May 8, 2008

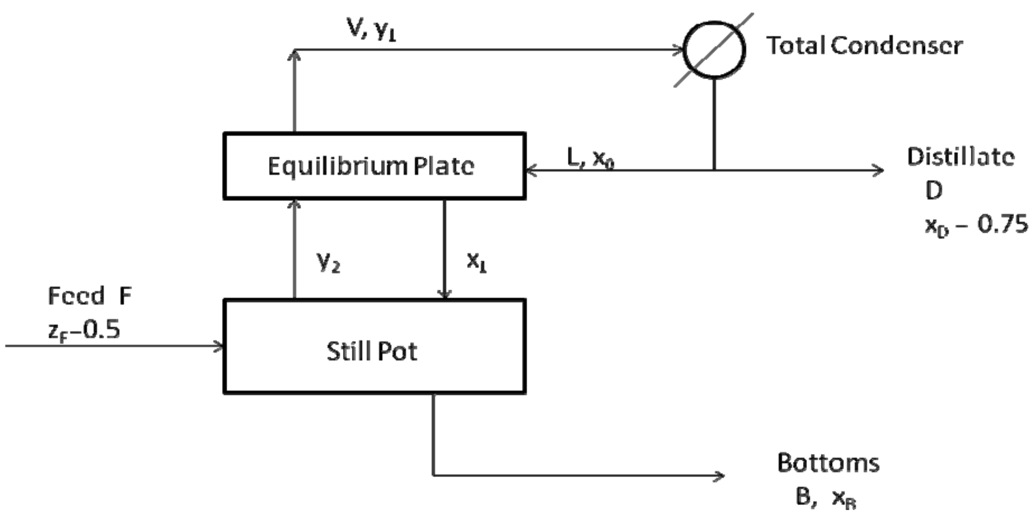
Problem 1.

1) A separation apparatus consists of a still pot, one theoretical plate, and a total condenser. The feed to this apparatus is a saturated liquid mixture of A and B at a pressure of 1 bar. The relative volatility of A to B is 2.5. Equilibrium is achieved in the still and its distillate composition is 75 mol % A. For each of the following situations, where possible, calculate the ratio of the molar flow rates of feed to the distillate. For each case, assume an initial feed of 100 moles/hour and equimolar composition.

a) The feed is to the still pot and no reflux is used.
b) The feed is to the still pot and a reflux ratio of 3 is used.
c) The feed is to the still pot and the minimum reflux ratio is used.
d) The feed is to the plate and a reflux ratio of 3 is used.
e) Repeat case b) after rust has destroyed the theoretical plate.

Solution:

A diagram of the process is given below.



a. The feed is to the still pot and no reflux is used.
Begin with an overall mass balance and component mass balance on the light key A.

$$F = D + B$$
$$Fz_F = Dx_D + Bx_B$$

Combining the two equations,

$$Fz_F = Dx_D + (F-D)x_B$$
$$F(z_F - x_B) = D(x_D - x_B)$$
$$\frac{F}{D} = \frac{x_D - x_B}{z_F - x_B} \qquad (1)$$

We are given the relative volatility of component A to B, $\alpha_{A,B}$ = 2.5. Using the definition of relative volatility,

$$\alpha_{A,B} = \frac{K_A}{K_B} = \frac{y_A/x_A}{y_B/x_B} = \frac{y_A(1-x_A)}{y_B(1-x_B)}$$

Rearranging,

$$y_A = \frac{\alpha_{A,B}x_A}{1+x_A(\alpha_{A,B}-1)} \qquad (2)$$

If no reflux is used (L = 0), then no liquid phase exists on Stage 1 so no mass transfer can occur and no further separation can be reached on Stage 1.

In that case, the vapor mole fraction of A leaving the still pot is the same as the vapor mole fraction of A leaving Stage 1. In other words, y_1=y_2. We can calculate the composition of the Bottoms because it is in equilibrium with the vapor leaving the still pot (y_2).

Since the total condenser does not change the composition of the streams, $y_2 = x_D = 0.75$.

$y_2 = y_1 = 0.75$.

Plugging into our equilibrium relationship Eq. 2,

$$y_{A,2} = 0.75 = \frac{2.5x_B}{1+x_B(2.5-1)}$$

$x_B = 0.545$

By inspection, we can see that this is not a feasible separation, because $z_F < x_D$ and $z_F < x_B$. We can attempt to calculate the ratio of the molar flow rates of feed to distillate using Equation 1.

$$\frac{F}{D} = \frac{x_D - x_B}{z_F - x_B}$$
$$\frac{F}{D} = \frac{0.75 - 0.545}{0.5 - 0.545}$$
$$\frac{F}{D} = -4.6 \qquad \text{Not possible}$$

In this case, the desired distillate composition cannot be reached with only one stage due to the inherent volatilities of A & B. It is not possible to calculate the ratio of F/D required.

b. The feed is to the still pot and a reflux ratio of 3 is used.

In part b, the theoretical stage can now be used to achieve further separation since reflux is present.

We can calculate the composition of the liquid in equilibrium with the vapor leaving the top stage using our equilibrium relationship as before.

$$y_{A,1} = 0.75 = \frac{2.5x_{A,1}}{1 + x_{A,1}(2.5-1)}$$
$$x_{A,1} = 0.545$$

From a mass balance around the total condenser and Stage 1, and assuming constant molar overflow (the flowrates of L and V do not change across stages), we find

$$y_2 = \frac{L}{V}x_1 + \frac{D}{V}x_D \quad (3)$$

Using the relationships R = L/D and V = L + D, we can rewrite Equation 3 as

$$y_2 = \frac{R}{R+1}x_1 + \frac{1}{R+1}x_D \quad (4)$$

Plugging in x_1 = 0.545, R = 3, and x_D =0.75 into Equation 4 , we find y_2 = 0.596. Using our equilibrium relationship to calculate $x_{B,}$

$$y_{A,2} = 0.596 = \frac{2.5x_B}{1 + x_B(2.5-1)}$$
$$x_B = 0.371$$

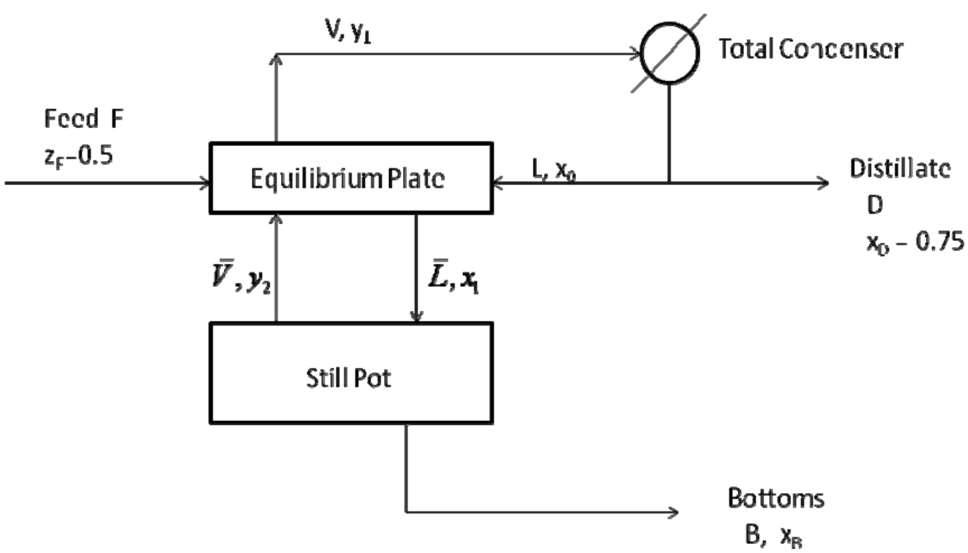
Plugging this bottoms composition into Equation 1,

$$\frac{F}{D} = \frac{0.75 - 0.371}{0.5 - 0.371} = 2.94 \qquad (5)$$

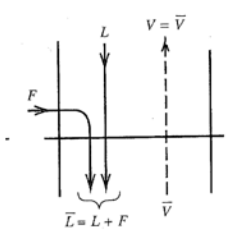
c. The feed is to the still pot and the minimum reflux ratio is used.

We find that it is not possible to calculate a ratio of molar flowrates of feed to distillate for the fundamental reason that at minimum reflux, a pinch point develops at the feed stage that requires an infinite number of stages to reach. Since our system has a finite number of stages, the separation is not possible.

d. The feed is to the plate and a reflux ratio of 3 is used.



A diagram of the process is given above. The section between the feed stage and the bottom stage is referred to as the stripping or exhausting section, and the liquid and vapor molar flowrates are identified with an overbar, $\bar{L}$ and $\bar{V}$. Because the feed is a saturated liquid, the entire portion of the feed goes into the stripping section. This can be visualized using Figure 7.7 in Seader and Henley, a portion of which is reprinted here:



Because the feed is a saturated liquid,

$\bar{L} = F + L$ and $\bar{V} = V$ (6) and (7)

From a mass balance around the still pot and using the relationships F = D + B and V = L +D,

$$\bar{L}x_1 = \bar{V}y_2 + Bx_B$$
$$(F + L)x_1 = Vy_2 + Bx_B$$
$$(F + L)x_1 = (L + D)y_2 + (F - D)x_B$$

Dividing by D,

$$x_1(\frac{F}{D} + R) = (R+1)y_2 + (\frac{F}{D} - 1)x_B \quad (8)$$

We can solve for x_1 using our equilibrium relationship for $y_1 = 0.75$, and as before, $x_1 = 0.545$. There are still three unknowns (F/D, x_B, y_2) in Equation 8, so we need two more relationships. They are provided from a component mass balance around the system and the equilibrium relationship.

$$\frac{F}{D} = \frac{x_D - x_B}{z_F - x_B} \quad (1)$$

and

$$y_2 = \frac{2.5x_B}{1 + x_B(2.5-1)} \quad (9)$$

Solving equations 1, 8, and 9 simultaneously for F/D, x_B, and y_2, we get

$$y_2 = 0.641, \frac{F}{D} = 4.02, x_B = 0.417$$

Or

$$y_2 = 0.683, \ \frac{F}{D} = 7.69, \ x_B = 0.463$$

e. Repeat case b) after rust has destroyed the theoretical plate.

If rust has destroyed the theoretical plate, then no mass transfer between the vapor and liquid phases can occur, and no separation is achieved on the theoretical plate. Despite there being liquid reflux, the mol fractions of the vapor and liquid streams exiting and entering the theoretical plate do not change.

As in part a., $y_1 = y_2 = 0.75$. Using our equilibrium relationship Eq. 2,

$$y_{A,2} = 0.75 = \frac{2.5x_B}{1 + x_B(2.5 - 1)}$$

$$x_B = 0.545$$

This leads to a ratio of F/D = -4.6, an unphysical result. Thus, the ratio of feed to distillate cannot be calculated since the desired separation is not possible.

Grading Scheme.

1. 1 point for Equation 1
2. 1 point for Equation 2
3. 1 point for $x_B = 0.545$ in part a, and showing that this leads to an unphysical F/D ratio either by calculation of F/D or by explaining that $x_B < z_F < x_D$ for a physical separation
4. 1 point for Equation 4
5. 1 point for F/D = 2.94 in part b.
6. 1 point for explaining in part c. that an infinite number of stages is required for the minimum reflux ratio to achieve the desired separation, so the separation is not for our system if $R = R_{min}$.
7. 1 point for Equation 8
8. 2 point for F/D = 4.02 and F/D = 7.69 in part d.
9. 1 point for explaining that no separation can occur on the plate if the theoretical plate is destroyed, and that for the same reasons as in part a., the separation is not possible.