# Homework 2: Problem 3.26

February 22, 2008

A steam turbine accepts 4500 kg/hr of steam at 60 bar and 500 °C, and exhausts it at 10 bar. Heat transfers to the surroundings ($T_{surr}$ = 300 K) at 69.86 kW.

(a) What is the maximum power?

The maximum power occurs when the process is reversible. We assume that the system is at a steady state. Since there is no accumulation of mass, $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. We can neglect the changes in potential and kinetic energy, so the energy balance simplifies to:

$$\dot{m}\left(\hat{h}_{in} - \hat{h}_{out}\right) + \dot{Q} + \dot{W}_{rev} = 0$$

From the Gibbs Phase Rule, since we have one component and one phase, we need two intensive properties to define the system completely. For the inlet stream, those two properties are the temperature and pressure. At 60 bar and 500 °C, the entropy is $\hat{s}_{in} = 6.8802$ kJ/kg K, and the enthalpy is $\hat{h}_{in} = 3422.1$ kJ/kg. For the outlet stream, we are given only the pressure, so we need to calculate one more intensive property. To determine this property, we use the entropy balance. Since the system is reversible, the total change in entropy must be zero:

$$\begin{aligned} \Delta S_{univ} &= \Delta S_{sys} + \Delta S_{surr} \\ &= 0 \end{aligned}$$

From which it follows that $\Delta S_{sys} = -\Delta S_{surr}$. Since the system is not adiabatic, we cannot assume that $\Delta S_{surr} = 0$. Instead we note that the change in entropy for the surroundings is related to the heat transfer between the system and the surroundings: $\Delta S_{surr} = -Q/T_{surr}$. The minus sign is because the heat transfer to the surroundings is the negative of the heat transfer to the system. Putting it all together, we can solve for the entropy of the exit stream:

$$\begin{aligned}
\hat{s}_{out} &= \hat{s}_{in} - \Delta S_{sys} \\
&= \hat{s}_{in} - \frac{-\dot{Q}}{\dot{m} T_{surr}} \\
&= 6.8802 \text{ [ kJ/kg-K ]} - \frac{69.86 \text{ [ kW ]}}{4500 \text{ [ kg/hr ]} 300 \text{ [ K ]}} \\
&= 6.8802 \text{ [ kJ/kg-K ]} - 0.1863 \text{ [ kJ/kg-K ]} \\
&= 6.6939 \text{ [ kJ/kg-K ] [ + 2 point ]}
\end{aligned}$$

Check the signs. The surroundings are getting warmer, so it makes sense that $\Delta S_{surr} > 0$. In order for the total entropy to be zero, it follows that $\Delta S_{sys} < 0$, which it is. Now that we have two intensive properties for the outlet stream, we can calculate the other properties, namely $T_{out}$ and $\hat{h}_{out}$. Either by interpolation from the steam tables or from NIST, we find that:

$$\begin{aligned}
\hat{h}_{out,NIST} &= 2827.5 \text{ [ kJ/kg ]} \\
\hat{h}_{out,s.t.} &= 2827.9 \text{ [ kJ/kg ]}
\end{aligned}$$

Solving the energy balance for the power yields:

$$\begin{aligned}
\dot{W}_{rev} &= \dot{m}\left(\hat{h}_{out} - \hat{h}_{in}\right) - \dot{Q} \\
&= (4500 \text{ [ kg/hr ] })(2827.9 - 3422.1 \text{ [ kJ/kg ] }) - -69.86 \text{ [ kW ]} \\
&= -672.9 \text{ [ kW ] [ + 2 points ]}
\end{aligned}$$

The work should be negative, since hopefully we are extracting work from the turbine.

(b) In the same way that we determined the exit enthalpy, the exit temperature is:

$$\begin{aligned}
T_{out,NIST} &= 199.7 \text{ [ K ] [ + 1 point ]} \\
T_{out,s.t.} &= 200 \text{ [ K ]}
\end{aligned}$$

(c) If the isentropic efficiency is actually 66.5%, then the actual power produced is:

$$
\begin{aligned}
\dot{W} &= \eta_{turbine}\dot{W}_{rev} \\
&= -447.5 \text{ [ kW ] } \text{ [ + 1 points ]}
\end{aligned}
$$

(d) You would expect the exit temperature to increase. Return to the energy balance: Assuming that the heat loss term is constant, then the only way for the work to be decreased is if $\hat{h}_{out}$ is increased, which would require a warmer temperature. Thus, the inefficiency implies that less enthalpy is being converted to usable work. [ + 2 points ]

(e) To determine the final temperature, rearrange the energy balance to solve for the final enthalpy:

$$
\begin{aligned}
\hat{h}_{out} &= \hat{h}_{in} + \frac{\dot{Q}+\dot{W}}{\dot{m}} \\
&= 3422.1 \text{ [ kJ/kg ] } + \frac{-69.86 \text{ [ kW ] } - 447.5 \text{ [ kW ]}}{4500 \text{ [ kg/hr ]}} \\
&= 3008.2 \text{ kJ/kg}
\end{aligned}
$$

which, according to NIST, occurs at $T_{out} = 279.8°\text{C}$, or by interpolation at $T_{out} = 280.2°\text{C}$ [ + 2 points ]