Quiz 1, Problem 3: Open System Analysis of a Combined Power Cycle

March 12, 2008

(a). Explain briefly why this design is an economical choice for a power plant compared to a traditional vapor (single) cycle.

Adding the gas turbine is more economical because it improves the overall efficiency of the system. The efficiency is related to the net work divided by the heat absorbed from the hot reservoir. Per given $\dot{Q}_H$, you increase the total power produced by adding the second cycle, and thus the efficiency. The gas turbine system has a high heat source, $\dot{Q}_H$, and it also has high-value exhaust heat, $\dot{Q}_C$. If you can capture this exhaust heat and extract more work from it, you will have a more efficient cycle. If this were a carnot process, you could think of it in terms of the Carnot efficiency: the combined cycle allows us to have a hotter T_H and a colder T_C than a single cycle. Thus, by increasing the ΔT, we increase the efficiency (for a Carnot process).

[if you didn't explain why it is more efficient, then -2 points].

(b) Determine the ratio of mass flow rates of the steam and air, $\dot{m}_{vap}/\dot{m}_{gas}$.

We begin with an overall energy balance. We assume that the system at steady state and that the change in kinetic and potential energy is negligible. Thus, for each stage in the the two cycles, the energy balance simplifies to:

$$\dot{m}\Delta H = \dot{Q} + \dot{W}$$

For the compressor, two turbines, and condenser, we assume that there is no heat loss. For the combustor, heat exchanger, and condenser, we assume that no work is being done.

Since there is no accumulation of energy within the heat exchanger, the heat transferred from the air to the steam must be equal in magnitude. The signs will differ, however, since the air cycle is rejecting heat and the steam cycle is accepting it:

$$
\begin{aligned}
\dot{Q}_{h.ex.,g.t.} &= -\dot{Q}_{h.ex.,v.c.} \\
\dot{m}_{gas}\left(\hat{h}_5 - \hat{h}_4\right) &= -\dot{m}_{vap}\left(\hat{h}_7 - \hat{h}_6\right)
\end{aligned}
$$

from which it follows that:

$$
\begin{aligned}
\frac{\dot{m}_{vap}}{\dot{m}_{gas}} &= \frac{\hat{h}_5 - \hat{h}_4}{\hat{h}_6 - \hat{h}_7} \\
&= \frac{400.98 - 858.02\ [\text{kJ/kg}]}{183.96 - 3138.3\ [\text{kJ/kg}]} \\
&= 0.1547
\end{aligned}
$$

[+ 10 points]

(c) Calculate the power produced by the turbine, $\dot{W}_{gas}$, and the vapor power cycle, $\dot{W}_{vap}$.

The net power output is the power output of the two turbines plus the power required to run the compressor and pump. For all four components, we assume that they run adiabatically. Thus, the specific work is balanced by the change in enthalpy.

$$
\begin{aligned}
\dot{W}_{net} &= \dot{W}_{gas} + \dot{W}_{comp} + \dot{W}_{vap} + \dot{W}_{pump} \\
&= \dot{m}_{gas}\left(\hat{h}_4 - \hat{h}_3\right) + \dot{m}_{gas}\left(\hat{h}_2 - \hat{h}_1\right) \\
&+ \dot{m}_{vap}\left(\hat{h}_8 - \hat{h}_7\right) + \dot{m}_{vap}\left(\hat{h}_6 - \hat{h}_9\right)
\end{aligned}
$$

Given a net power of -45 MW, we can use the result from (b) and solve for the mass flow rates:

$$
\begin{aligned}
\dot{m}_{vap} &= \frac{\dot{W}_{net}}{\frac{\dot{m}_{gas}}{\dot{m}_{vap}}\left[\left(\hat{h}_4-\hat{h}_3\right)+\left(\hat{h}_2-\hat{h}_1\right)\right]+\left(\hat{h}_8-\hat{h}_7\right)+\left(\hat{h}_6-\hat{h}_9\right)} \\
&= \frac{-45\ [\mathrm{MW}]}{\frac{1}{0.1547}\left[858.02-1515.42+669.79-300.19\right]+2104.74-3138.3+183.96-173.88\ [\mathrm{kJ/kg}} \\
&= \frac{-45000\ [\mathrm{kJ/s}]}{-1860.34+-1023.48\ [\mathrm{kJ/kg}]} \\
&= 15.60\ [\mathrm{kg/s}]
\end{aligned}
$$

Once we have the steam flow rate, we can easily calculate the air flow rate:

$$
\begin{aligned}
\dot{m}_{gas} &= \frac{\dot{m}_{gas}}{\dot{m}_{vap}}\dot{m}_{vap} \\
&= \frac{1}{0.1547}\times 15.60\ [\mathrm{kg/s}] \\
&= 100.84\ [\mathrm{kg/s}]
\end{aligned}
$$

The power of the gas turbine is:

$$
\begin{aligned}
\dot{W}_{gas} &= \dot{m}_{gas}\left(\hat{h}_4-\hat{h}_3\right) \\
&= 100.84\ [\mathrm{kg/s}]\ \left(858.02-1515.42\ [\mathrm{kJ/kg}]\ \right) \\
&= -66291.2\ \mathrm{kJ/s}
\end{aligned}
$$

The power of the steam turbine is:

$$
\begin{aligned}
\dot{W}_{vap} &= \dot{m}_{vap}\left(\hat{h}_8-\hat{h}_7\right) \\
&= 15.60\ [\mathrm{kg/s}]\ \left(2104.74-3138.3\ [\mathrm{kJ/kg}]\ \right) \\
&= -16123.5\ [\mathrm{kJ/s}]
\end{aligned}
$$

The negative sign indicates that work is being done by the system.

[If you forgot to include $\dot{W}_{comp}$ or $\dot{W}_{pump}$, it is -4 each. If you forgot to include the minus sign for $\dot{W}_{vap}$ or $\dot{W}_{gas}$, it is -1 each].

(d) What is the overall efficiency?

The overall efficiency is:

$$\eta = \left| \frac{\dot{W}_{net}}{\dot{Q}_H} \right|$$

This equation requires the heat flux into the system from the combustor, which is given by:

$$\begin{aligned} \dot{Q}_H &= \dot{m}_{gas} \left(\hat{h}_3 - \hat{h}_2 \right) \\ &= 100.84 \text{ [kg/s] } (1515.42 - 669.79 \text{ [kJ/kg] }) \\ &= 85273.3 \text{ [kJ/s]} \end{aligned}$$

Thus, the overall efficiency is:

$$\begin{aligned} \eta &= \left| \frac{\dot{W}_{net}}{\dot{Q}_H} \right| \\ &= \frac{|-45 \text{ [MW] }|}{85273.3 \text{ [kJ/s]}} \\ &= 0.5277 \end{aligned}$$

[If your efficiency was greater than 1 and you failed to mention that that cannot happen, then -1 point. If your efficiency was 0.9, then you probably should have lost a point, because that is way too high, but I didn't take one off.].

(e) How much more efficient is the combined cycle than a single vapor cycle?

The heat absorbed from the hot reservoir for a single vapor cycle is:

$$\begin{aligned} \dot{Q}_{h.ex.} &= \dot{m}_{vap} \left(\hat{h}_7 - \hat{h}_6 \right) \\ &= 15.60 \text{ [kg/s] } (3138.3 - 183.96 \text{ [kJ/kg] }) \\ &= 46087.7 \text{ [kJ/s]} \end{aligned}$$

The net work for the vapor cycle is:

$$
\begin{aligned}
\dot{W}_{net,vap} &= \dot{W}_{vap} + \dot{W}_{pump} \\
&= \dot{m}_{vap}\left(\hat{h}_8 - \hat{h}_7\right) + \dot{m}_{vap}\left(\hat{h}_6 - \hat{h}_9\right) \\
&= 15.60 \text{ [kg/s] } (2104.74 - 3138.3 + 183.96 - 173.88 \text{ [kJ/kg] }) \\
&= -15966.3 \text{ [kJ/s]}
\end{aligned}
$$

$$
\begin{aligned}
\eta_{vap} &= \left|\frac{\dot{W}_{net}}{\dot{Q}_H}\right| \\
&= \frac{|-15966.3 \text{ [kJ/s] }|}{46087.7 \text{ [kJ/s]}} \\
&= 0.3464
\end{aligned}
$$

We are asked to compare the two efficiencies:

$$
\begin{aligned}
\%\eta &= \frac{\eta - \eta_{vap}}{\eta_{vap}} \\
&= \frac{0.5277 - 0.3464}{0.3464} \\
&= 0.5233
\end{aligned}
$$

So the combined cycle is 52% more efficient than a single vapor cycle.

[If you failed to make a comparison, it is -2 points. If you made a comparison but didn't quantify it, then -1 point. Most of you quantified it incorrectly, but that's ok.].

COMMENTS:

Most students lost 2 points on part (a) for not stating that it is more efficient and explaining why. It isn't enough just to state that you're getting more work. What we're looking for is the ability to extract more work from the heat supplied – i.e. efficiency.

Most students did fine are (b). Please note that the mass and/or molar flow rates should not be negative.

Most students forgot to include $\dot{W}_{comp}$ and $\dot{W}_{pump}$. Of those who did include those two terms, probably half got the sign wrong. The signs will take care of themselves. It is a mistake to guess the signs or use absolute values. Also, about half of the students failed to note that work being done by the system has a negative sign.

Several students made mistakes on the efficiency. In many cases, students failed to use the heat exchanger for $\dot{Q}_H$ for part (e). If you used the wrong $\dot{W}_{net}$ or the wrong $\dot{Q}_H$, you lost 5 points. In some cases, students used the wrong equation for the efficiency. You cannot assume that this is a carnot cycle, so that equation will not work.