0.333564       | 0.055358       | 0.132894 |
| 93.206  | 0.3579 | 0.4779 | 1.284584   | 1.10271    | 0.250435       | 0.09777        | 0.152409 |
| 95.017  | 0.405  | 0.5135 | 1.243455   | 1.130403   | 0.217894       | 0.122575       | 0.161179 |
| 96.365  | 0.448  | 0.5512 | 1.223754   | 1.139986   | 0.201923       | 0.131016       | 0.162782 |
| 97.646  | 0.5052 | 0.5644 | 1.125956   | 1.250774   | 0.118633       | 0.223763       | 0.170651 |
| 98.462  | 0.5432 | 0.6174 | 1.155098   | 1.199924   | 0.144186       | 0.182258       | 0.161577 |
| 99.811  | 0.6332 | 0.6772 | 1.101788   | 1.278053   | 0.096934       | 0.245338       | 0.151369 |
| 99.95   | 0.6605 | 0.6926 | 1.081772   | 1.31678    | 0.078601       | 0.275189       | 0.145343 |
| 100.278 | 0.6945 | 0.7124 | 1.061697   | 1.373566   | 0.059869       | 0.31741        | 0.138548 |
| 100.467 | 0.7327 | 0.7383 | 1.044897   | 1.431181   | 0.043919       | 0.3585         | 0.128006 |
| 100.999 | 0.7752 | 0.7729 | 1.03937    | 1.484582   | 0.038615       | 0.395133       | 0.11876  |
| 101.059 | 0.7922 | 0.7876 | 1.037025   | 1.50297    | 0.036356       | 0.407443       | 0.113468 |
| 99.877  | 0.908  | 0.8959 | 1.017144   | 1.644352   | 0.016999       | 0.497346       | 0.061191 |
| 99.799  | 0.9448 | 0.9336 | 1.017866   | 1.746713   | 0.017708       | 0.557736       | 0.047518 |
| 96.885  | 1      | 1      | 1          | #DIV/0!    | 0              | #DIV/0!        | 0        |



d) Margules eqn.  $\ln \gamma_1 = x_2^2 [A_{12} + 2(A_{21} - A_{12})x_1]$  ,  $\ln \gamma_2 = x_1^2 [A_{21} + 2(A_{12} - A_{21})x_2]$

$$\frac{G^E}{x_1 x_2 RT} = A_{21} x_1 + A_{12} x_2$$

$$= \ln \gamma_2^\infty x_1 + \ln \gamma_1^\infty x_2$$

$\therefore$  Plot  $\frac{G^E}{x_1 x_2 RT}$  vs.  $x_1$  and get  $\ln \gamma_2^\infty$  as  $x_1 = 1$  intercept  
 $\ln \gamma_1^\infty$  as  $x_1 = 0$  intercept

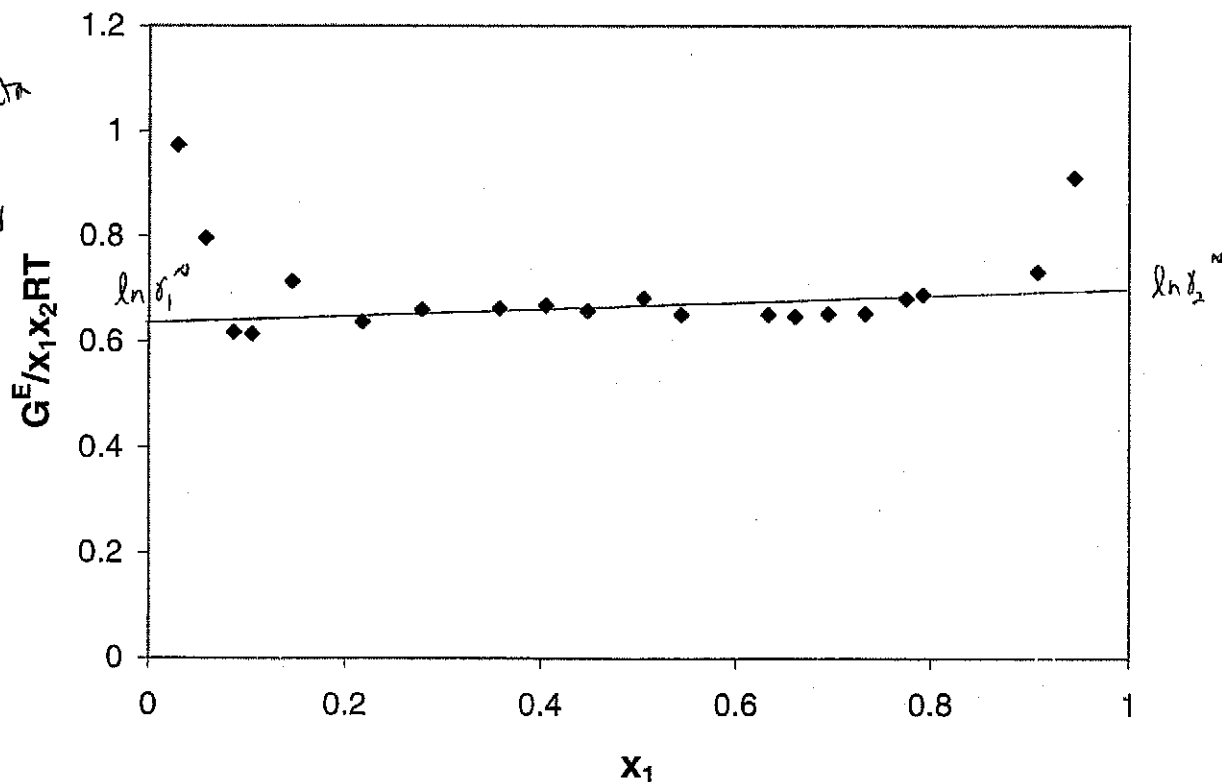
From best fit line

$$\ln \gamma_1^\infty \approx 0.63$$

$$\ln \gamma_2^\infty = 0.71$$

$$\frac{G^E}{RT} = (0.71 x_1 + 0.63 x_2) x_1 x_2$$

From data

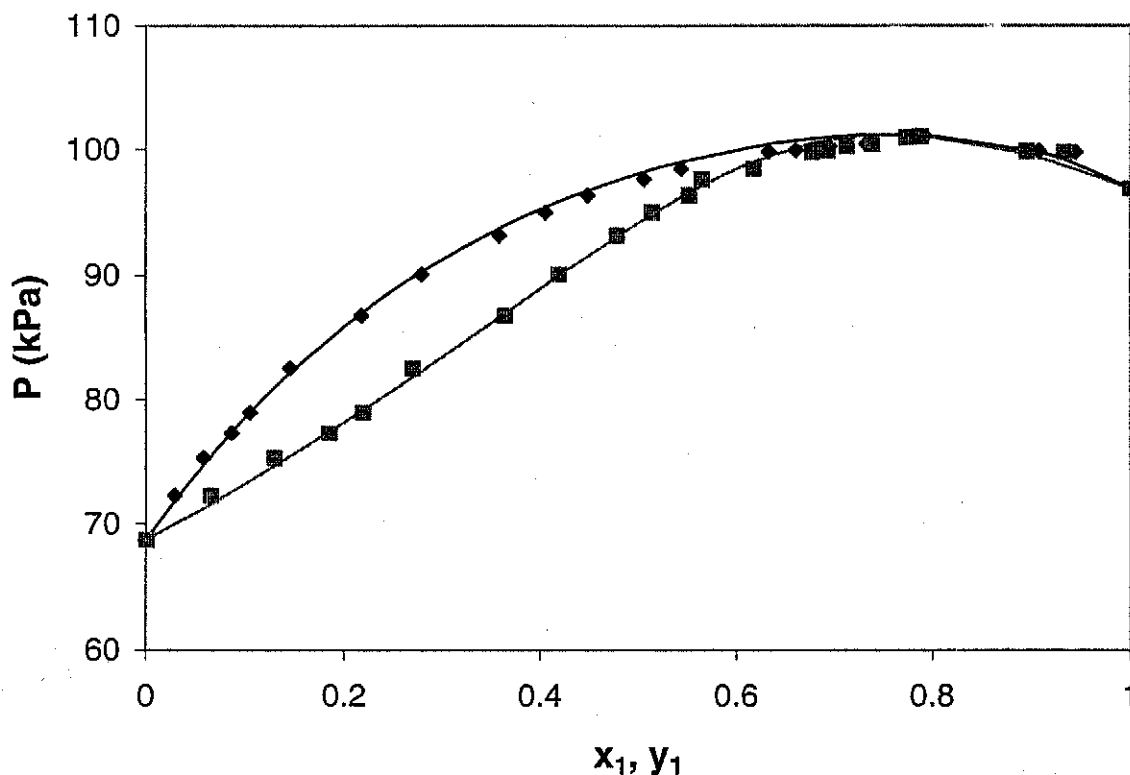


From Margules

| $x_1$  | $G^E/RT$ | $\ln \gamma_1$ | $\ln \gamma_2$ | $P_{\text{Margules}}$ | $y_1$    |
|--------|----------|----------------|----------------|-----------------------|----------|
| 0      | 0        | 0.63           | 0              | 68.728                | 0        |
| 0.0287 | 0.017626 | 0.598689       | 0.000457       | 71.84595              | 0.070428 |
| 0.057  | 0.034108 | 0.568337       | 0.001817       | 74.67727              | 0.130547 |
| 0.0858 | 0.049955 | 0.538003       | 0.00415        | 77.32868              | 0.184101 |
| 0.1046 | 0.059789 | 0.518515       | 0.006201       | 78.94248              | 0.215608 |
| 0.1452 | 0.079635 | 0.477306       | 0.012085       | 82.13629              | 0.276045 |
| 0.2173 | 0.110108 | 0.40725        | 0.027612       | 86.93554              | 0.363903 |
| 0.2787 | 0.131129 | 0.350972       | 0.046184       | 90.27144              | 0.424882 |
| 0.3579 | 0.151359 | 0.283354       | 0.077786       | 93.73384              | 0.491112 |
| 0.405  | 0.159622 | 0.245977       | 0.100843       | 95.41286              | 0.525934 |
| 0.448  | 0.16466  | 0.213805       | 0.124774       | 96.73077              | 0.555679 |
| 0.5052 | 0.167586 | 0.174031       | 0.161005       | 98.19789              | 0.593196 |
| 0.5432 | 0.167107 | 0.149595       | 0.187931       | 99.00607              | 0.617338 |
| 0.6332 | 0.158088 | 0.098392       | 0.261138       | 100.4228              | 0.674057 |
| 0.6605 | 0.15312  | 0.084795       | 0.286047       | 100.7155              | 0.691606 |
| 0.6945 | 0.145455 | 0.069169       | 0.318878       | 100.9878              | 0.714002 |
| 0.7327 | 0.134866 | 0.053389       | 0.358203       | 101.165               | 0.740183 |
| 0.7752 | 0.120594 | 0.038105       | 0.405049       | 101.1878              | 0.771065 |
| 0.7922 | 0.114143 | 0.032677       | 0.424717       | 101.1407              | 0.784074 |
| 0.908  | 0.058696 | 0.006562       | 0.573233       | 99.76766              | 0.88757  |
| 0.9448 | 0.036798 | 0.00238        | 0.625896       | 98.84916              | 0.928233 |
| 1      | 0        | 0              | 0.71           | 96.885                | 1        |

$$P = \gamma_1 x_1 P_1^{\text{sat}} + \gamma_2 x_2 P_2^{\text{sat}}$$

$$\text{then } y_1 = \frac{\gamma_1 x_1 P_1^{\text{sat}}}{P}$$



c)

van Laar eqn.

$$\ln \gamma_1 = A'_{12} \left( 1 + \frac{A'_{12} x_1}{A'_{21} x_2} \right)^{-2}$$

$$\ln \gamma_2 = A'_{21} \left( 1 + \frac{A'_{21} x_2}{A'_{12} x_1} \right)^{-2}$$

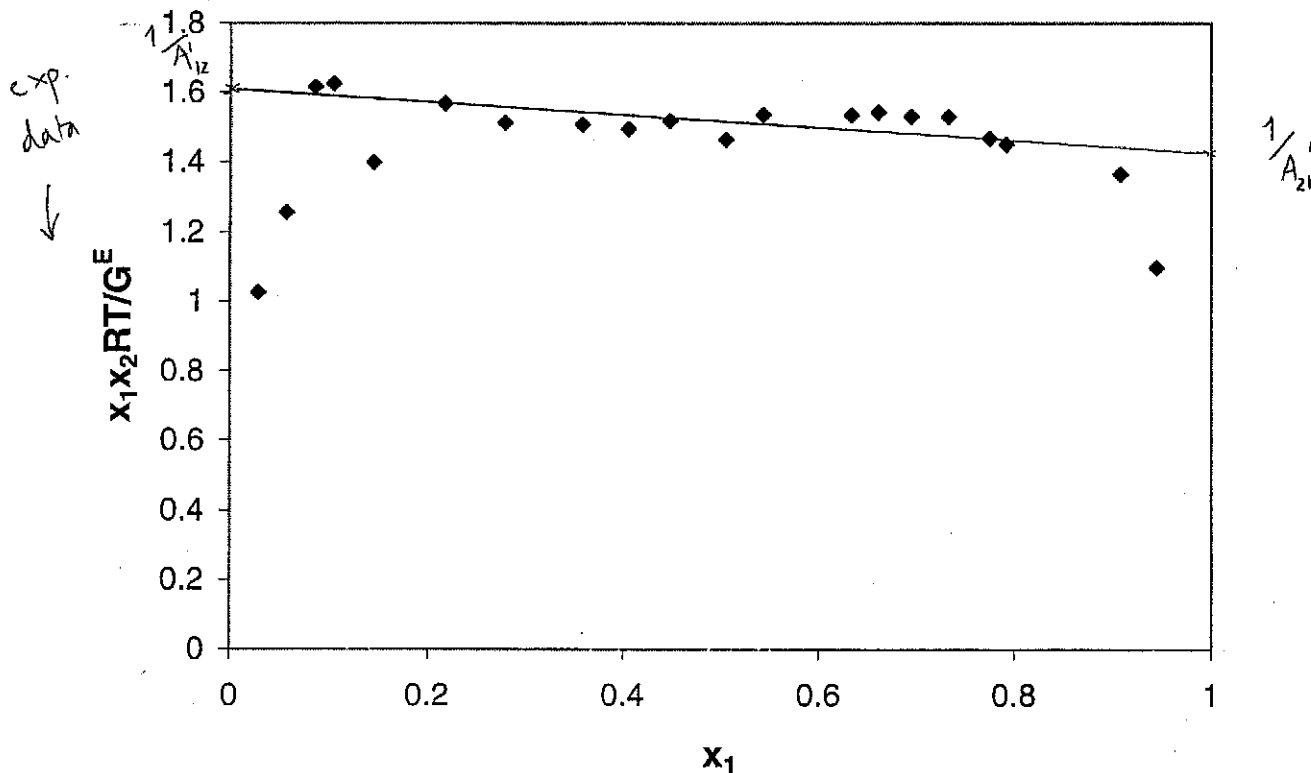
| $x_1$  | $G^E/RT$ | $x_1 x_2 RT/G^E$ |
|--------|----------|------------------|
| 0      | 0        | #DIV/0!          |
| 0.0287 | 0.027154 | 1.026618         |
| 0.057  | 0.04281  | 1.255576         |
| 0.0858 | 0.04853  | 1.616297         |
| 0.1046 | 0.057649 | 1.624632         |
| 0.1452 | 0.088665 | 1.399841         |
| 0.2173 | 0.10849  | 1.567703         |
| 0.2787 | 0.132894 | 1.51268          |
| 0.3579 | 0.152409 | 1.507833         |
| 0.405  | 0.161179 | 1.495077         |
| 0.448  | 0.162782 | 1.519184         |
| 0.5052 | 0.170651 | 1.464819         |
| 0.5432 | 0.161577 | 1.535698         |
| 0.6332 | 0.151369 | 1.534386         |
| 0.6605 | 0.145343 | 1.542835         |
| 0.6945 | 0.138548 | 1.531382         |
| 0.7327 | 0.128006 | 1.530011         |
| 0.7752 | 0.11876  | 1.467372         |
| 0.7922 | 0.113468 | 1.450797         |
| 0.908  | 0.061191 | 1.365167         |
| 0.9448 | 0.047518 | 1.097546         |
| 1      | 0        | #DIV/0!          |

$$\frac{x_1 x_2}{G^E/RT} = \frac{x_1}{A'_{21}} + \frac{x_2}{A'_{12}} \quad \therefore \text{Plot } \frac{x_1 x_2}{G^E/RT} \text{ vs. } x_1$$

intercept at  $x_1 = 1$  gives  $1/A'_{21}$   
 $x_1 = 0$  gives  $1/A'_{12}$

$$\therefore A'_{12} \approx \frac{1}{1.6} = 0.625$$

$$A'_{21} \approx \frac{1}{1.4} = 0.714$$

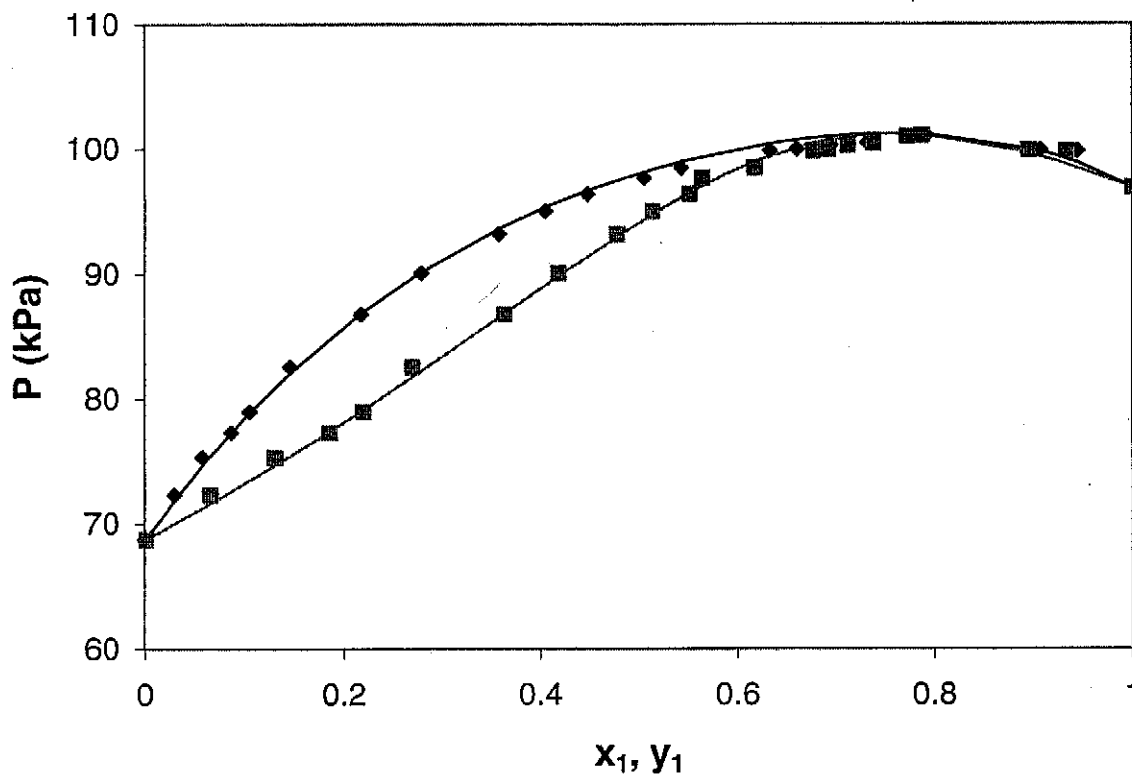


from van Laar

| $x_1$  | $\ln \gamma_1$ | $\ln \gamma_2$ | $P_{\text{van Laar}}$ | $y_1$    |
|--------|----------------|----------------|-----------------------|----------|
| 0      | 0.625          | 0              | 68.728                | 0        |
| 0.0287 | 0.593881       | 0.000454       | 71.82149              | 0.070114 |
| 0.057  | 0.563763       | 0.001803       | 74.63191              | 0.130031 |
| 0.0858 | 0.533706       | 0.004115       | 77.26543              | 0.183461 |
| 0.1046 | 0.514415       | 0.006145       | 78.8694               | 0.214925 |
| 0.1452 | 0.473668       | 0.011964       | 82.0467               | 0.275343 |
| 0.2173 | 0.404503       | 0.027292       | 86.83105              | 0.363341 |
| 0.2787 | 0.348998       | 0.045609       | 90.16592              | 0.42454  |
| 0.3579 | 0.28231        | 0.076776       | 93.63766              | 0.491104 |
| 0.405  | 0.245419       | 0.099533       | 95.3257               | 0.526121 |
| 0.448  | 0.213634       | 0.123177       | 96.65301              | 0.556031 |
| 0.5052 | 0.174275       | 0.159032       | 98.13336              | 0.593731 |
| 0.5432 | 0.150048       | 0.185729       | 98.95038              | 0.617965 |
| 0.6332 | 0.099118       | 0.258557       | 100.3875              | 0.674783 |
| 0.6605 | 0.085544       | 0.283423       | 100.6863              | 0.692325 |
| 0.6945 | 0.069912       | 0.316268       | 100.9661              | 0.714686 |
| 0.7327 | 0.054084       | 0.355715       | 101.1517              | 0.740794 |
| 0.7752 | 0.038703       | 0.402862       | 101.1838              | 0.771556 |
| 0.7922 | 0.033226       | 0.422706       | 101.1404              | 0.784507 |
| 0.908  | 0.006726       | 0.573541       | 99.78569              | 0.887555 |
| 0.9448 | 0.002447       | 0.627447       | 98.86628              | 0.928134 |
| 1      | 0              | 0.714          | 96.885                | 1        |

$$P = \gamma_1 x_1 P_1^{\text{sat}} + \gamma_2 x_2 P_2^{\text{sat}}$$

$$y_1 = \frac{\gamma_1 x_1 P_1^{\text{sat}}}{P}$$





f) you can read this of the P-x-y diagram or calculate them using

$$y_1 = \frac{\gamma_1 x_1 P_1^{\text{sat}}}{P} = \frac{\gamma_1 x_1 P_1^{\text{sat}}}{\gamma_1 x_1 P_1^{\text{sat}} + \gamma_2 x_2 P_2^{\text{sat}}}$$

and using either Margules or van Laar to get

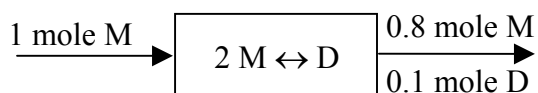
$\gamma_1$  and  $\gamma_2$  as a function of  $x_1$  and  $x_2$

Margules  $x_1 = 0.759$

van Laar  $x_1 = 0.761$

**38)**

Using 1 mole of M feed as a basis:



We know the final composition because we are told 20% of M is converted to D, i.e. 0.2 mole of M is consumed and 0.1 mole of D is produced (by stoichiometry).

Reaction equilibrium equation:

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = K = \prod_i \left(\frac{\hat{f}_i}{f_i^{\circ}}\right)^{v_i}$$

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = \prod_i (\gamma_i x_i)^{v_i} \quad (\text{for liquids at low to moderate } P)$$

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = (\gamma_M x_{M_i})^{v_M} (\gamma_D x_{D_i})^{v_D} = \frac{(\gamma_D x_{D_i})^1}{(\gamma_M x_{M_i})^2}$$

According to the reaction, the stoichiometric coef.:  $v_M = -2$  (reactant) and  $v_D = 1$  (product).

We are given  $\Delta G_{\text{rxn}}^{\circ}$  at temperature of interest (298 K). What about the other side of the eqn?

$$x_M = n_M/n_{\text{total}} = 0.8 \text{ mol}/(0.8 \text{ mol}+0.1 \text{ mol}) = 0.889. \quad x_D = 1 - x_M = 0.111.$$

$$G^E/RT = A x_M x_D$$

$$\rightarrow \ln \gamma_M = A x_D^2 \quad \text{and} \quad \ln \gamma_D = A x_M^2 \quad (\text{see pset J if you don't know why this is so})$$

$$\rightarrow \gamma_M = \exp(A x_D^2) \quad \gamma_D = \exp(A x_M^2)$$

Putting everything together:

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = \frac{(\gamma_D x_{D_i})}{(\gamma_M x_{M_i})^2} = \frac{\exp(A x_M^2) x_D}{(\exp(A x_D^2) x_M)^2}$$

$$\exp\left(-\frac{-1000 \text{ J/mol}}{(8.314 \text{ J/mol K})(298 \text{ K})}\right) = \frac{\exp(A \cdot 0.889^2) 0.111}{(\exp(A \cdot 0.111^2) 0.889)^2}$$

The only unknown in the equation above is A. We can re-arrange the equation to solve it

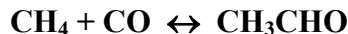
$$A = \mathbf{3.09 \text{ (ans)}}$$

39)

Here it is... The last homework problem of them all...

Unfortunately (or perhaps rather suitably) it is a rather long one. However, let's see if we can break it down to several parts that are easier to understand.

We have the reaction:



at 260°C and 100 bar. Gases are non-ideal but form ideal solution.

To save time, let's abbreviate:



Information given: 1 mole of CO enters with 1 mole of Me and 4 moles of N<sub>2</sub>.

a) Final compositions of reaction mixture as a function of  $\varepsilon$ .

$$y_i = \frac{n_i}{n} = \frac{n_{i,0} + v_i \varepsilon}{n_0 + v \varepsilon} \quad (\text{derived in lecture})$$

$y_i$  = mole fraction of component  $i$ ;  $v_i$  = stoichiometric coef. of component  $i$ ;

$n_{i,0}$  = initial number of moles of component  $i$ ;  $n_0$  = initial total number of moles =  $\sum n_{i,0}$ ;  $v = \sum v_i$

Making a table like the following usually helps:

| Component      | $n_{i,0}$<br>(moles) | $v_i$<br>(from rxn) | $n_i = n_{i,0} + v_i \varepsilon$<br>(moles) | $y_i = n_i/n$                           |
|----------------|----------------------|---------------------|--|---|
| Me             | 1                    | -1                  | $1 - \varepsilon$                            | $(1 - \varepsilon) / (6 - \varepsilon)$ |
| CO             | 1                    | -1                  | $1 - \varepsilon$                            | $(1 - \varepsilon) / (6 - \varepsilon)$ |
| Ac             | 0                    | 1                   | $\varepsilon$                                | $\varepsilon / (6 - \varepsilon)$       |
| N <sub>2</sub> | 4                    | 0                   | 4  | $4 / (6 - \varepsilon)$                 |
| Total          | $n_0 = 6$            | $v = -1$            | $n = 6 - \varepsilon$                        | 1                                       |

The answers for this part are in the rightmost column of the above table

b)

For reaction equilibrium: 
$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = K = \prod_i \left(\frac{\hat{f}_i}{f_i^{\circ}}\right)^{v_i}$$

Most problems require us to 'fill in the blanks' for both sides of the equations. Let's start with the right hand side.

For all gas reaction, we derived in class, for standard reference of pure species at 1 atm:

$$\prod_i \left(\frac{\hat{f}_i}{f_i^{\circ}}\right)^{v_i} = \prod_i \left(\frac{y_i \phi_i P}{1 \text{ bar}}\right)^{v_i} \quad \text{This is a general expression for gas-phase reaction.}$$

The problem states that the gases are not ideal ( $\phi_i$  is not 1); but the gases form ideal solution.

This allows us to say that  $\hat{\phi}_i = \phi_i$ . This is because for ideal solution of gases,  $\hat{f}_i = y_i f_i = y_i \phi_i P$  (The fugacity of comp.  $i$  in the mixture is fugacity of pure  $i$  multiplied by its fraction in the mixture). So rewriting the expression above:

$$\prod_i \left( \frac{\hat{f}_i}{f_i^o} \right)^{v_i} = \prod_i \left( \frac{y_i \phi_i P}{1 \text{ bar}} \right)^{v_i} \quad \text{expression for ideal solution of non-ideal gases}$$

We have the  $y_i$  and  $v_i$  for all the species (done in part a).  $P = 100 \text{ bar}$  (given).  $T = 260^\circ\text{C} = 533 \text{ K}$ . What about the  $\phi_i$ 's?  $\phi$  of pure species; we can get through generalized correlation.

| Component | Pc<br>(bar) | Tc<br>(K) | $\omega$ | P <sub>r</sub><br>(P/P <sub>c</sub> ) | T <sub>r</sub><br>(T/T <sub>c</sub> ) | ( $\phi^o$ ) | ( $\phi^1$ ) | $\phi$<br>$= (\phi^o)(\phi^1)^\omega$ |
|-----------|-------------|-----------|----------|---------------------------------------|---------------------------------------|--------------|--------------|---------------------------------------|
| Me        | 45.99       | 190.6     | 0.012    | 2.17                                  | 2.80                                  | 1.01         | 1.14         | 1.01                                  |
| CO        | 34.99       | 132.9     | 0.048    | 2.86                                  | 4.01                                  | 1.04         | 1.13         | 1.04                                  |
| Ac        | 55.50       | 466.0     | 0.291    | 1.80                                  | 1.14                                  | 0.63         | 1.11         | 0.65                                  |

from Appendix B

P = 100 bar  
T = 533 K

App. E  
(with interpolations)

We see that even at this high pressure, but because the temperature is also high, we have almost ideal behavior for methane and CO. The acetaldehyde cannot be assumed to be ideal however, since  $\phi_{Ac} = 0.65$ .

Now we can write down all the knowns for the right hand side of our equilibrium equation:

$$\prod_i \left( \frac{\hat{f}_i}{f_i^o} \right)^{v_i} = \prod_i \left( \frac{y_i \phi_i P}{1 \text{ bar}} \right)^{v_i} = \left( \frac{y_{Me} \phi_{Me} P}{1 \text{ bar}} \right)^{v_{Me}} \left( \frac{y_{CO} \phi_{CO} P}{1 \text{ bar}} \right)^{v_{CO}} \left( \frac{y_{Ac} \phi_{Ac} P}{1 \text{ bar}} \right)^{v_{Ac}}$$

with  $v_{Me} = -1$ ,  $v_{CO} = -1$ ,  $v_{Ac} = 1$  (and re-arranging):

$$\prod_i \left( \frac{\hat{f}_i}{f_i^o} \right)^{v_i} = \left( \frac{y_{Ac} \phi_{Ac}}{(y_{Me} \phi_{Me})(y_{CO} \phi_{CO})} \right) \left( \frac{1 \text{ bar}}{P} \right)$$

Writing in expressions for  $y_i$  from part a, and also  $\phi_i$  values from the table above:

$$\prod_i \left( \frac{\hat{f}_i}{f_i^o} \right)^{v_i} = \left( \frac{\frac{\varepsilon}{6-\varepsilon}(0.65)}{\frac{1-\varepsilon}{6-\varepsilon}(1.01) \frac{1-\varepsilon}{6-\varepsilon}(1.04)} \right) \left( \frac{1 \text{ bar}}{P} \right) = \left( \frac{(0.65)\varepsilon(6-\varepsilon)}{(1.01)(1.04)(1-\varepsilon)^2} \right) \left( \frac{1 \text{ bar}}{P} \right)$$

$$\prod_i \left( \frac{\hat{f}_i}{f_i^o} \right)^{v_i} = \frac{0.618 \varepsilon (6-\varepsilon)}{(1-\varepsilon)^2} \left( \frac{1 \text{ bar}}{P} \right) \quad (*)$$

Why isn't  $N_2$  included in the expression above? We can include it, but  $v_{N_2} = 0$ . So the term drops out anyway.

Not so bad now, was it? Okay... now for the other part of the equation.

Reminder:  $\exp\left(-\frac{\Delta G_{rxn}^o(T)}{RT}\right) = K = \prod_i \left( \frac{\hat{f}_i}{f_i^o} \right)^{v_i}$

We know how to get  $\Delta G^\circ_{\text{rxn}}$  at 298 K (from data in Appendix C). To get  $\Delta G^\circ_{\text{rxn}}$  at a different temperature, we derived in class (and also in the textbook equation 13.18):

$$\frac{\Delta G^\circ(T)}{RT} = \frac{\Delta G^\circ(T_o) - \Delta H^\circ(T_o)}{RT_o} + \frac{\Delta H^\circ(T_o)}{RT} + \frac{1}{T} \int_{T_o}^T \frac{\Delta C_p^\circ}{R} dT - \int_{T_o}^T \frac{\Delta C_p^\circ}{R} \frac{dT}{T}$$

(where the  $\Delta$ 's means  $\Delta_{\text{rxn}}$ )  $T_o = T$  reference = 298 K for our case.  $\Delta C_p = \sum \nu_i C_{p,i}$ .

From appendix C:

| $C_p^{\text{ig}}/R = A + BT + CT^2 + DT^{-2}$ |   |  |                        |                                       |  |                       |
|---|---|--|------------------------|---------------------------------------|--|-----------------------|
| Component                                     | $\Delta H^\circ_f$ at $T_o=298$ K<br>(J/mol)  | $\Delta G^\circ_f$ at $T_o=298$ K<br>(J/mol)   | A                      | B                                     | C                                      | D                     |
| Me  | -74,520                                       | -50,460  | 1.702                  | $9.081 \cdot 10^{-3}$                 | $-2.164 \cdot 10^{-6}$                 | 0                     |
| CO  | -110,525                                      | -137,169                                       | 3.376                  | $0.557 \cdot 10^{-3}$                 | 0                                      | -3,100                |
| Ac  | -166,190                                      | -128,860                                       | 1.693                  | $17.978 \cdot 10^{-3}$                | $-6.158 \cdot 10^{-6}$                 | 0                     |
| $\Delta$                                      | $\Delta H^\circ_{\text{rxn}}(T_o)$<br>=18,855 | $\Delta G^\circ_{\text{rxn}}(T_o)$<br>= 58,769 | $\Delta A =$<br>-3.385 | $\Delta B =$<br>$8.340 \cdot 10^{-3}$ | $\Delta C =$<br>$-3.994 \cdot 10^{-6}$ | $\Delta D =$<br>3,100 |

Explanation on the  $\Delta$  row: Just a shorthand where  $\Delta M = \sum \nu_i M_i$  where M is variable of interest.

e.g.  $\Delta G^\circ_{\text{rxn}} = \Delta(\Delta G^\circ_f) = \sum \nu_i \Delta G^\circ_{f,i}$ .

$$\frac{\Delta G^\circ(T)}{RT} = \frac{\Delta G^\circ(T_o) - \Delta H^\circ(T_o)}{RT_o} + \frac{\Delta H^\circ(T_o)}{RT} + \frac{1}{T} \int_{T_o}^T \frac{\Delta C_p^\circ}{R} dT - \int_{T_o}^T \frac{\Delta C_p^\circ}{R} \frac{dT}{T}$$

$$\frac{\Delta G^\circ(T)}{RT} = \frac{\Delta G^\circ(T_o) - \Delta H^\circ(T_o)}{RT_o} + \frac{\Delta H^\circ(T_o)}{RT} + \frac{1}{T} \int_{T_o}^T (\Delta A + \Delta B \cdot T + \Delta C \cdot T^2 + \Delta D \cdot T^{-2}) dT - \int_{T_o}^T (\Delta A + \Delta B \cdot T + \Delta C \cdot T^2 + \Delta D \cdot T^{-2}) \frac{dT}{T}$$

$$\frac{\Delta G^\circ(T)}{RT} = \frac{\Delta G^\circ(T_o) - \Delta H^\circ(T_o)}{RT_o} + \frac{\Delta H^\circ(T_o)}{RT} + \frac{1}{T} \left[ \Delta A(T - T_o) + \frac{\Delta B}{2}(T^2 - T_o^2) + \frac{\Delta C}{3}(T^3 - T_o^3) - \Delta D \left( \frac{1}{T} - \frac{1}{T_o} \right) \right] - \left[ \Delta A \ln \left( \frac{T}{T_o} \right) + \Delta B(T - T_o) + \frac{\Delta C}{2}(T^2 - T_o^2) - 2\Delta D \left( \frac{1}{T^2} - \frac{1}{T_o^2} \right) \right]$$

Long, but doable. We know all the values for the above equation.  $T=298$  K and  $T_o = 533$  K.

If we do the calculation, we get:

$$\frac{\Delta G^\circ(533 \text{ K})}{RT} = 20.47 \quad (**)$$

Having calculated this, we can return to our equilibrium equation:

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = K = \prod_i \left(\frac{\hat{f}_i}{f_i^{\circ}}\right)^{v_i}$$

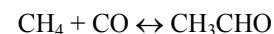
Putting together what we calculated before, (\*) and (\*\*) above:

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = K = \frac{0.618 \varepsilon (6 - \varepsilon) \left(\frac{1 \text{ bar}}{P}\right)}{(1 - \varepsilon)^2}$$

$$\exp(-20.47) = 1.29 \cdot 10^{-9} = \frac{0.618 \varepsilon (6 - \varepsilon) \left(\frac{1 \text{ bar}}{100 \text{ bar}}\right)}{(1 - \varepsilon)^2}$$

This is a quadratic equation in  $\varepsilon$  which we can solve:

$\varepsilon = 3.50 \cdot 10^{-8}$  ... practically  $\approx 0$ . What does this mean? Why?



This means our reaction does not proceed very far before it reaches equilibrium. The equilibrium constant  $K = 1.29 \cdot 10^{-9}$  is quite small, suggesting reaction to the left (producing methane and CO) is favored over reaction producing acetaldehyde.

We could not have known this before we calculated the  $K$  and  $\varepsilon$ . So it's not really pointless. Besides, the point is to make sure you know how to do this type of calculation. Anyway, for the mole fraction, we can calculate using  $\varepsilon$  (using expressions we got in part a):

**$y_{\text{methane}} = 0.167$ ;  $y_{\text{CO}} = 0.167$ ;  $y_{\text{Ac}} = 6 \cdot 10^{-8} \approx 0$ ;  $y_{\text{N}_2} = 0.666$ .**

Recapping:

$$\exp\left(-\frac{\Delta G_{\text{rxn}}^{\circ}(T)}{RT}\right) = K = \prod_i \left(\frac{\hat{f}_i}{f_i^{\circ}}\right)^{v_i}$$

- 1) We write the reaction equilibrium equation:
- 2) We figure out both sides of the equation, writing in terms of what we know (given or data from back of the book) and what we don't know (what we try to solve).
- 3) If the temperature is not 298 K and the value for  $\Delta G_{\text{rxn}}^{\circ}$  or  $K$  are not given,  $\Delta G_{\text{rxn}}^{\circ}(T)$  must be calculated using the  $C_p$ 's like above.
- 4) For the right hand side, we express the fugacities depending on the phase of the species (liquid, gas, solid, solution).
- 5) The expression of fugacities will usually involve some molar fraction ( $y_i$ ,  $x_i$ ). Usually it's easier to write down the molar fraction in terms of one variable (for one reaction), namely  $\varepsilon$ . This requires some stoichiometry and, sometimes, material balance.
- 6) Finally, if we have reaction equilibrium, we equate the lefthand side and the righthand side, and solve for what we don't know.

(For the case of more than one reactions (say  $n$  reactions), we have  $n$  equilibrium equations. Also, we have  $n$  of  $\varepsilon$ :  $\varepsilon_{\text{rxn } 1}$ ,  $\varepsilon_{\text{rxn } 2}$ , ...  $\varepsilon_{\text{rxn } n}$ . We haven't practiced with this. We may do this to study for the final)