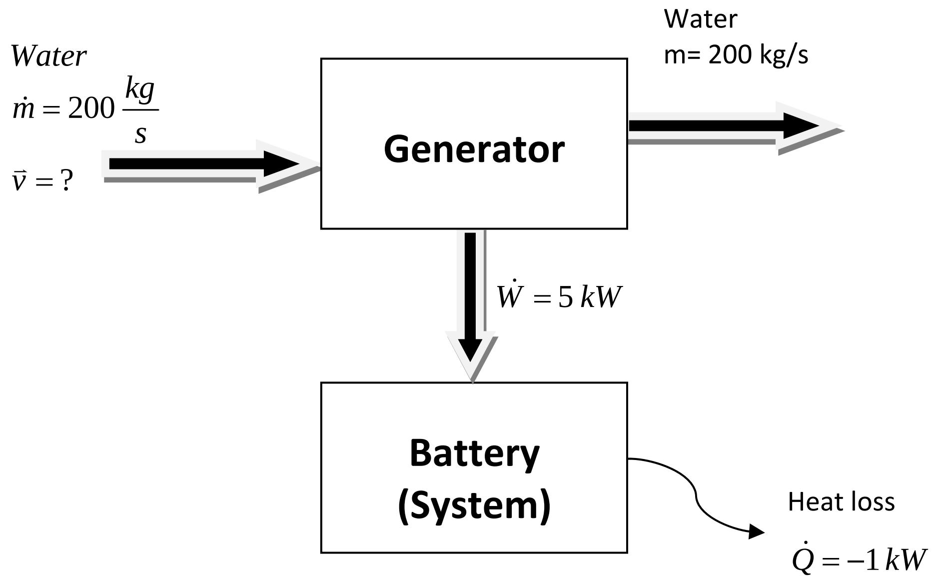
Spring 2008 10.213 Chemical and Biological Thermodynamics
Problem Set #2
Solution to Problem 1

1.



a. Part a asks how much energy is stored in the battery over 10 hours. This suggest that we define the system as the battery and apply the First Law of Thermodynamics to that system. No mass enters or leaves the system, so it is a closed system. The first law is

$$\frac{dU}{dt} + \frac{dE_K}{dt} + \frac{dE_P}{dt} = \dot{Q} + \dot{W} \qquad (1.1)$$

We are not given any information relating to the kinetic or potential energy of the system, so we can assume that the terms related to these energies in (1.1) can be neglected. The resulting form of the equation is

$$\frac{dU}{dt} = \dot{Q} + \dot{W} \qquad \text{[1 pt for reduced form of 1}^{st}\text{ Law]} \quad (1.2)$$

The generator transfers 5 kW of power to the battery, so W = + 5 kW. This work is positive because energy is transferred from the surroundings (generator) to the system (battery).

Energy is leaving the battery in the form of heat, so Q = -1 kW. The heat loss is negative in this case because the heat is transferred from system (battery) to the surroundings.

Substituting in (1.2)

$$\frac{dU}{dt} = -1\,kW + 5\,kW \qquad (1.3)$$

$$\frac{dU}{dt} = 4\,kW \qquad \text{[2 points]} \qquad (1.4)$$

Integrating from 0 to 10 hours,

$$\int_{U_1}^{U_2} dU = \int_{t_1}^{t_2} 4\,kW dt$$

$$U_2 - U_1 = 4kW \int_0^{10hr} dt$$

$$\Delta U = 4\frac{kJ}{s} \times (10hr - 0) \times \frac{3600s}{1\,hr}$$

$$\Delta U = 1.44 \times 10^5 kJ \qquad \text{[2 points]}$$

b. The kinetic energy of the water stream is converted to electrical energy and stored in a generator. The efficiency of the conversion from Kinetic energy → Electrical energy is 50%.

$$\eta = efficiency\ of\ conversion\ of\ Kinetic\ Energy \rightarrow Electrical\ Energy$$

$$\eta = \frac{W}{E_K} = \frac{(Electrical\ energy\ gained)}{(Kinetic\ energy\ required)}$$

$$\eta = 0.5$$

$$W = 5\,kW$$

$$Substituting\ in\ values,$$

$$E_K = 10kW \qquad \text{[2 points]}$$

The kinetic energy of the water is converted to electrical energy, so the amount of kinetic energy of the water decreases at a rate of 10 kJ/s . Assuming the final velocity of the water is zero,

$$\Delta E_K = \frac{1}{2}\dot{m}v_f^2 - \frac{1}{2}\dot{m}v_i^2$$

$$-10\frac{kJ}{s} = 0 - \frac{1}{2}\dot{m}v_i^2 \qquad \text{[1 point]}$$

$$-10\frac{kJ}{s} = -\frac{1}{2} * 200\frac{kg}{s} * v_i^2$$

$$v_i = 10\frac{m}{s} \qquad \text{[1 point]}$$

In an actual turbine, the exiting water will have some final velocity, and the frictional resistance to the turning of blades will cause energy loss in the form of heat. [1 point]