Solution for Problem 6.21 **10.213**

Consider a mixture of species 1, 2, and 3. The following equation of state is available for the vapor phase:

$$Pv = RT + P^2\left[A(y_1 - y_2) + B\right]$$

Where $\dfrac{A}{RT} = -9.0\times10^{-5}\left[\dfrac{1}{atm^2}\right]$, $\dfrac{B}{RT} = 3.0\times10^{-5}\left[\dfrac{1}{atm^2}\right]$

Consider a vapor mixture with

$n_1 = 1$, $n_2 = 2$, and $n_3 = 2$

P = 50 atm, T= 500K

Find:

(i) v, molar volume of the vapor mixture
(ii) V, Total volume of vapor mixture
(iii) v_1, molar volume of species 1
(iv) v_2, molar volume of species 2
(v) v_3, molar volume of species 3
(vi) $\overline{V}_1$, partial molar volume of species 1 in the vapor mixture

Solution:

(i) v, molar volume of the vapor mixture

Using the given equation of state and rearranging,

$$v = \frac{RT}{P} + P\left[A(y_1 - y_2) + B\right]$$

$$v = \frac{(0.08206\frac{Latm}{molK})(500K)}{50atm} + (50atm)\left[(-9.0\times10^{-5})(\frac{1}{5}-\frac{2}{5}) + (3.0\times10^{-5})\right](0.08206\frac{Latm}{molK})(500K)$$

(1 point)

$$\boxed{v = 0.9191\frac{L}{mol}}$$ (1 point)

(ii) V, Total volume of vapor mixture

$$V = n_{Total}v$$ (1/2 point)

$$V = (5mol)(0.9191\frac{L}{mol})$$

$$\boxed{V = 4.595L}$$ (1/2 point)

(iii) v_1, molar volume of species 1

Assuming that there is only species 1 present, $y_1 = 1$, $y_2 = 0$, $y_3 = 0$

(1/2 point)

$$v_1 = \frac{RT}{P} + P\left[A(y_1 - y_2) + B\right]$$

$$v_1 = \frac{(0.08206\frac{Latm}{molK})(500K)}{50atm} + (50atm)\left[(-9.0\times10^{-5})(1-0) + (3.0\times10^{-5})\right](0.08206\frac{Latm}{molK})(500K)$$

$$\boxed{v_1 = 0.6975\frac{L}{mol}}$$ (1/2 point)

(iv) *v_2, molar volume of species 2*

Assuming that there is only species 2 present, $y_1 = 0$, $y_2 = 1$, $y_3 = 0$

(1/2 point)

$$v_2 = \frac{RT}{P} + P\left[A(y_1 - y_2) + B\right]$$

$$v_2 = \frac{(0.08206\frac{Latm}{molK})(500K)}{50atm} + (50atm)\left[(-9.0\times10^{-5})(0-1) + (3.0\times10^{-5})\right](0.08206\frac{Latm}{molK})(500K)$$

$$\boxed{v_2 = 1.0668\frac{L}{mol}}$$ (1/2 point)

(v) *v_3, molar volume of species 3*

Assuming that there is only species 3 present, $y_1 = 0$, $y_2 = 0$, $y_3 = 1$

(1/2 point)

$$v_3 = \frac{RT}{P} + P\left[A(y_1 - y_2) + B\right]$$

$$v_3 = \frac{(0.08206\frac{Latm}{molK})(500K)}{50atm} + (50atm)\left[(-9.0\times10^{-5})(0-0) + (3.0\times10^{-5})\right](0.08206\frac{Latm}{molK})(500K)$$

$$\boxed{v_3 = 0.8821\frac{L}{mol}}$$ (1/2 point)

(vi) $\overline{V_1}$*, partial molar volume of species 1 in the vapor mixture*

$$\overline{V_1} = \left(\frac{\partial V}{\partial n_1}\right)_{T,P,n_2,n_3} \qquad \text{Equation (6.29)}$$

Using the given equation of state,

$$V = n_{Total}v = (n_1 + n_2 + n_3)v$$

$$V = (n_1 + n_2 + n_3)\frac{RT}{P} + (n_1 + n_2 + n_3)P\left[A\left(\frac{n_1}{n_1 + n_2 + n_3} - \frac{n_2}{n_1 + n_2 + n_3}\right) + B\right]$$

$$V = (n_1 + n_2 + n_3)\frac{RT}{P} + PA(n_1 - n_2) + (n_1 + n_2 + n_3)PB$$

(2 points for correct expression of V)

Differentiating V with respect to n_1 and keeping other factors constant,

$$\overline{V_1} = \left(\frac{\partial V}{\partial n_1}\right)_{T,P,n_2,n_3}$$

$$\overline{V_1} = \frac{RT}{P} + PA + PB$$

(1 point for correct evaluation of $\overline{V_1}$)

$$\overline{V_1} = \frac{(0.08206\frac{Latm}{molK})(500K)}{50atm} + (50atm)(-9.0\times10^{-5} + 3.0\times10^{-5})(0.08206\frac{Latm}{molK})(500K)$$

$$\boxed{\overline{V_1} = 0.6975\frac{L}{mol}} \qquad \text{(1 point)}$$