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Problem 1 (5.16 part a only)

We are told that propane is being expanded in an isentropic turbine (such as one used in an ideal Rankine or Carnot cycle). Propane is fed to the turbine at 350 °C and 600 cm³/mol (State 1) and leaves at atmospheric pressure (P = 1 atm). Since we need another intensive property to fully specify the state of the exhaust, we can use the fact that the process is isentropic to provide a 2nd constraint on the system.

Relating entropy to the independent variables T and V, we get

$$ds = \frac{c_v}{T} dT + \left(\frac{\partial P}{\partial T} \right)_v dv$$

Setting ds = 0 and using the van der Waals equation for PvT behavior, we get

$$\left(\frac{\partial P}{\partial T} \right)_v = \frac{R}{v-b}$$

$$ds = 0 = \frac{c_v}{T} dT + \frac{R}{v-b} dv \quad (\text{Eq. 1})$$

The heat capacity is a function of both temperature and volume under nonideal conditions, so we have to construct a path so that the temperature is changing at ideal conditions. For S(T,v), we will expand the gas from v₁ → infinity, isochorically change the temperature from T₁ to T₂ in the ideal state, and then isothermally compress the gas from Infinity → v₂.

$$\Delta S = \int_{\infty}^{v_2} \left(\frac{\partial P}{\partial T} \right)_{T=T_2} dV + \int_{T_1}^{T_2} \frac{C_v^{IG}}{T} dT + \int_{v_1}^{\infty} \left(\frac{\partial P}{\partial T} \right)_{T=T_1} dV$$

Since $\left(\frac{\partial P}{\partial T} \right)_v = \frac{R}{v-b} \neq f(T)$, we can combine the dV integrals to get an expression

$$\Delta S = \int_{v_1}^{v_2} \left(\frac{\partial P}{\partial T} \right)_v dV + \int_{T_1}^{T_2} \frac{C_v^{IG}}{T} dT \quad (\text{Eq. 2})$$

Using the data from Koretsky, we can determine C_v^{IG}

$$c_v^{IG} = C_p^{IG} - R = R(A + BT + CT^2) - R \quad (\text{Eq. 2b})$$

where

$$A = 1.213$$

$$B = 28.785 * 10^{-3}$$

$$C = -8.824 * 10^{-6}$$

Integrating Equation 1 between state 1 and state 2,

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$$\int_{T_1}^{T_2} \frac{R(A-1)}{T} + BR + CRTdT + \int_{v_1}^{v_2} \frac{R}{v-b} dv = 0 \quad (\text{Eq. 2c})$$

$$R(A-1) \ln \left(\frac{T_2}{T_1} \right) + BR(T_2 - T_1) + \frac{CR}{2} (T_2^2 - T_1^2) + R \ln \frac{(v_2 - b)}{(v_1 - b)} = 0 \quad (\text{Eq. 3})$$

All the variables in Equation 3 are known except T_2 and v_2 . We need another equation relating T and v . For that, we can use the van der Waals equation, and apply it to state 2:

$$P_2 = \frac{RT_2}{v_2 - b} - \frac{a}{v_2^2} \quad (\text{Eq. 4})$$

Now we have two equations (3 & 4) and two unknowns. We can solve them simultaneously using Matlab.

```
%10.213 Problem Set #5 Problem 5.16 a
%By Sarah Bashadi 3/12/08
```

```
function prob516
```

```
clc
```

```
xo=[450 2000];
```

```
Sol= fsolve(@vdw,xo);
```

```
Sol
```

```
return
```

```
function Y = vdw(x)
```

```
R=82.057; %cm^3 atm/mol K
```

```
%Cv constants
```

```
A=1.213;
```

```
B=28.785*10^-3;
```

```
C=-8.824*10^-6;
```

```
T1=623.15; %K
```

```
V1=600; %cm3/mol
```

```
P2= 1; %atm
```

```
%replace variables
```

```
T2=x(1);
```

```
V2=x(2);
```

```
%vdW interaction parameters
```

```
a=92*10^5; % atm cm6/mol^2
```

```
b=91; % cm3/mol
```

```
%ds=0
```

```
Y(1)=R*(A-1)*log(T2/T1)+B*R*(T2-T1)+C*R/2*(T2^2-T1^2)+R*log((V2-b)/(V1-b));
```

```
%vdw equation
```

```
Y(2)=P2-R*T2/(V2-b)+a/V2^2;
```

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return

Matlab returns a solution of $T = 448 \text{ K}$ and $v = 36636 \text{ cm}^3/\text{mol}$, which demonstrates the reduction in temperature and expansion of the gas as it goes through a turbine.

Alternate method:

Starting from Eq. 1,

$$ds = 0 = \frac{c_v}{T} dT + \frac{R}{v-b} dv \quad (\text{Eq. 1})$$

Instead of constructing an alternate path so that the temperature change is occurring at ideal conditions, we can develop an expression for C_v that applies at non-ideal conditions.

Using the results from Ex. 5.3 in Koretsky, we can say that

$$c_v^{real} = c_v(v, T) = c_v^{ideal\ gas} + \int_{v_{ideal\ gas}}^{v_1} T \left(\frac{\partial^2 P}{\partial T^2} \right) dv \quad (\text{Eq. 2})$$

However for the van der Waals EOS,

$$\frac{\partial^2 P}{\partial T^2} = 0$$

Simplifying equation 2,

$$c_v^{real} = c_v^{ideal\ gas} = R(A + BT + CT^2) - R$$

where

$$A = 1.213$$

$$B = 28.785 * 10^{-3}$$

$$C = -8.824 * 10^{-6}$$

We can then calculate the change in entropy over the path from $S(T_1, V_1)$ to $S(T_2, V_2)$ directly, so that

$$\int_{T_1}^{T_2} \frac{R(A-1)}{T} + BR + CRT dT + \int_{v_1}^{v_2} \frac{R}{v-b} dv = 0$$

We can proceed to integrate and solve the problem as before.

Grading Scheme:

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- 1.) 1 point for computing $\left. \frac{\partial P}{\partial T} \right)_v = \frac{R}{V-b}$
- 2.) 1 point for $ds=0$.
- 3.) 2 points for Eq. 2
- 4.) 1 point for Eq. 2b
- 5.) 1 point for Eq. 2c
- 6.) 1 point for Eq. 3
- 7.) 1 point for Eq. 4
- 8.) 2 points for correct final volume and temperature