Homework 9: Problem 3

May 8, 2008

We are asked to concentrate the fruity ester ethyl 2-methyl butyrate (butanoic acid, $C_7H_{14}O_2$), by sending the vapor from the flash tank to a distillation column. The mole fraction of the ester in the feed is $z_F = 10^{-7}$. The vapor stream from the column will be condensed into the essence, which must recover at least 90% of the ester and may contain no more than 1% of the water present in the feed. The relative volatility of the ester to water is 4.0 at 100 ° C.

To begin, combine the two constraints with a mass balance to solve for the mole fraction of the ester in the distillate x_D and the ratio of the distillate molar flow rate to the inlet molar flow rate, $\phi = D/F$:

$$\begin{aligned} 0.9 z_F F &= x_D D \\ 0.01\,(1 - z_F)\,F &= (1 - x_D)\,D \end{aligned}$$

From which it follows that:

$$\begin{aligned} \phi &= 0.9 z_F + 0.01\,(1 - z_F) \\ &= 0.01 \\ &\text{and} \\ x_D &= \frac{0.9 z_F}{\phi} \\ &= 9 \times 10^{-6} \end{aligned}$$

[+1 point]

From these values, we can do a similar mass balance on the bottom stream to find the mole fraction of the ester in the bottom, $x_B = (z_F - x_D\phi) / (1-\phi) = 1.010 \times 10^{-8}$. Next, we use the relative volatility information to calculate the vapor-liquid equilibrium curve:

$$\alpha = \frac{y_e/x_e}{y_w/x_w}$$

or

$$y_e = \frac{\alpha x_e}{1 + x_e(\alpha - 1)}$$

where e is for ester and w is for water.

Part (a)

We are asked to calculate the minimum ratio of steam required by the reboiler to the steam in the inlet stream. This is equivalent to asking for $V_{B,min}$, the minimum boil-up ratio.

To calculate the minimum boil-up ratio, we must first find the q-line. Recall that the variable q is related to the feed condition. We assume that the feed for this system is a saturated vapor, since it is the vapor stream from a flash tank. From Figure 7.8 on page 261 of Seader and Henley, a saturated vapor gives a q-value of 0, which gives a horizontal line for the q-line. Next, find the point where the q-line intersects the VLE curve (x_2,y_2), starting from the 45°-line at (z_F,z_F). Since the slope of the q-line is zero, it follows that $y_2 = z_F$. Rearranging the VLE curve equation, we get:

$$\begin{aligned} x_2 &= \frac{y_2}{\alpha - y_2(\alpha - 1)} \\ &= 2.5 \times 10^{-8} \end{aligned}$$

The curve for the stripping section is:

$$OCS = \frac{V_B + 1}{V_B} x - \frac{x_B}{V_B}$$

This curve must intersect the q-line. We need to find the point along the q-line that minimizes V_B. This point occurs where the q-line intersects the VLE curve:

$$
\begin{aligned}
V_B &= \frac{x - x_B}{y - x} \\
V_{B,min} &= \frac{\frac{z_F}{\alpha - z_F(\alpha-1)} - x_B}{z_F - \frac{z_F}{\alpha - z_F(\alpha-1)}} \\
&= 0.1987
\end{aligned}
$$

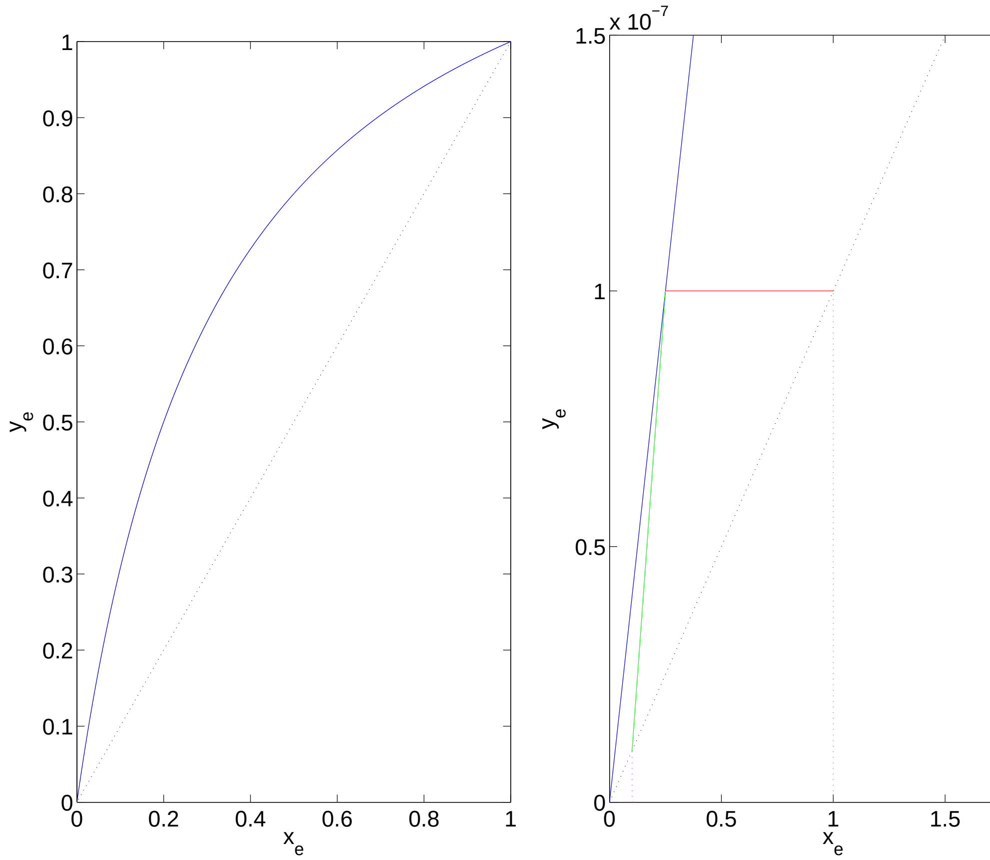
[+ 2 points]

The amount of steam required to be produced by the reboiler is equal to $(1 - x_B) V_{B,min} B$. The ratio of this amount to the inlet steam is:

$$
\begin{aligned}
\frac{\text{minimum amount of steam produced}}{\text{feed stream steam}} &= \frac{(1 - x_B) V_{B,min} B}{(1 - z_F) F} \\
&= \underbrace{\frac{1 - x_B}{1 - z_F}}_{\approx 1} V_{B,min} (1 - \phi) \\
&= 0.1987 \times 0.99 \\
&= 0.1967
\end{aligned}
$$

[+ 3 points]

The VLE curve and the minimum operating line are show in the figure below: the VLE curve is in blue; the 45 $^\circ$ line is the dotted black line; the minimum operating curve is the green line; the q-line is in red. All the action occurs near $x_e \approx 10^{-7}$, so the plots zoom in on that region.



part (b)

The operating reboil ratio is 50% greater than the minimum value, so the actual operating reflux ratio is $V_B = 0.298$. Using this boil-up ratio, we can construct the operating line of the stripping section, OCS. We need to find where the point where OCS line intersects the q-line. $(\tilde{x}_2, \tilde{y}_2)$. The equations for these two lines are:

$$\begin{aligned} OCS &= \frac{V_B + 1}{V_B} x - \frac{x_B}{V_B} \\ q - line &= z_F \end{aligned}$$

Setting these two equations equal and solving for $\tilde{x}_2$, we find:

$$
\begin{aligned}
\tilde{x}_2 &= \left(z_F + \frac{x_B}{V_B}\right)\frac{V_B}{V_B+1} \\
&= 3.074\times 10^{-8} \\
\tilde{y}_2 &= 1\times 10^{-7}
\end{aligned}
$$

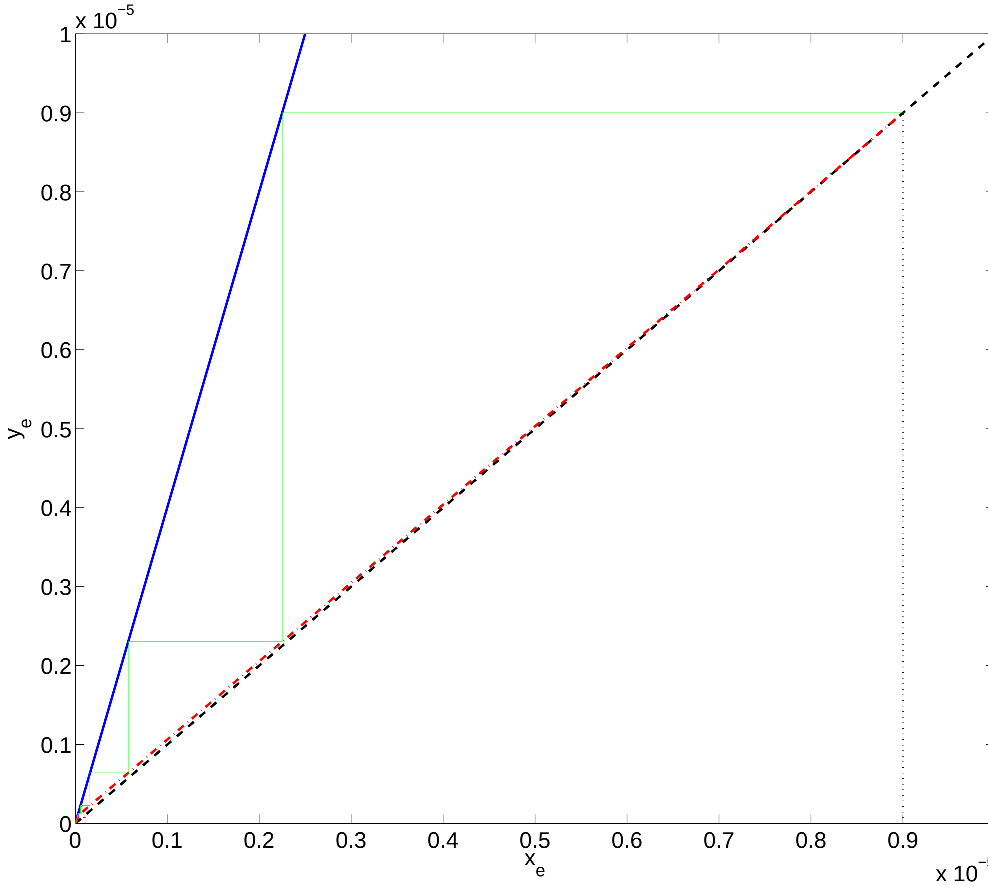
The operating line for the rectifying section section, OCR, starts at (x_D,x_D) and intersects the q-line at ($\tilde{x}_2$, $\tilde{y}_2$). From these points, we can calculate the reflux ratio:

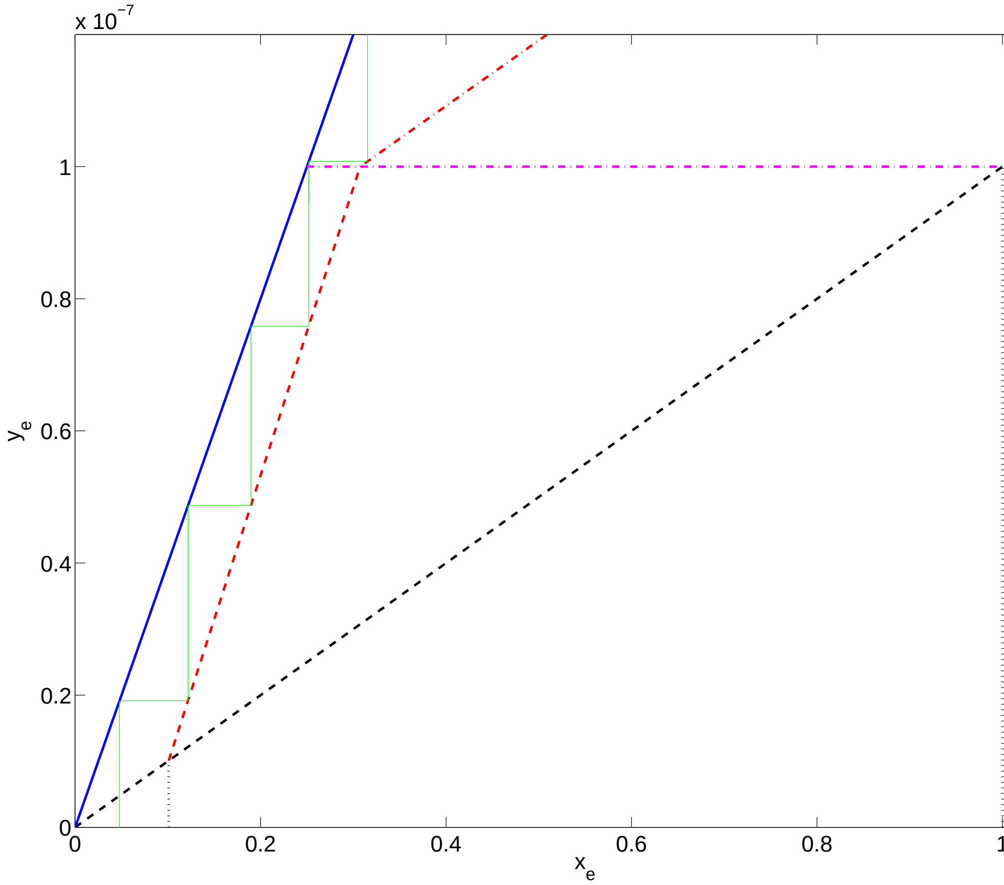
$$
\begin{aligned}
OCS &= \frac{R}{R+1}x + \frac{x_D}{R+1} \\
&\text{or} \\
R &= \frac{x_D - z_F}{z_F - \tilde{x}_2}
\end{aligned}
$$

Which yields: $R = 128.5$.

At this point, we can calculate the VLE curve and the OCR, OCS, and q-lines, so we have everything we need to solve the problem. Starting at (x_D,x_D) on the OCR line, create a series of step lines between the VLE curve and the OCR line. Once you cross the q-line, create a series of steps between the VLE curve and the OCS line. Once these steps cross x_B, stop. The results are plotted in the figures below. From the plots, we see that six (6) stages are required in the rectifying section, and three (3) stages are required in the stripping section. Thus, nine (9) stages are required to obtain the desired result.

The VLE curve is in blue; the 45 $^\circ$ line is the dashed black line; the OCR and OCS lines are dash-dot red; the q-line is dash-dot magenta, and the steps are in solid green. The rectifying occurs between $10^{-7} \leq x_e \leq 9\times 10^{-6}$, and the stripping section occurs between $10^{-8} \leq x_e \leq 10^{-7}$, as show in the two plots below. The first plot has six step above the q-line, the second plot has three steps below the q-line.


x 10^−5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y_e
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
x_e
x 10



[+ 5 points]