

Homework 3: Problem 3.44

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We are asked to modify the example refrigeration system described in Figure 3.9 on page 143. Using only one compressor and evaporator, design a system with two refrigeration units: one at $-15\text{ }^{\circ}\text{C}$ (we'll call it the freezer) and a second chamber at $5\text{ }^{\circ}\text{C}$ (we'll call it the fridge). Note that this is quite similar to what you have at home (unless your fridge is fancy, in which case it may well have dual compressors). As an additional constraint, the refrigeration capacity of the two chambers should be equal: $\dot{Q}_{\text{freezer}} = \dot{Q}_{\text{fridge}}$.

Throughout the problem, you will assume that the system behaves ideally. In other words, you can neglect the need for a finite temperature difference for the heat exchangers, and you can assume that the compressor works reversibly.

You might be tempted to split the working fluid into two streams and then run the two chambers in parallel, since in theory you could maintain the same refrigeration capacity by controlling the fraction of flow to each unit. This is a bad idea, however, because eventually you'd have to blend the two streams, which would have different states, and this would cause difficulties for the compressor.

Instead, it is better to run the two chambers in series. Conceptually, we describe the process as splitting the Evaporator in Figure 3.9a into two separate evaporators in series, with a second throttle between them. The first throttle – between States 4 and 1 – drops to an intermediate pressure, and the second throttle – between States X and Y – drops to the low pressure.

Begin at State 4 on the figure below. State 4 is a saturated liquid at high pressure; since it has just left the condenser, it is at the reservoir temperature T_{res} . When the working fluid passes through the first throttle and becomes State 1, the

enthalpy stays constant, but the pressure and temperature both drop. The fluid is no longer a high-pressure saturated liquid; it is a lower-pressure two-phase flow. We must determine the pressure so that the temperature of the working liquid is at (or just below) the temperature of the cooling unit. The two-phase intermediate-pressure flow enters the first evaporator (which corresponds to the fridge) with the same enthalpy as h_4 , and it exits at a higher enthalpy h_X . The temperature remains constant at T_{fridge} , but the cooling is achieved by the evaporation of the liquid phase. Whether or not all the liquid is evaporated will be determined later. The working fluid then passes through a second throttle, dropping isenthalpically to a lower pressure and temperature, and then enters the second evaporator (which corresponds to the freezer) at state Y. Once again, we must determine the lower pressure so that the temperature of the fluid is equal to the temperature of the freezer. The working fluid emerges from the second evaporator as a saturated vapor at state 2. We must ensure that all of the fluid is a saturated vapor, because any liquid phase could severely damage the compressor. The rest of the cycle remains unchanged from Figure 3.9 in your text: The low-pressure saturated vapor is compressed into a high-pressure vapor (State 3), which then passes through the condenser and emerges as a high-pressure saturated liquid (State 4) before entering the first throttle to repeat the process.

The constraints are:

$$\begin{aligned} T_{fridge} &= 278\text{K} \\ T_{freezer} &= 258\text{K} \\ T_{res} &= 298\text{K} \end{aligned}$$

$T_{res} = 25^\circ\text{C}$ is not stated in the problem, but it should be room temperature. Additionally, we know that the two cooling units must share the same cooling capacity, from which it follows that:

$$\begin{aligned} \dot{Q}_{freezer} &= \dot{Q}_{fridge} \\ \dot{n}(h_X - h_1) &= \dot{n}(h_2 - h_Y) \end{aligned}$$

Since we assume that the throttle process is isenthalpic – $h_X = h_Y$ and $h_4 = h_1$ – the intermediate enthalpy is:

$$h_X = \frac{1}{2}(h_4 + h_2)$$

For the following analysis, R134a will be used. Other working fluids will be considered at the end. Before we can determine the work and cooling capacity, we must first determine the three pressures of the cycle. To determine the states, we start with State 2. We know that State 2 must be a saturated vapor, and we know its temperature (since it emerges from the freezer at 258 K). To determine the pressure, find the pressure at the NIST web page at which the fluid is saturated at 258 K: 0.16293 MPa. Next, we look at State 4, which is a saturated liquid at 298 K. Once again, we look for the pressure at which the fluid is saturated at 298 K: 0.66241 MPa. The results are tabulated below:

T [K]	P [MPa]	h_l [kJ/mol]	h_v [kJ/mol]	s_l [kJ/mol-K]	s_v [kJ/mol-K]
258	0.16293	18.360	$39.745 = h_2$	94.362	$177.25 = s_2$
298	0.66241	$23.909 = h_4$	42.063	114.20	175.12

From the data in the table, we now have enough information to specify the enthalpy of the working fluid as it emerges from the fridge evaporator:

$$\begin{aligned}
 h_X &= \frac{1}{2}(h_4 + h_2) \\
 &= \frac{1}{2}(23.909 + 39.745) \\
 &= 31.827 \text{ kJ/mol } [+ 2 \text{ points }]
 \end{aligned}$$

As we the previous two cases, we must determine the pressure at which the liquid is saturated; unlike the previous two cases, however, we don't know the exact phase of the liquid. What we know is the temperature, the enthalpy, and that it must be either saturated liquid, saturated vapor, or both.

T [K]	P [MPa]	h_l [kJ/mol]	h_v [kJ/mol]	s_l [kJ/mol-K]	s_v [kJ/mol-K]
278	0.34500		40.965		176.05
278	0.34784	21.075	40.956	104.44	175.96
278	0.35000	21.075		104.44	

Given that the fluid must be at 278 K and saturated, it follows that the pressure must be 0.34784 MPa. At any other pressure, the fluid would either be pure liquid or pure vapor, and the enthalpy would not match. If we look at the value for h_X , we see that it lies between the values for saturated liquid and saturated vapor at 278 K and 0.34784 MPa, so it is clear that not all of the liquid evaporated.

Now that we have determined all three pressures, we have enough information to characterize the system completely. State 3 is the isentropic compression of the low-pressure saturated vapor to a heated vapor. Returning again to the tables, we determine the temperature at which $s_3 = s_2$, given that $P = 0.66241$ MPa:

T [K]	P [MPa]	h_l [kJ/mol]	h_v [kJ/mol]	s_l [kJ/mol-K]	s_v [kJ/mol-K]
304.2	0.66241		42.707 = h_3		175.25 = s_3

The specific work required by the compressor is:

$$\begin{aligned}
 \frac{\dot{W}_C}{\dot{n}} &= h_3 - h_2 \\
 &= 42.707 - 39.745 \text{ kJ/mol} \\
 &= 2.962 \text{ [kJ/mol] } [+ 2 \text{ points }]
 \end{aligned}$$

The specific heat loads for the evaporators are:

$$\begin{aligned}
 \frac{\dot{Q}_{fridge}}{\dot{n}} &= h_X - h_4 \\
 &= 31.827 - 23.909 \text{ kJ/mol} \\
 &= 7.918 \text{ [kJ/mol] } [+ 2 \text{ points }] \\
 \frac{\dot{Q}_{freezer}}{\dot{n}} &= h_2 - h_X \\
 &= 39.745 - 31.827 \text{ kJ/mol} \\
 &= 7.918 \text{ [kJ/mol] } [+ 2 \text{ points }]
 \end{aligned}$$

The specific heat load for the condenser is:

$$\begin{aligned}
 \frac{\dot{Q}_H}{\dot{n}} &= h_4 - h_3 \\
 &= 23.909 - 42.707 \text{ kJ/mol} \\
 &= -18.798 \text{ [kJ/mol] } [+ 2 \text{ points }]
 \end{aligned}$$

The coefficient of performance is:

$$\begin{aligned}
COP &= \frac{\dot{Q}_{fridge} + \dot{Q}_{freezer}}{\dot{W}_C} \\
&= \frac{2 \times 7.918 \text{ [kJ/mol]}}{2.962 \text{ [kJ/mol]}} \\
&= 5.34
\end{aligned}$$

And the complete system is:

Stage	T [K]	P [MPa]	phase
1	278	0.34874	liquid + vapor
X	278	0.34874	liquid + vapor
Y	258	0.16293	liquid + vapor
2	258	0.16293	saturated vapor
3	304.2	0.66241	heated vapor
4	298	0.66241	saturated liquid

The Ts diagram will be similar to that in Figure 3.9b. However, there will be an intermediate horizontal line, corresponding to the intermediate pressure. Starting again at State 4 in the upper left, as the fluid passes through the throttle, it cools down to 278 K and gains entropy. As it passes through the fridge evaporator, the temperature remains constant at 278 K, but the entropy increases. Similarly, as the fluid passes through the second throttle, it cools down to 258 K and gains entropy. In the freezer evaporator, the temperature is constant, and the entropy continues to increase. Since we assume that the compressor is reversible, there is no change in the entropy, but there is a sharp increase in the temperature, shown by the vertical line. As this hot vapor enters the condenser, it cools down to a saturated vapor at 298 K, then it condenses into a saturated liquid, all the while losing entropy.

In determining the best fluid for your fridge/freezer, there are a couple of things to keep in mind. First, you would like to select a fluid that will not require a large difference between the high and low pressures, since a larger pressure difference will require more compression, which will decrease the COP. Second, you'd like to avoid having a low-end pressure that is too low or a high-end pressure that is too high, since these extrema can place real-world mechanical constraints on the system (like the choice of compressor, valves, etc).

If we consider another common working fluid, ammonia, we can compare its performance to R134a.

T [K]	P [MPa]	h_l [kJ/mol]	h_v [kJ/mol]	s_l [kJ/mol-K]	s_v [kJ/mol-K]
250	0.23467	4.6610	$27.033 = h_2$	20.625	$107.34 = s_2$
278	0.51298	6.2273	27.426	26.444	102.70
298	0.99861	$7.8357 = h_4$	27.700	31.984	98.641
359.7			$30.540 = h_3$		$107.34 = s_3$

Based on these values, we can see why R134a is a superior working fluid. First, for ammonia, even though the low-end pressure is higher, the high-end pressure is higher still; thus the difference in pressures is greater, so the system will have to be more robust mechanically, and it will require a stronger compressor. Second, the work required by the compressor will be:

$$\begin{aligned}
 \frac{\dot{W}_C}{\dot{n}} &= h_3 - h_2 \\
 &= 30.540 - 27.033 \text{ kJ/mol} \\
 &= 3.507 \text{ kJ/mol}
 \end{aligned}$$

So nearly 20 % more work will be required by the system. Also, note how much higher the temperature is at State 3. You probably don't want the back of your fridge to be 190 °F.

Finally, when you work on this problem, keep in mind the various ideality assumptions. In actuality, $T_{fridge} < 5 \text{ °C}$, $T_{freezer} < -15 \text{ °C}$, and $T_{hot} > 25 \text{ °C}$, since you will need a finite temperature gradient for heat transfer to work. This will require a lower low-end pressure and a higher high-end pressure. Also, the compressor will not be reversible, so there will be an irreversible loss of efficiency there.

