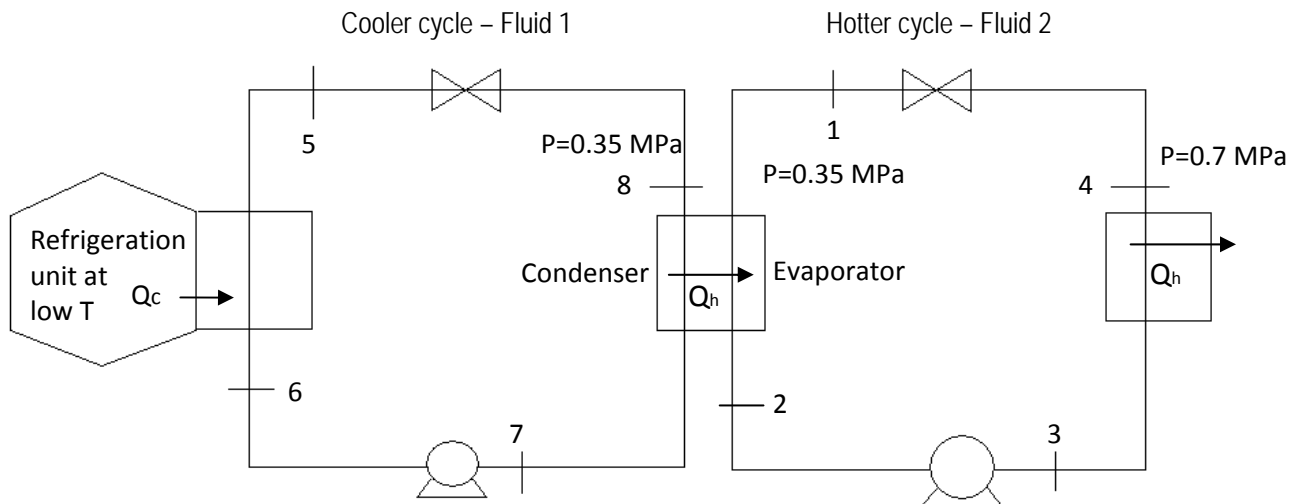


10.213 Chemical and Biological Thermodynamics

Solution to Problem set #3

Problem 3.42 (parts a-d only)

3.42



Properties of saturated R-134a from <http://webbook.nist.gov/chemistry/fluid>

Pressure (MPa)	Temperature (K)	h_L (kJ/mol)	h_v (kJ/mol)
0.12	250.84	17.412	39.295
0.35	278.18	21.099	40.967
0.70	299.86	24.181	42.159

Given: Flowrate in hotter cycle, $\dot{n}_2 = 0.5 \frac{\text{mol}}{\text{s}}$

- To determine the flow rate in the cooler cycle, we find the quantity that relates the two cycles, Q_h , or the amount of heat ejected from the cooler cycle to the hotter cycle. This heat is absorbed by the refrigerant 134a as it enters the evaporator at a flowrate of 0.5 mol/s. The amount of heat absorbed is the difference between the enthalpy of the fluid in states 1 and 2. The enthalpy of the fluid in state 1 is equal to the enthalpy of the fluid in state 4 because no work or heat has been added to the fluid as it is throttled.

$$\dot{Q}_H = \dot{n}_2(h_2 - h_1)$$

$$h_1 = h_4$$

$$\dot{Q}_H = \dot{n}_2(h_2 - h_4) \quad [0.5 \text{ point}]$$

In state 2, the fluid is a saturated vapor at the lower operating pressure, 0.35 MPa.

In state 4, the fluid is a saturated liquid at the higher operating pressure, 0.7 MPa.

$$h_2 = 40.967 \text{ kJ/mol} \quad [0.5 \text{ point}]$$

$$h_4 = 24.181 \text{ kJ/mol} \quad [0.5 \text{ point}]$$

$$\dot{Q}_H = 0.5 \frac{\text{mol}}{\text{s}} \left(40.967 \frac{\text{kJ}}{\text{mol}} - 24.181 \frac{\text{kJ}}{\text{mol}} \right)$$

$$\dot{Q}_H = 8.39 \text{ kW} \quad [0.5 \text{ point}]$$

\dot{Q}_H is also equal to the amount of heat loss by the fluid in the cooler cycle as it enters the condenser and changes from a superheated vapor (state 7) to a saturated liquid (state 8).

As the superheated fluid leaves the compressor, the fluid pressure has increased to the higher operating pressure. For an ideal case, this compression occurs isentropically, or $s_7 = s_6$. (Note that the fluid remains a vapor during the compression.) Using these criteria, the temperature and enthalpy of the fluid in state 7 is found from NIST tables:

Vapor phase data on Saturation Curve:

Temperature (K)	Pressure (MPa)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Entropy (J/mol*K)	Phase
250.84	0.12000	37.310	39.295= h_6	177.89 = s_6	vapor

Next we generate an isobaric properties table at 0.35 MPa to find the temperature at which the fluid entropy matches state 6 ($s_6 = 177.89$)

Isobaric Data for P = 0.35000 MPa

Temperature (K)	Pressure (MPa)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Entropy (J/mol*K)	Phase
283.00	0.35000	39.282	41.417	177.55	vapor
284.00	0.35000	39.364	41.510 = h_7	177.88 = s_7	vapor
285.00	0.35000	39.446	41.603	178.21	vapor

Fluid 1 exits the compressor at a pressure of 0.35 MPa and temperature of 284 K.

$$\dot{Q}_H = \dot{n}_1(h_8 - h_7)$$

$$h_8 = 21.099 \text{ kJ / mol} \quad [0.5 \text{ point}]$$

$$h_7 = 41.510 \text{ kJ / mol} \quad [0.5 \text{ point}]$$

$$-8.39 \text{ kW} = \dot{n}_1 \left(21.099 \frac{\text{kJ}}{\text{mol}} - 41.510 \frac{\text{kJ}}{\text{mol}} \right)$$

(\dot{Q}_H is negative because heat is removed as the stream goes from a superheated to a saturated vapor.

$$\boxed{\dot{n}_1 = 0.411 \text{ mol / s}} \quad [0.5 \text{ point}]$$

- b. The rate of heat removal from the refrigeration unit is equal to the differences in enthalpy of the fluid between states 5 & 6.

$$\dot{Q}_C = \dot{n}_1(h_6 - h_5)$$

$$h_5 = h_8 \quad (\text{No work or heat added})$$

$$\dot{Q}_C = \dot{n}_1(h_6 - h_8) \quad [0.5 \text{ point}]$$

State 6: Fluid 1 is saturated vapor at 0.12 MPa.

State 8: Fluid 1 is a saturated liquid at 0.35 MPa.

$$h_6 = 39.295 \text{ kJ / mol} \quad [0.5 \text{ point}]$$

$$\dot{Q}_C = 0.411 \frac{\text{mol}}{\text{s}} \left(39.295 \frac{\text{kJ}}{\text{mol}} - 21.099 \frac{\text{kJ}}{\text{mol}} \right)$$

$$\boxed{\dot{Q}_C = 7.48 \text{ kW}} \quad [0.5 \text{ point}]$$

(\dot{Q}_C is positive because heat is transferred from the refrigeration unit to the fluid.)

- c. The amount of work performed on the fluid in each compressor is equal to the difference in enthalpy of the fluid as it enters and exits the compressor.

For the cooler cycle:

$$\dot{W}_{C1} = \dot{n}_1(h_7 - h_6)$$

We've already determined the enthalpy of the fluid in states 6 and state 7.

$$\dot{W}_{C1} = \dot{n}_1(h_7 - h_6) \quad [0.5 \text{ point}]$$

$$\dot{W}_{C1} = 0.411 \frac{\text{mol}}{\text{s}} \left(41.510 \frac{\text{kJ}}{\text{mol}} - 39.295 \frac{\text{kJ}}{\text{mol}} \right)$$

$$\boxed{\dot{W}_{C1} = 0.91 \text{ kW}} \quad [0.5 \text{ point}]$$

For the hotter cycle:

$$\dot{W}_{C2} = \dot{n}_2(h_3 - h_2) \quad [0.5 \text{ point}]$$

The same strategy used in part a.) is applied to determine the fluid properties in state 3.

The saturation properties of r134a at 0.35 MPa

Temperature (K)	Pressure (MPa)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Entropy (J/mol*K)	Phase
278.18	0.35000	38.884	40.967=h₂	175.95=s₂	vapor

Isobaric table of r134a at 0.7 MPa:

Temperature (K)	Pressure (MPa)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Entropy (J/mol*K)	Phase
301.50	0.70000	40.214	42.333	175.63	vapor
302.00	0.70000	40.260	42.386	175.80	vapor
302.50	0.70000	40.306	42.438=h₃	175.97=s₃	vapor

Fluid 2 exits the compressor as a superheated vapor at 0.7 MPa and 302.5 K. The work required for compression is:

$$\dot{W}_{c2} = \dot{n}_2 (h_3 - h_2)$$

$$h_3 = 42.438 \text{ kJ/mol} \quad [1 \text{ point}]$$

$$\dot{W}_{c2} = 0.5 \frac{\text{mol}}{\text{s}} (42.438 \frac{\text{kJ}}{\text{mol}} - 40.967 \frac{\text{kJ}}{\text{mol}})$$

$$\boxed{\dot{W}_{c2} = 0.74 \text{ kW}} \quad [0.5 \text{ point}]$$

- d. The COP measures the performance of the refrigeration cycle by comparing the amount of heat removed from the refrigeration unit to the total amount of work required. These quantities were calculated in parts b. and c., so the COP can directly be determined as:

$$COP = \frac{\dot{Q}_c}{\dot{W}_{c1} + \dot{W}_{c2}} \quad [1 \text{ point}]$$

$$COP = \frac{7.48 \text{ kW}}{0.91 \text{ kW} + 0.74 \text{ kW}}$$

$$\boxed{COP = 4.53} \quad [1 \text{ point}]$$