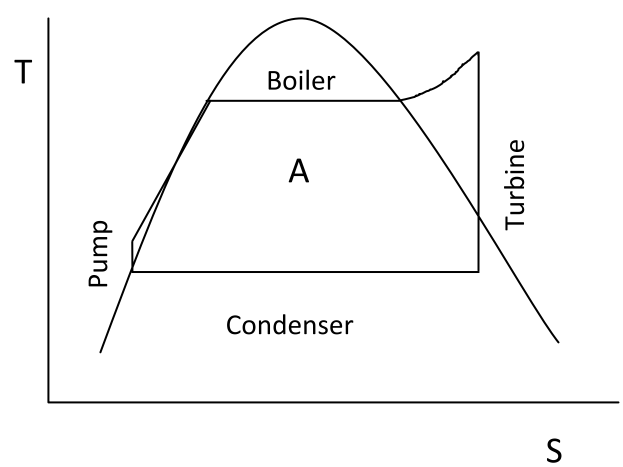
Problem 1.

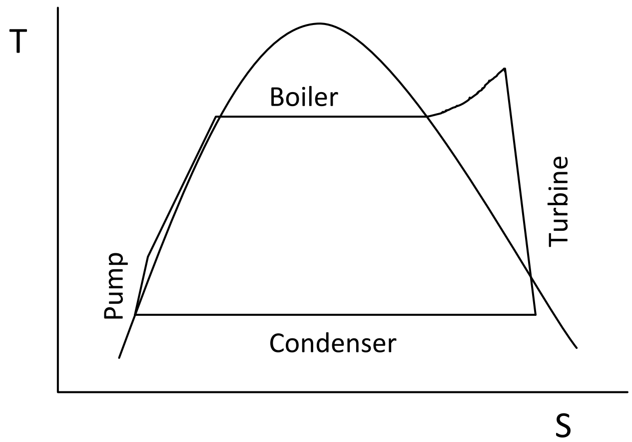
Sketch a T-S diagram of an ideal Rankine cycle with superheating. Show how the cycle would deviate from the ideal case if irreversibilities are introduced in the turbine and compressor. What effect does this modification have on the net power output of the cycle?

The ideal Rankine vapor power cycle which involves superheating is given below. In an ideal cycle, the compressor and turbine are assumed to be reversible and thus isentropic processes (represented by a vertical line). The net work extracted from an ideal Rankine cycle is the area within the diagram (A).



If irreversibilities are introduced into the turbine and compressor, the process is no longer isentropic. Entropy is generated as the fluid passes through the turbine and compressor. The T-s diagram for an irreversible process would look as follows:

Non-isentropic expansion (turbine) and compression (pump):



Since we are no longer assuming a reversible process, the area enclosed doesn't represent W_{net}, because $dQ \neq TdS$. However, in the turbine, less energy is extracted from the fluid as can be seen on the diagram from the higher quality of fluid. The irreversible process is less efficient and lowers the net power output of the cycle.

Grading:

- 3 points for correct ideal Rankine cycle T-S diagram

- 2 points each for showing entropy generation in pump and turbine (4 points total)

- 3 points for indicating that net power output of an irreversible cycle is lower

b. Sketch a design for a heat pump based on geothermal energy.

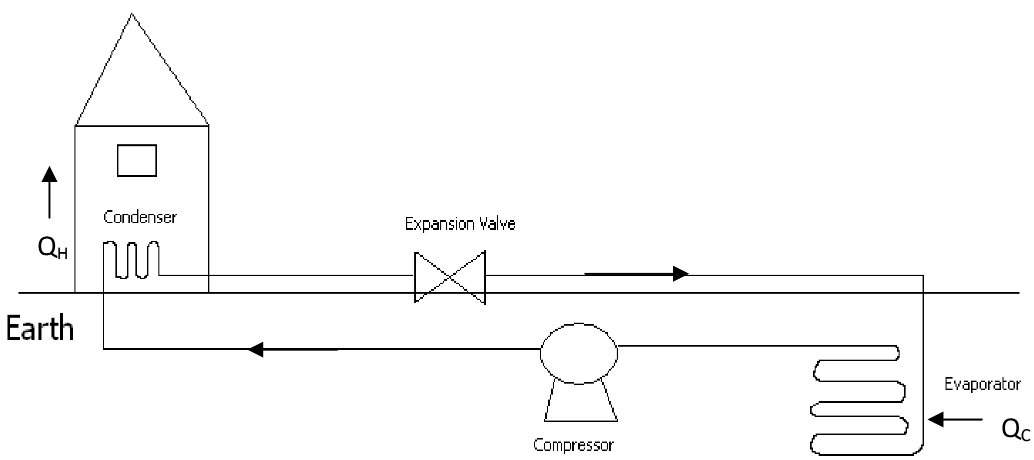
A heat pump uses work to transfer energy from a cold reservoir to a hot reservoir. With a geothermal heat pump, heat from the earth is transferred into the home through a refrigerant working fluid. The stages of the process are as follows:

1.) A refrigerant flows through pipes underground, absorbing heat from the earth, causing it to evaporate and become a low temperature, low pressure gas.

2.) The refrigerant then passes through a compressor where it becomes a superheated gas.

3.) The refrigerant then contacts air within your house, losing heat that goes on to raise the temperature of your house, in a condenser-type process.

4.) The refrigerant (which is still at high pressure) is then sent through an expansion valve and the process is repeated.



To calculate the coefficient of performance for a heat pump, we compare the amount of heat delivered to the amount of work put into the system.

$$C.O.P = \frac{Q_H}{W_{Net}} = \frac{Q_H}{Q_H - Q_c}$$

In the limit of an ideal Carnot cycle, we can say

$$C.O.P = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

The maximum COP possible using a geothermal pump is

$$C.O.P.(Geothermal) = \frac{298K}{298K - 283K}$$
$$C.O.P.(Geothermal) = 19.9K$$

In Boston, using a traditional air-source heat pump, the maximum COP possible is

$$C.O.P.(Boston) = \frac{298K}{298K - 269.26K}$$
$$C.O.P.(Boston) = 10.38$$

In Atlanta, the maximum COP possible from a traditional air-source heat pump is

$$C.O.P.(Atlanta) = \frac{298K}{298K - 280.37K}$$
$$C.O.P.(Atlanta) = 16.9$$

Thus, a geothermal heat pump provides the most benefit in colder climates.

Grading:

- 2 points for showing an evaporator→compressor → condenser →valve system

- 3 points for indicating how a geothermal system can be created from these components (i.e. not sufficient to just have the components, must explain how it would work)

- 1 point for C.O.P = Q_H/W_{net} or C.O.P. = $Q_H/(Q_H - Q_C)$

- 1 point for C.O.P = $T_h/(T_H-T_C)$

- 1 point each for correct calculation of C.O.P in Boston, Atlanta, and using a geothermal pump (total 3 points)

c. You have decided to take a job as a venture capitalist, and you are looking to start a company based on sustainable energy. An inventor claims to have developed a power cycle capable of delivering a net work output of 410 kJ for an energy input by heat transfer of 1000 kJ. The system undergoing the cycle receives the heat transfer from hot gases at a temperature of 500 K and discharges energy by heat transfer to the atmosphere at 300 K. Should you invest in his invention?

We begin by calculating the efficiency of the cycle the inventor has claimed to design. The net work produced is 410 kJ = W_{net}, and the amount of heat required is 1000 kJ = Q_H. The efficiency is then

$$\eta = \frac{W_{net}}{Q_H} = \frac{410kJ}{1000kJ} = 0.41$$

We can compare this to the efficiency of a Carnot cycle operating at the same conditions, between T_c = 300 K and T_H = 500 K. The Carnot cycle represents the most efficient cycle possible and thus any real system cannot exceed the Carnot efficiency.

$$\eta = \frac{T_H - T_C}{T_H} = \frac{500\ K - 300\ K}{500\ K} = 0.40$$

Because the inventor's claim for efficiency exceeds the Carnot efficiency at the operating conditions, the process is not physically possible, and you should not invest in his invention.

Grading:

- 2 pts for efficiency of Carnot Cycle

- 2 pts for efficiency of inventor's cycle

- 1 pt for stating that the inventor's process is impossible because it exceeds the maximum possible Carnot cycle efficiency