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Measuring Growth and Change In Metropolitan Form

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Abstract

Though urban expansion is key to explaining rapid economic growth, accelerating resource consumption, and social transformation in numerous emerging regions of the world, confusion persists around an empirical basis for measuring the growth and change of metropolitan areas. Numerous cities in the Global South simply do not know how and how fast they are growing. Policies addressing energy consumption and economic growth are consequently based on partial evidence.

Recent advancements in spatial analysis tools and remote-sensing have made it possible to track changes in a large number of metropolitan areas around the world using consistent methods. Taking advantage of these developments, this paper discusses seven fundamental urban form and land-use metrics that can be used to track growth and change in cities around the world – *Size*, *Coverage*, *Polycentricity*, *Compactness*, *Discontiguity*, *Expandability*, and *Land-Use Mix*. We analyze previous analogous metrics in the literature and propose important improvements made possible by novel GIS routines. The metrics are implemented in a new open-source toolbox for ArcGIS called the Metropolitan Form Analysis Toolbox.

A longitudinal analysis of these metrics over time can tell us how the patterns of metropolitan areas are changing – is a city growing more compact or sprawling, are its centers consolidating around particular locations or is employment becoming more scattered? Coupling these changes with social, economic and environmental indicators of corresponding cities allows one to estimate what factors explain the types and speeds of growth we observe in different cities. Having a measurable basis for tracking growth and change in metropolitan form thus not only reveals what the current urban expansion trends are, but also helps illuminate where these trends come from, where they might be headed, and how informed policy could affect their evolution.

Keywords: *metropolitan form, GIS, urban extent, urbanization, shape metrics.*

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and wealth distribution. Understanding the growth and change of urban economies requires understanding the growth and change of their metropolitan areas.

Unlike Western cities, whose 19th and 20th century expansions are by now relatively well documented, the extents and growth patterns of metro areas in the Global South have so far been under studied. Metropolitan areas often leap across municipal boundaries, making it difficult to track for any one mayor or governor. Numerous rapidly developing cities in the Global South simply do not know how and how fast they are growing. This leads to poorly informed policies and public investment decisions. Lacking an empirical understanding of metropolitan growth, argues David E. Dowall (1995), leads to a “blind flight” for local governments and a failure to effectively deal with rapid population change and land development.

Recent advances in remote sensing technology, however, have opened up new possibilities for a systemic study of urban forms and their expansion patterns within and across a large sample of cities. Commercial satellite imagery can be used to approximate the physical growth of cities by categorizing land use changes over time (Limhoff, Lawrence et al., 1997; Bartholomé & Belward 2005; Schneider, Friedl et al., 2010; CIESIN, 2013). These developments have made it possible for cities in data-poor regions to monitor their expansion in an economical and relatively accurate manner¹. Repetitive capture of such data has made it possible to compare urban growth and change across time. As more research is emerging in this area, it becomes important to implement reliable metrics to capturing metropolitan growth and change.

This paper addresses this need and implements eight metrics that characterize physical properties of metropolitan form. The metrics – *Size, Density, Coverage, Polycentricity, Compactness, Discontiguity, Expandability, and Land-Use Mix* – aim to capture different and complementary qualities of metropolitan areas. We have automated the application of these metrics in a new open-source ArcGIS toolbox called the *Metropolitan Form Analysis Toolbox*, which can facilitate otherwise labor-intensive growth measurements through fast computer routines, making their application available to more cases around the world. The work on the toolbox is in progress, expected to be released later in 2013.

A body of literature on metropolitan form, goes back more than a century (Kohl, 1841; Christaller, 1933), but efforts to systemically evaluate metropolitan form using empirical data across cities have been scarce. Numerous studies have looked at specific aspects of metropolitan growth, such as sprawl (Mieszkowski & Mills, 1993; Edmonston & Guterbock, 1984, Haar & Lindsay, 1986; Krakover, 1992), or the extent of individual infrastructure elements (Bettencourt, Lobo et al., 2007), but not the overall evolution of a city’s physical landscape. Morphological studies of city form have addressed the latter, but data availability has typically restricted such studies to relatively small towns or neighborhoods (Conzen, 1960; Whitehead, 1981&1987; Siksna,1996; Moudon, 1986).

There is a tradition of environmental shape analysis in geography, where the basis for a number of metrics that are applicable to studying the shape of metropolitan areas, has been established (Gibbs, 1961; Clark and Boyce, 1964; Lo, 1980; Austin, 1981). Traditional concepts of shape analysis have been more recently combined with the computational power of GIS, allowing their application to large geographic datasets (Wentz, 1997; Angel, Parent et al., 2010; Parent, Civco et al., 2009). The GIS based methods investigate the shapes of urban areas, measuring the distribution of continuous urbanized areas around the metropolitan center, the shape characteristics urban edges, the relationship between a city’s area and perimeter, and the ease with which the area can be traversed or circumvented. A comprehensive study of metropolitan growth across a large sample of cities was developed by Angel, Sheppard and Civco (2005). Taking advantage of remote sensing data mentioned above, the authors studied the spatial extents of 3,943 cities over two time intervals – 1990

¹ We discuss some of the challenges in using such data in the discussion section below.

and 2000. They analyzed how the built-up area, density, contiguity and compactness of these cities changed over a decade and whether and how strongly they were affected by population growth, income growth, climate, vehicle ownership, land quality and other factors. Alan Betaud and Malpezzi (2003) has also analyzed a number of cities by population density and compared their 'dispersion'. The present work builds upon these efforts and attempts to further improve the metropolitan form metrics with a few important additions.

Critics of urban shape analysis have alerted that examining the changes in metropolitan boundaries is not sufficient for revealing what is actually taking place within them – the distribution of population and businesses, the quality of centers or combination of uses cannot be understood by analyzing only the shape of a city's extent (Prosperi, Moudon et al., 2009). Metrics of metropolitan form should not only focus on their outer shape, but also on intra-urban distribution of land uses, people and centers.

These concerns have been addressed by urban designers and planners, who typically deal with more fine-grain attributes of urban form – parcels, buildings, land uses, street networks – and who tend to study metropolitan form from the inside out. New Urbanists have used Patrick Geddes' idea of a "transect" to describe how the physical pattern of a city changes as distance increases from the center (Geddes 1915; Duany 2002). Kevin Lynch has suggested that the critical qualities to observe in any metropolitan area should include the distribution of building stock, the pattern of circulation and communication infrastructure, and the distribution of employment (Lynch 1991). He further argues that such data should be evaluated according to at least six performance dimensions: 1) the amount of opportunities and choices for goods, services and facilities that the layout makes available to different individuals, 2) the intensity of interaction that it produces for its users, 3) the initial investment costs of development, 4) the continuous operating costs of development, 5) the capacity of the environment to grow and change, and 6) the 'imageability' or ease with which the users of the metropolitan area can comprehend its structure. Embedded in Lynch's approach, is a conviction that metropolitan forms are not neutral products of complex social and economic processes, but also conscious policy and design choices that can make city environments perform better or worse. Lynch's work on metropolitan form steers the metrics towards normative evaluations that can help analysts or stakeholders comprehend not only how, but "how well" a city is growing. More research is needed on such normative analyses of city form. But before performance analysis can be implemented, reliable metrics describing the formal properties of the underlying cities are needed.

2 Metropolitan Form Analysis Toolbox

Although the literature on metrics or methods of capturing these qualities is scarce, still, most of the measures we use in the toolbox are derived from previous applied studies. We also propose a few new metrics – *polycentricity*, *land-use mix*, *expandability* – that have not been widely operationalized in comparative studies of metropolitan growth patterns in the past. Our aim has been to evaluate existing measures and to propose improvements that would allow them to capture more nuanced and more useful aspects of metropolitan form.

The metrics are selected such that in addition to the overall pattern of growth, the intra-urban changes could be identified and described. Size, coverage, discontiguity, compactness² and expandability target the properties of the overall shape of the built-up urban area. Polycentricity and land-use mix, on the other hand, focus on qualities within a metropolitan boundary. In the following we explain the specifications of the metrics that are implemented in the toolbox and demonstrate their application on

² Compactness can also capture internal qualities of a city, depending on the units of analysis that are employed.

sample data from Los Angeles, Singapore, Jakarta, Guangzhou and Chengdu, using data from Schneider, Friedl et al. (2009, 2010), LA County and Singapore.

2.1 Size and Density

The total built-up land cover of a city, the cover of a particular land use category (e.g. industry) and the density of resources that these areas accommodate (population, buildings, etc.) are probably the most fundamental and commonly used measures of urban form. Major trends of change can be apprehended by observing snapshots of urban land cover over time. Figure 1 illustrates the change in the area of urban extent of Chengdu and Guangzhou in the decade of 2000 to 2010. The figure shows that Chengdu grew by 755 square kilometers, doubling its size, while Guangzhou grew by 1426 km², 30% of its year 2000 area. The analysis allows us to categorize the observed growth by type – blue indicates continuous edge expansion, green infill development and red leapfrog development. Edge expansion formed the largest share in both cases, but there was notably more leapfrog growth in Guangzhou (15% of all expansion) than Chengdu (7%). One of the causes for more scattered growth in Guangzhou could be the rugged, unbuildable terrain around the city (see the coverage metric).

The size metric simply estimates the area of each urbanized polygon that forms part of a metro area and computes the combined area of all such polygons. Size can also be used to estimate the total area of a particular land use if such data is available. Density, on the other hand, estimates the amount of particular elements of a city per unit area of land in each urbanized polygon. Albeit basic, size and density offer valuable insights in comparing cities. A simultaneous comparison of different density measures can lead to a rather nuanced comparison of urban form. Comparing residential density, unit density, floor area ratios (FAR) and building coverage side by side, for instance, can illustrate important typological differences in urban form (Pont and Haupt, 2010).

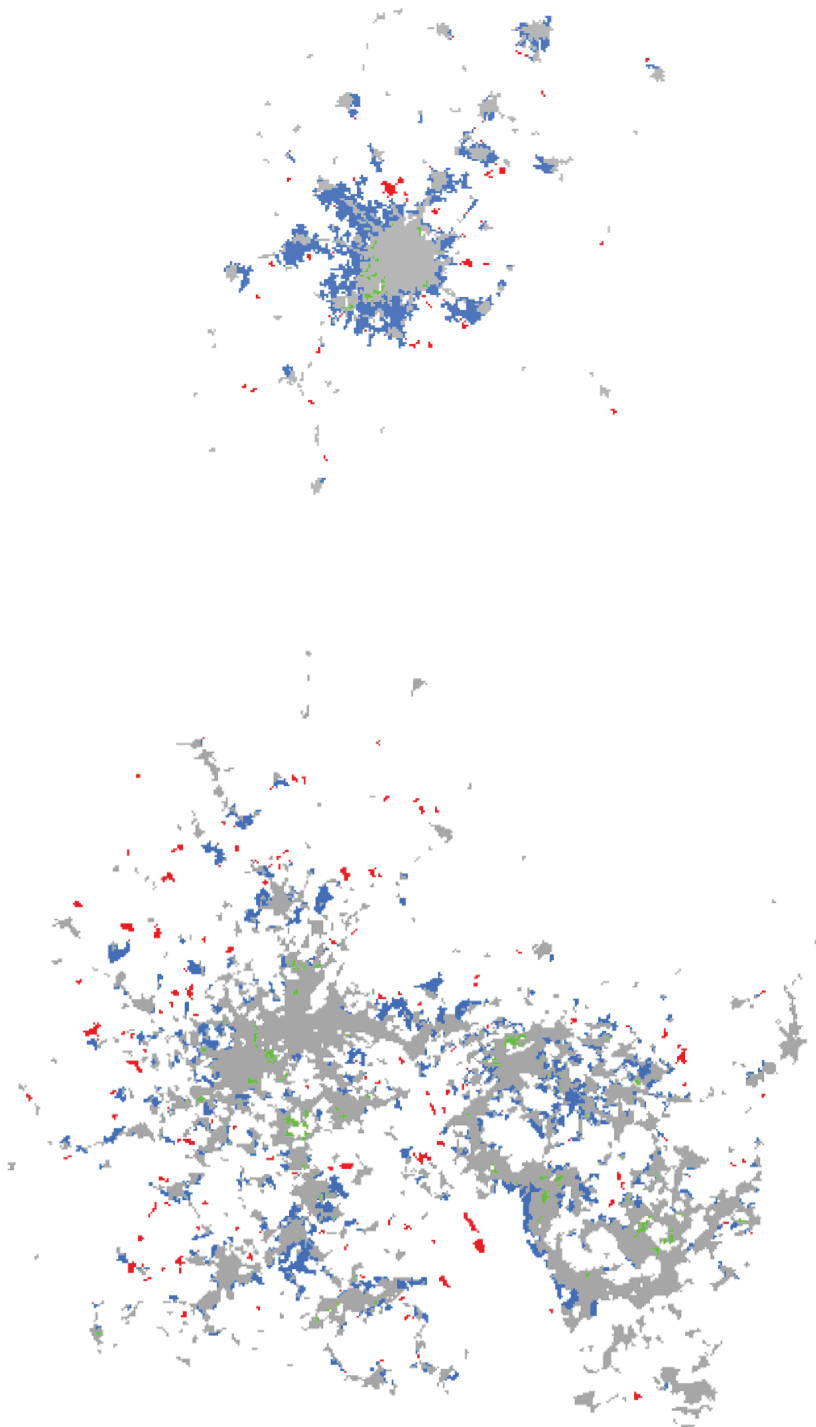


Figure 1: (Above) Size change in Chengdu 2000-2010. Leapfrog growth (red): 58 km²; Infill growth: 22 km²; Edge growth: 675 km². Total growth: 755 km². Total area in 2000: 755 km²; Total area in 2010: 1500 km².

(Below) Size change in Guangzhou 2000-2010. Leapfrog growth (red): 215 km²; Infill growth (green): 100 km²; Edge growth (blue): 1,111 km²; Total growth: 1,426 km². Total area in 2000: 4651 km²; Total area in 2010: 6077 km². Source: (Schneider, Friedl et al., 2009, 2010).

2.2 Coverage

Coverage illustrates how large a share of the total urban extent, or a sub-area of this extent, is covered by a given land use type. Coverage is another form of a density measure, whose numerator captures only the ground area of different uses. While most commonly estimated for building footprints – the percentage of the urban area that is covered by buildings – coverage can also be estimated for any land-use (LU_n) type over the area of study (A):

$$C_n = \frac{A[LU_n]}{A}$$

Two cities with similar population density and FAR can have vastly different urban forms. Parts of Singapore and Manila, for instance, both have high population densities and district-wide FAR, but the former is largely made up of high-rise buildings and the latter of high-density low-rise urban fabric. How both types of configurations can yield a similar population density and FAR is largely explained by their differences in building coverage. In Singapore, building coverage is generally low, and ample green space permeates between tall residential slabs. In low-rise settlements of Manila, buildings and circulation paths between them cover almost all ground.

In addition to building coverage, we propose a new measure that estimates the coverage of a city's urbanized land over the buildable land within its convex hull (Figure 2). The convex hull of a metropolitan area is defined as the smallest convex polygon that can accommodate all parts of the metropolitan extent. Not all land within the convex hull of a city is necessarily developable. Slopes that are too steep to build on, water bodies, natural reserves etc. can be said to be “unbuildable”. We get the “buildable” area of the convex hull by subtracting unbuildable areas.

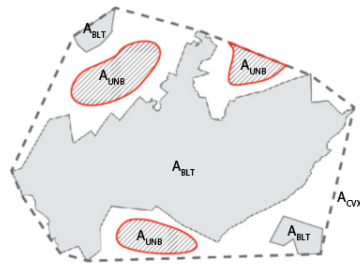


Figure 2: the convex hull (A_{CVX}) around developed polygons (A_{BLT}) and unbuildable land within the convex hull (A_{UNB}).

Estimating coverage within the convex hull minus unbuildable areas offers a consistent way of measuring how much land is urbanized in each city. The exact shape of the hull and the extent of the unbuildable area is relative to each city, but the estimation remains comparable since remaining vacant land within the hull can be said to be vacant *by choice*.

If the area of the convex hull around the developed polygons is A_{CVX} , the unbuildable area within the convex hull A_{UNB} , and the total area of all built-up polygons A_{BLT} , then the built-up coverage C_{BLT} is estimated as follows:

$$C_{BLT} = \frac{A_{BLT}}{(A_{CVX} - A_{UNB})}$$

The higher the built-up coverage, the more continuously a city is developed and the less leap-frog development it has. Cities with strong growth boundaries like Italian hill-towns, have a built-up coverage close to 100%.

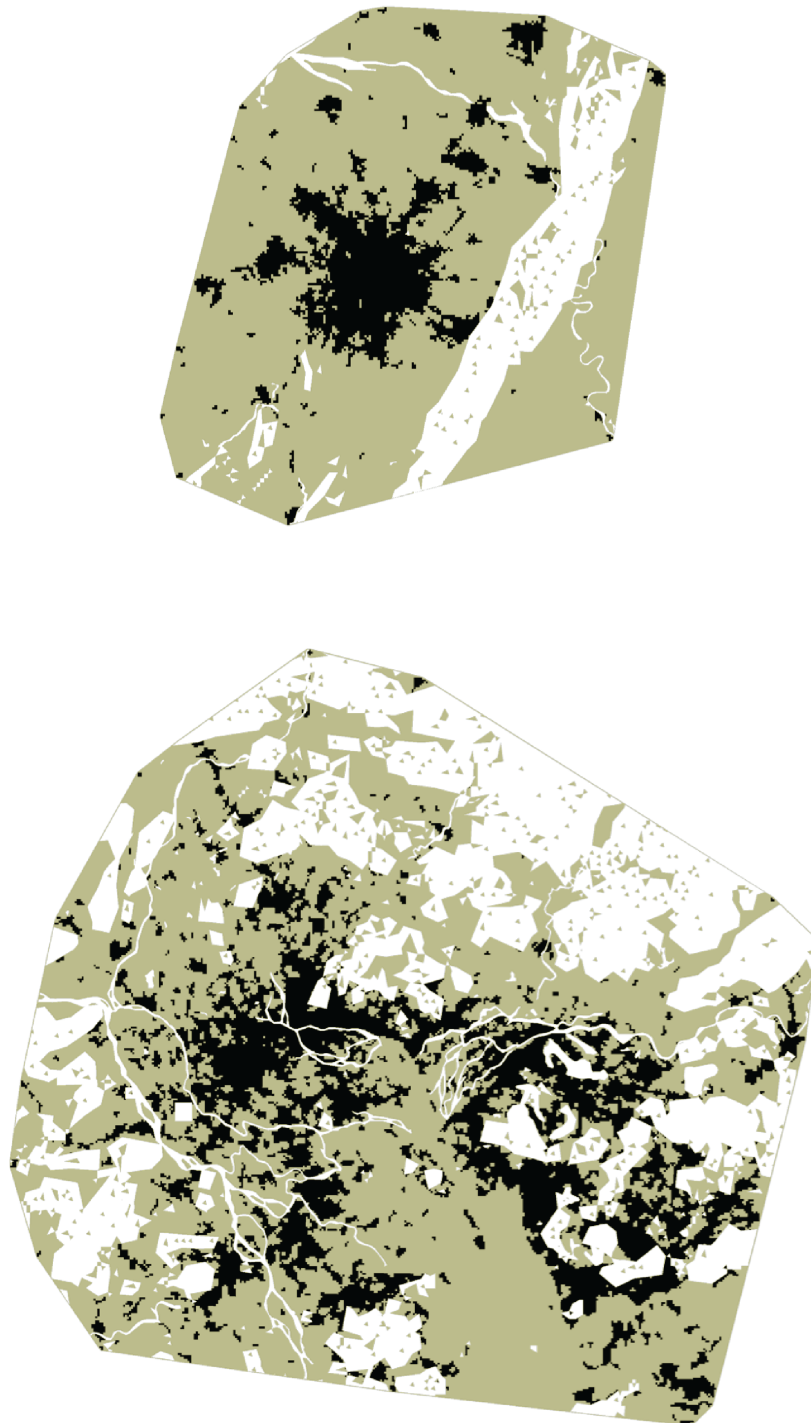


Figure 3: (Above) Coverage in Chengdu 2010. Built up area within the convex hull (black): 1,500 km²; Buildable area within the convex hull (green): 21786 km²; Unbuildable area within the convex hull (white): 10298 km². Coverage = 0.15%.

(Below) Coverage in Guangzhou 2010. Built up area within the convex hull (black): 6,077 km²; Buildable area within the convex hull (green): 10278 km²; Unbuildable area within the convex hull (white): 1889 km². Coverage = 0.28 %. Source: (Schneider, Friedl et al., 2009, 2010).

Figure 3 illustrates the urbanized coverage in Chengdu and Guangzhou. Even though the continuous central core of Chengdu visually suggests that Chengdu has a more compact metro area, the urbanized coverage is actually higher in Guangzhou (28%) than Chengdu (15%). This is because much of the terrain within the convex hull of Guangzhou, marked by the white areas in the figure, is unbuildable. Developing in valleys and plateaus, Guangzhou has covered more of the buildable land within its convex hull.

2.3 Polycentricity

The economic activities of a city are usually concentrated in one or more employment centers. Urban centers emerge from transportation cost savings rendered by proximity, from economies of scale in production and service activities, and from spatial externalities between different space users in a city (Fujita & Ogawa, 1982). Despite the continued dominance of the mono-centric urban model (Alonso 1964, Muth 1969, Mills 1967), its primary assumption that jobs are clustered in a single dominant location is becoming increasingly inconsistent with the spatial distribution of employment in contemporary metropolitan areas. Research has shown how increases in traffic congestion, land prices and rents, commuting costs and air pollution can jointly shift economies of scale to diseconomies of scale and lead to the emergence of multiple sub-centers (McMillen and Smith, 2003). Even the most mono-centric cities, such as Las Vegas, NV or Baltimore, MD have smaller sub-centers outside the central business district (CBD).

Quantifying polycentricity is challenging since defining a center is far from obvious and the number of centers is not necessarily the only yardstick for polycentricity. There is no universal definition for urban centers; centers can have blurry boundaries and polycentricity can be relative to the size of a city as well as the resolution of the lens with which we examine an urban landscape. An area can be a center locally, but not large enough to qualify as a center at the scale of the city or in comparison to other cities.

Polycentricity has been described at both an intra-urban level (polycentric cities), and inter-urban level (polycentric urban regions). The studies of intra-urban polycentricity have been mainly focused on identifying centers based on analysis of job density. Even at the intra-urban level, three approaches to distinguishing centers can be detected (Masip, 2011).

The first group (e.g. McDonald, 1987) defines a sub-center simply as the second peak beyond CBD, the census tract that has higher employment density than its neighboring tracts, regardless of its share in the total number of employment of the city. This approach fails to recognize the fact that several contiguous tracts can jointly form a local peak. The second group uses a lower cut-off criterion to identify sub-centers. Some have defined sub-centers by an absolute lower cut-off. Giuliano and Small (1991), for instance, defined sub-centers as census tracts with a density of more than 10 employees per acre and at least 10,000 jobs. Others have used relative cut-off criteria. García-López (2007, 2008) and Muñiz and García-López (2009) defined sub-centers as “zones with a density higher than the metropolitan average and at least 1% of metropolitan employment.” The third group of methods uses the standard mono-centric model to identify sub-centers. For instance, McDonald and Prather (1994) defined sub-centers as areas with positive residuals that are significant at a 95% confidence level – areas with densities significantly higher than the expected density in the mono-centric model, based on the bell-curve distance from the CBD. The main disadvantage of this method is that it presumes the existence and location of a CBD. Some in this last group use a weighted regression for smoothing the natural logarithm of employment density, which allows for local variation in the flattening rate of the gradient. These studies have been mostly interested in finding the number of centers that fit the proposed definitions, and do not take into account the overall share of jobs that are in centers and the relative size balance between centers³.

³ A city where only 10% of jobs are located in centers (the rest are dispersed) would thus be considered as polycentric as a city where 90% of the jobs are in centers. Likewise, a city with one large center and three small centers would be considered as polycentric as a city with four equally large centers. We have tried to address these shortcomings in the proposed metric for polycentricity below.

At the inter-urban level, identifying centers has not been the main challenge, as centers are usually marked by cities. The focus of these studies has instead been on measuring polycentricity based on the relative size balance between centers. The Entropy Index (Limtanakool, Schwanen et al., 2009), for instance, examines how uniformly commuting flows are distributed among centers. It is defined as:

$$EI = - \sum_{i=1}^L \frac{(z_i) \ln(z_i)}{\ln(L)}$$

where EI is the entropy index, Z_i the ratio of the number of trips from i to the total number of trips within the region, and L is the number of cities. EI is constrained between 0 and 1. It is undefined if all trips involve only one city, and 1 if trips are equally distributed among all cities.

Our proposed polycentricity metric is inspired by both intra- and inter-urban polycentricity metrics. We build upon the existing density-based methods in order to identify centers, adding some important improvements and extend the concept of spatial interaction between centers to include a size balance consideration into our polycentricity measure, analogous to Limtanakool's index above.

Detecting Centers

The definition of centers is based on the distribution of jobs in a city (Figure 4, top left). The specification thus requires the availability of spatial employment data, though different resolutions (zip, tract, block, address) can be used. In order to first distinguish urban centers in the data, we implement the following three definitions:

- 1) Job density at urban centers has to be higher than two standard deviations from the mean density in the entire city (Figure 4, top right). This threshold is relative to each city and not universally defined.
- 2) If several adjacent polygons pass the above cut-off threshold, then these neighboring polygons are grouped together to form joint centers (Figure 4, bottom left).
- 3) A center must contain n % or more of all jobs in a city (Figure 4, bottom right). While many places in a city can have a high number of jobs clustered on a small area of land, an area should not be considered a center if it does not contain an adequate percentage of total jobs in the city. Note that n should not be fixed for all city sizes, but the minimum cluster size threshold should vary along with the size of the city. We have experimentally determined that " $n=10/\sqrt{\text{population}}$ " yields suitable values that adjust intuitively to city size. In a city of 100,000 jobs, it yields a cutoff at 3.16% (3,162 jobs), in a city of 1,000,000 jobs n becomes 1% (10,000 jobs) and in a city of 10,000,000 jobs n is 0.32% (31,622 jobs). Ultimately, n can be refined on actual data by testing the polycentricity measure on multiple cities and regions.

The third step thus counts the total number of jobs found in each cluster and eliminates clusters that do not pass the appropriate minimum percentage test, adjusted to city size. We conclude the procedure of detecting centers by assigning an ID to each center that is found and summing the total number of jobs in each center.

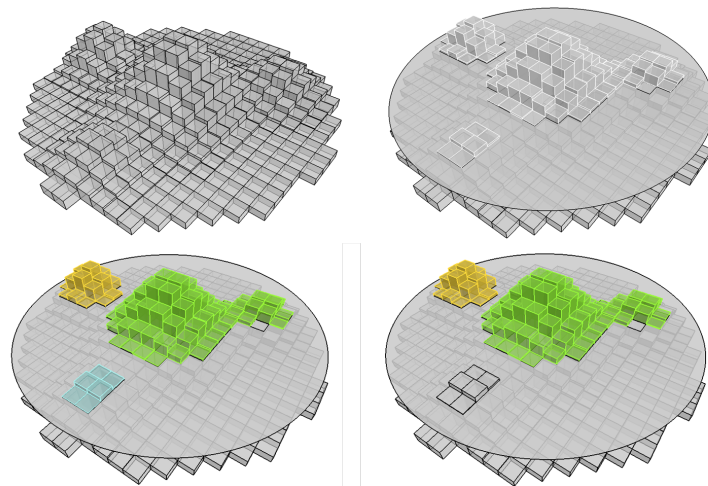


Figure 4: (Top left): A raster dataset of employment density measures in a hypothetical polycentric city. (Top right): Employment density at urban centers has to be higher than two standard deviations from the mean density in the entire city. (Bottom left): If several adjacent polygons pass the 2 Std. Dev. cut-off threshold, then these neighboring polygons or raster cells are grouped together to form joint centers. (Bottom right): A center must contain n % or more of all jobs in a city, where $n = 10/\sqrt{\text{population}}$. In a city of 100,000 jobs, it yields a cut-off at 3.16% (3,162 jobs), in a city of 1,000,000 jobs, n becomes 1% (10,000 jobs), and in a city of 10,000,000 jobs, n is 0.32% (31,622 jobs).

Estimating Polycentricity

Having defined the centers, we propose a polycentricity metric whose value depends simultaneously on the number of centers, the sizes of the centers, and the relative size distribution between centers that are found in a city. We consider a city to be more polycentric if a) it has more centers b) a greater share of its total job pool is located in its centers and c) the less any one of its center dominates, and the more equally balanced the sizes of its different sub-centers are. We, thus, define polycentricity PC as:

$$PC = HI \times N \times R_c$$

where PC is the polycentricity index, HI the homogeneity index, N the number of centers, and R_c the ratio of the total amount of jobs found in all centers to the total amount of jobs in the city. HI measures the degree to which the sizes of centers are homogenous. We define HI by using Limtanakool et al (2009) Entropy Index:

$$HI = - \sum_{i=1}^L \frac{(z_i) \ln(z_i)}{\ln(L)}$$

, where Z_i is the ratio of the number of jobs at center i to the total number of jobs in all centers, and L the number of centers in the city. HI is constrained between 0 and 1; it is 1 if jobs are equally distributed among all centers and tends to zero if the share of jobs in one center tends to 100%. The index is undefined if all jobs are in a single center as the denominator would be zero.

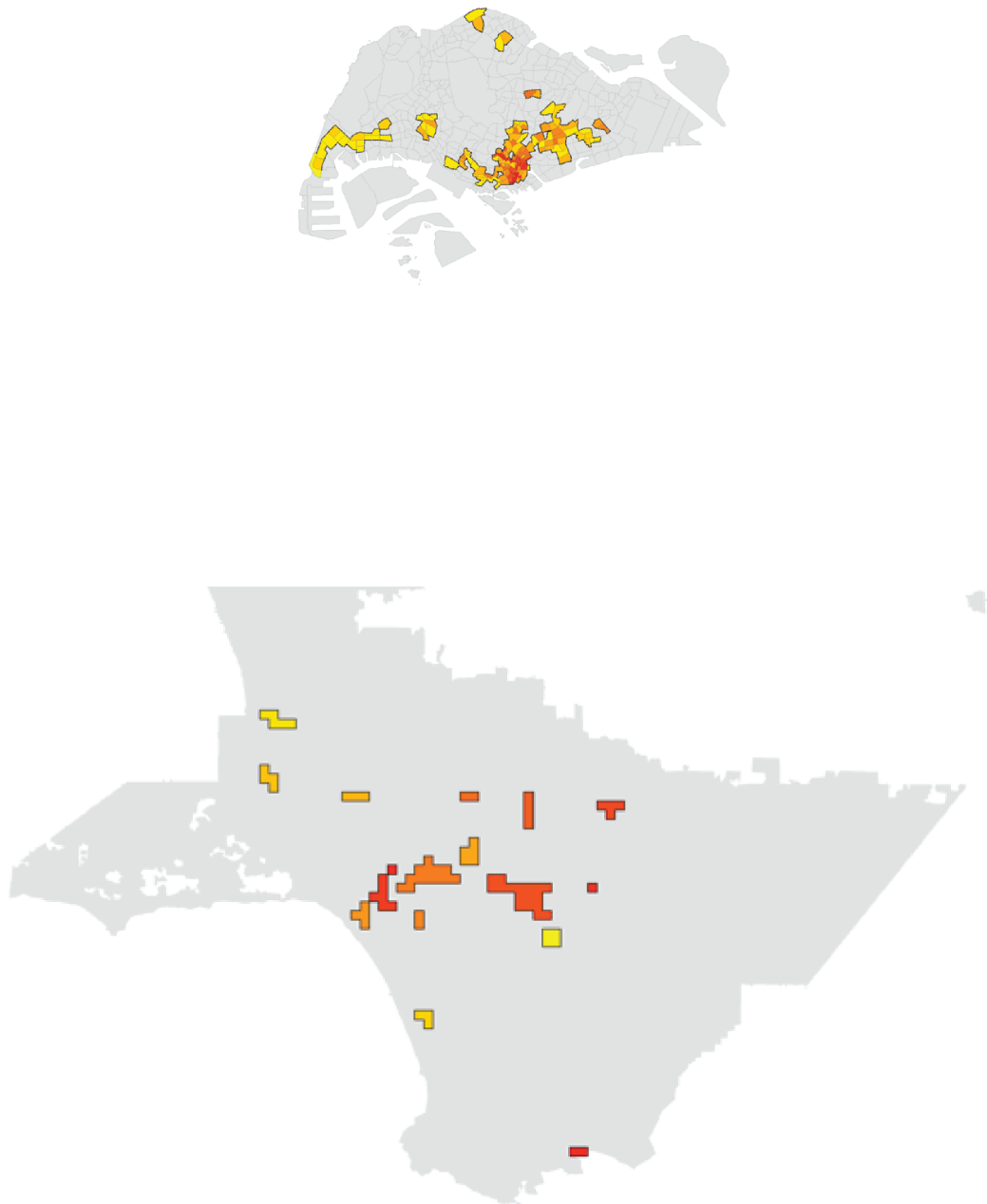


Figure 5: (Above) Polycentricity in Singapore 2010. Singapore has 7 centers with employment densities at least 2 Std. Dev. over the city-wide mean. Polycentricity = 1.67.

(Below) 1 Polycentricity in Los Angeles county 2010. LA county has 17 centers with employment densities at least 2 Std. Dev. over the city-wide mean. Polycentricity = 3.70. Source: LA County and Singapore employment data.

Figure 5 illustrates the polycentricity estimates for Singapore (1.67) and Los Angeles County (3.70). Seven centers can be detected in Singapore with employment densities that are two standard

deviations above the mean. In LA County, the number of such centers is 17. The size balance between the centers is more evenly distributed in LA, whereas in Singapore the downtown core accommodates significantly higher employment densities than other sub-centers. Lastly, the total share of employment located in these centers is 53% in Singapore and 25% in LA, favoring greater polycentricity in Singapore. But since two out of three ingredients of the index suggest that LA County is more polycentric, it obtains a higher overall score of the two.

2.4 Compactness⁴

Compactness, and its inverse quality – dispersion⁵ – measure the degree to which the resources of a city – people, buildings, jobs etc. – are spatially spread out; the closer they are located to each other, the more compact the city is.

One of the most commonly used measure for capturing the compactness of a city is areal density – the quantity of resources per unit area of land. Areal density assumes, however, that resources are uniformly distributed throughout the study area – it neither distinguishes internally homogenous or heterogeneous distributions, contiguous or discontinuous developments, nor the effects that the shape of development can have on its dispersion (Figure 6).

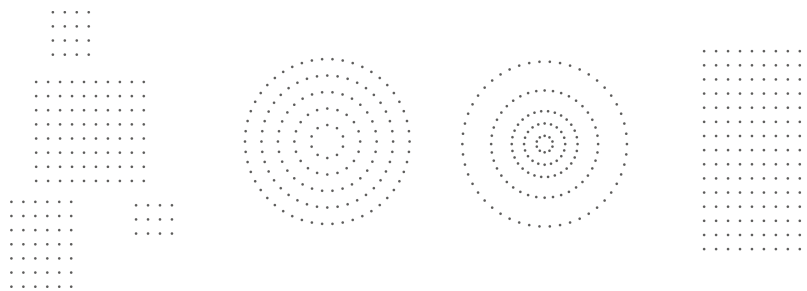


Figure 6: The internal distribution of a city's resources can vary considerably, which remain invisible to a density measure. All configurations have similar densities.

Another way of capturing the compactness of a city's development is achieved by measuring the availability of open space around each piece of developed land. The openness index (Burchfield, Overman et al., 2006), for instance, measures dispersion as the average percentage of open space in the immediate square kilometer around each residential development. As openness, too, is basically a density measure it comes with the same shortcomings in capturing Compactness mentioned above.

Bertaud and Malpezzi (2003) define dispersion – the inverse quality of compactness – as the ratio between the average distance from the centroids of population tracts to the CBD and the average distance of the same population from the centroid of a hypothetical circular city of the same size⁶. In their measurements of 48 cities they assume the CBD to be at the geometrical centroid of all population tracts of a metro area.

The main shortcoming of this measure is that it only captures the relationship between developed areas and the

4 The term compactness has been mostly used by the group metrics that examine the similarity of the boundary of the metropolitan area to a circular disk; however, as a universally agreed terminology does not yet exist in the literature, we use Compactness for our proposed metric.

5 Dispersion has also been used to characterize urban sprawl. The concept of sprawl, however, involves a number of factors beyond spatial form and remains poorly defined in literature.

6 The average distance of a uniformly distributed population from the center of a disk is equal to two third of the radius of the disk.

city center and overlooks their spatial relationships with respect to each other. This may be problematic in cities where the CBD is lacking or unimportant (e.g. Randstadt in Holland) or where the CBD is not at the geometric centroid of the tracts (e.g. Singapore). The metric is more appropriate for a symmetrical monocentric city than a polycentric metropolis. Second, while this measure weighs distances to the center by population, it is not affected by the absolute changes of density in the city – if the relative size balance in tracts is kept constant, but the population in each tract is increased at the same rate, the dispersion index would remain constant⁷.

In geometry, compactness is defined by how a shape corresponds to a circular disk. If all else equal, spatial distributions are most compact if their aggregate collection forms a circular shape. Circular distributions, as widely witnessed in nature, have the smallest perimeter-to-area ratio of any two-dimensional geometric shape. Capitalizing on this property of circular shapes, a number of researchers have described the observed dispersion of urban resources (people, development, buildings) in comparison to a perfectly circular distribution of the same amount of resources.

Angel et al. (2005) have defined compactness (the opposite of dispersion) in a similar vein, but instead of comparing the observed development to a hypothetical circle, they compare it to the actual availability of developable land in that city. Whereas Bertaud's index relies on distance measurements in defining dispersion, Angel's measure relies on area measurements. Angel et al. account for geographical constraints in the observed area, arguing that "compactness should be restricted to buildable areas, in the sense that a city located on a coast, on a mesa cut up by steep gorges, or in a valley surrounded by steep cliffs can be very compact even if it does not resemble a full disk" (p. 68, *Ibid*). Their compactness measure is thus defined as the ratio of the observed built-up area to the observed buildable area within "the circle of minimum radius encompassing the consolidated built-up area of the city." The circle, which is used only for the purposes of restricting the geographical extent that is compared, can be defined either as "the minimum radius encompassing the consolidated built-up area of the city" – called the outer circle – or restricted to the main built up area of the city – which we might call the inner circle (Figure 7).

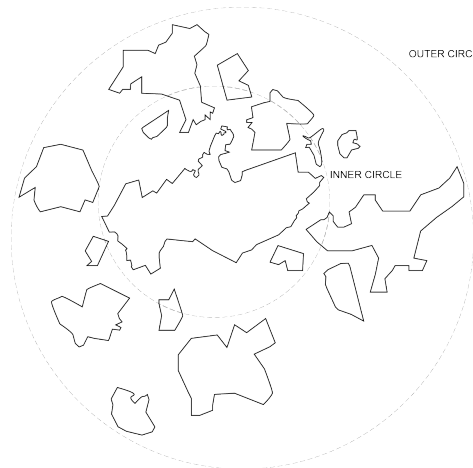


Figure 7: The circle of minimum radius encompassing the consolidated built-up area of the city.
Source: Angel et al. (2005).

A key challenge to the index is that if the outer circle is used, a small and consolidated built-up area located far from the main built-up area can significantly impact the radius of the circle, increasing the circle's reference area exponentially. If the calculation of the index is restricted to only the largest continuous built-up area — as performed by Angel's team — then the index may return unreasonably high compactness values in cities with satellite towns that have significant developments outside of the main built-up area (e.g. Paris, Singapore, Seoul).

⁷ However, since the distances from census tracts to the CBD is weighted by population, keeping the total population constant, but changing the arrangements of density between census tracts does affect the outcome.

Proposed measure of compactness

A compactness index should capture the degree to which the resources of a city (e.g. people, buildings, jobs, etc.) are spread out. Put alternatively, the metric should capture how accessible different parts of the city are accessible to each other. There is an analogous measure in transportation research, called “Gravity” (Hansen, 1959). The gravity index of a location is proportional to the total amount of resources available to that location and inversely proportional to the travel cost of reaching them:

$$G_i = \sum_{j \in G - \{i\}} \frac{W[j]}{e^{\beta \cdot d[i,j]}}$$

, where G_i is the gravity index for location i , $W[j]$ the size or attractiveness of the destination j , and $d[i,j]$ the distance between locations i and j , and beta is the exponent that controls the effect of distance decay between i and j . Distance $d[i,j]$ can be measured from the centroid of polygon i to the centroid of polygon j .

Computing the gravity index from each built-up polygon to all other polygons in a metropolitan area and taking the mean result across all polygons, can thus capture how compactly the resources of a metropolitan region are situated with respect to each other (Figure 8).

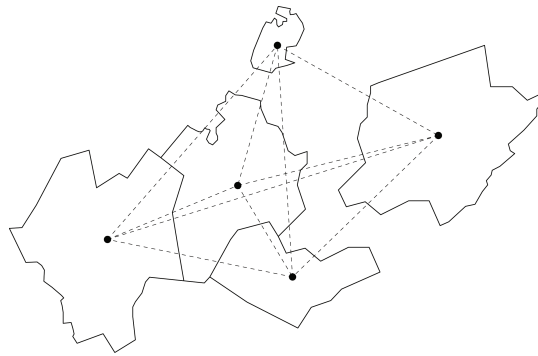


Figure 8: The proposed compactness metric is based on distance measurements between the centroids of built-up areas.

If weighted by the size or other respective properties of the resources, then the spatial relationships between larger destinations have a proportionately stronger effect on the index than smaller destinations.

$$G = \frac{\sum_{i \in G} W[i] \cdot G_i}{\sum_{i \in G} W[i]}$$

The key improvement of the proposed compactness measure is its flexibility for using different units of analysis based on the availability of data, computational limitation, or the objectives of the analysis. Rather than using the built-up polygons of the metropolitan area as the unit of analysis, for instance, compactness can be measured based on the raster dataset of the metropolitan extent, where raster cells are the units of analysis. Using raster cells allows one to capture very subtle variations in the compactness across different cities.⁸ Using census tracts, weighted by their population, buildings or business locations as the unit of analysis, allows compactness to be measured for a particular type of land use.

The weighted average gravity measure is obviously impacted by the total amount of the resources in the city. It would not make sense to get a higher average gravity in a larger city that has more people than in a smaller city

⁸ It is similar to the Cohesion index used by Parent, Civco et al. (2009).

that has less people, even if the latter is more compactly distributed. The mean gravity measure discussed above, therefore cannot be used to compare compactness across cities of different size unless it is reasonably normalized.

We include three reasonable ways to proceed with the normalization in the Metropolitan Form Analysis toolbox. Two of the methods normalize the mean gravity index by the same measure in a reference case. In one of these methods, the reference case is a circular city with its number of resources (population, jobs, or built-up pixels) similar to the actual city, and uniformly distributed. The density of the reference city ρ is a fixed value across all cities — for example $\rho=1000$ units per square kilometer. As ρ is a constant, the area can be derived from the given population in every city (Figure 9).

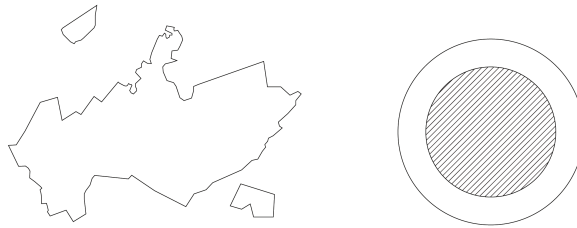


Figure 9: Normalization by a reference case. The area of the larger circle – reference case defined by Bertaud et al – is equal to the total built-up area. The area of our proposed reference disk, the disk, is determined by the total population, and a reference density, ρ , which is treated as a constant.

In the second method, geographic constraints are super-imposed on the reference case, which, in this case, is a polygon derived by subtracting the unbuildable land from the circle C . The reference circle shares the same geometric center as the observed city, such that the remaining area is equal to the area of the reference case calculated by the method discussed above (Figure 10).

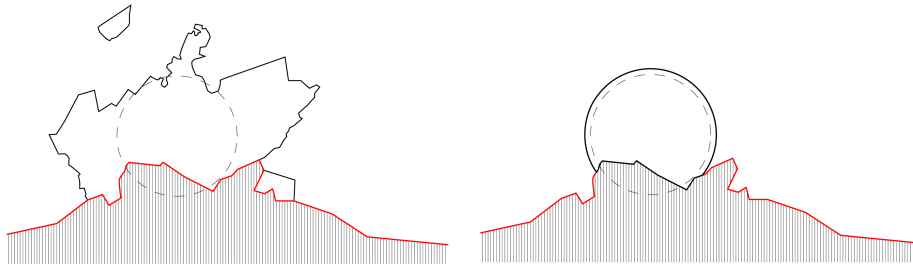


Figure 10: Normalization by a reference case, accounting for geographic constraints. The dashed circle is the reference case without accounting for the geographic constraints, and its area is equal to the area of the reference case polygon that accounts for the geographic constraints.

Finally, the mean gravity index across all observed polygons can also be normalized by the total amount of resources (i.e. population) in the city. This can be defined as follows:

$$G_{i-norm} = e \cdot \frac{G_i}{W_{total}}$$

, where W_{total} is the total amount of resources in the city (e.g. population). The compactness index C is then given as the weighted average of the normalized gravity indices of all polygons i :

$$C = \frac{\sum_{i \in G} W[i] \cdot G_{i-norm}}{\sum_{i \in G} W[i]}$$

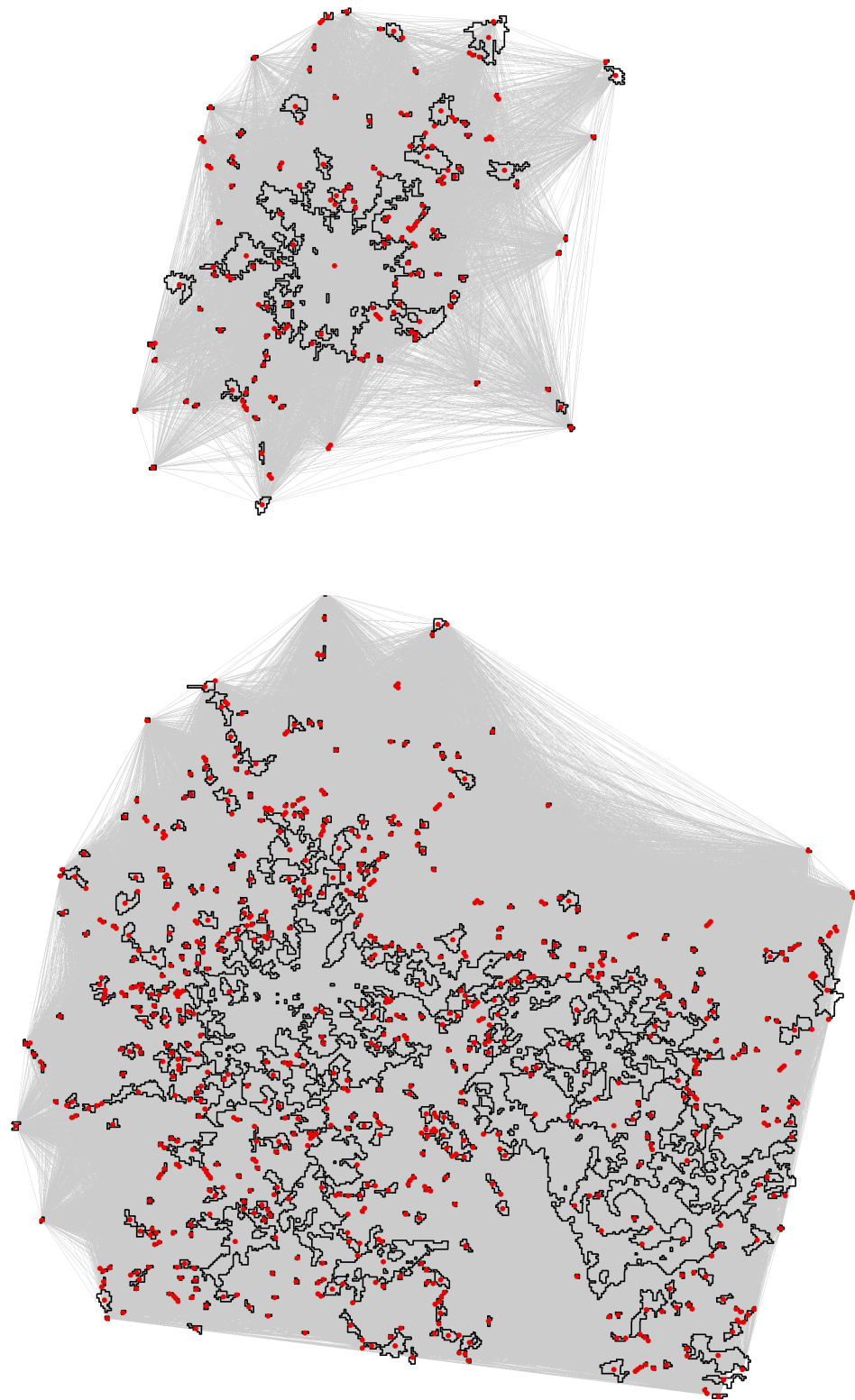


Figure 11: (Above) Compactness in Chengdu 2010: 1.10, Change from 2000: -4%.

(Below) Compactness in Guangzhou 2010: 1.09, Change from 2000: -15%. Normalization by total population. Source: (Schneider, Friedl et al., 2009, 2010).

2.5 Expandability

A key determinant of a city's growth is the availability of buildable land in its vicinity. The availability of space not only affects the rate of possible growth but also its character. Cities that are constrained by geographic features, such as water bodies or steeply sloped land, grow very differently from those with no barriers around them. The former, for instance, leave no room for leapfrog development and set serious physical limits on sprawl; the latter allow for spatially spread-out and fragmented growth. The Expandability metric aims to capture these constraints by quantifying the availability of buildable land beyond the urban extent within the non-urban realm. Quantifying expandability is fundamental to explaining sprawl, segregation, density and land prices.

The key challenge in quantifying the expandability of a city is to define a reasonable zone for the measurement of the buildable area around existing urban clusters, which could be analyzed consistently for expansion across cities.

City authorities are interested in knowing how much developable land is available in their administrative area. Actual growth may occur, however, well beyond the existing administrative boundaries. The size of administrative boundaries in different cities can range widely and may occasionally constitute an area many times as large as the current urban extent. Studies of land supply and land demand management (e.g. Hopkins and Knaap 2000) have used the urban growth boundary as a limit for land supply. Growth boundaries, however, are inadequate for our purpose since few cities have legally implemented urban growth boundaries. Furthermore, the definition and regulation of growth boundaries varies widely across cities.

Albert Saiz's (2010) *Geographic Determinants of Housing Supply* is one of the rare studies on measuring developable lands that has disregarded administrative boundaries. Saiz's search area constitutes a 50-kilometer radius from the centroid of the city. But as Wendell Cox (2011) has rightly pointed out, an invariant search radius makes results incomparable in cities of different size. While in larger cities a 50-kilometer radius may barely cover the built-up area, for smaller towns it may contain several times their existing urban extent. Angel's compactness metric (2005), also estimates the proportion of developed land over all developable land and uses a circle around all built-up areas, or a smaller circle around the main built-up area. Though the circle is relative to a city's size and shape, it is also extremely sensitive to small outlying developments or narrow "peninsulas" stretching out from the main built-up area.

In order to specify a measure that captures the availability of buildable land in a consistent way across cities of different size, we propose an expandability metric that satisfies the following conditions:

- a) The search radius should be measured from the edges of urban extent rather than its centroid.
- b) The search area should be relative to city size.

The zone in which our proposed metric measures the buildable area is an idealized expansion of the existing built area, which is found by offsetting the boundaries of all built-up polygons so far that the total built-up area of each polygon doubles^{9&10}. We compute expandability index as the ratio between the area of buildable parts of this idealized expansion (A_e) and the existing built area (A_b):

$$\text{Expandability} = A_e / A_b$$

, where A_e is double the existing urbanized area minus the unbuildable area (Figure 12).

9 The particular choice of 100% expansion roughly matches the average 20-year growth that was observed across 66 cities in Angel's study (2005). The radius can be adjusted as needed by the analyst to reflect typical annual growth, 5-year growth or other growth of existing built area.

10 The offset radius at which we precisely double the area of A_b cannot be mathematically pre-determined, but it can be found in a simple automated iteration of offsets that check the expanded area against the original area until the right radius is found.

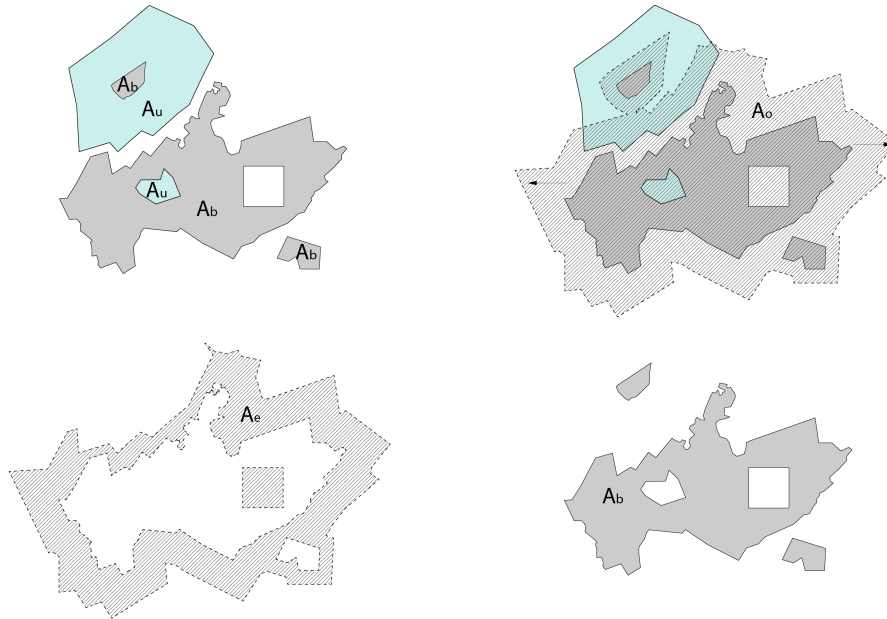


Figure 12 (Top left) Existing built-up area A_b and unbuildable area A_u . (Top right) The Idealized expansion, A_o ; the area of A_o is twice the existing built-up area A_b . (Bottom left) The expansion area A_e is found by subtracting all the unbuildable areas A_u from the idealized offset area A_o . (Bottom right) The final expandability metric is computed as the ratio between the expansion area A_e and the existing built area A_b .

When we consider areas that are unbuildable, we can categorize such land into two groups:

- 1) Land that is unbuildable due to natural obstacles, such as mountains or water.
- 2) Land that is unbuildable due to human policy choices, such as parks, protected areas or urban growth boundaries.

The two types of unbuildable land can have a different effect on a city's growth. Those areas that are put aside as unbuildable due to conscious policy regulation could have a higher risk of being re-zoned for building than natural obstacles. But even natural obstacles are not set in stone – Singapore has expanded its shorelines by 20% in 40 years, and a number of cities have historically leveled mountains to make way for urbanization. These risks can be integrated into our Expandability index by considering the financial and technological capacity of the city and allowing a small fraction of the obstacles to be overturned each year.

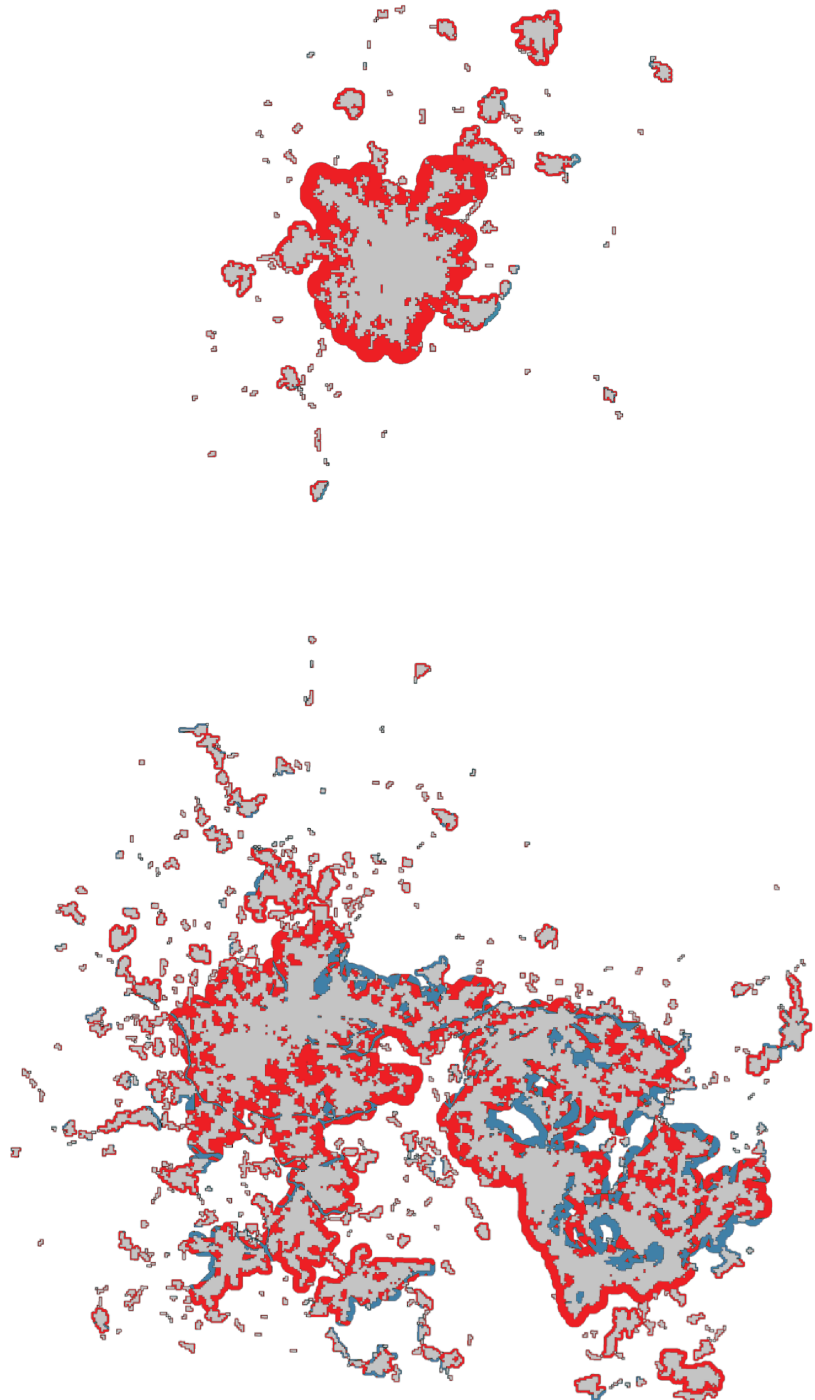


Figure 13: (Above) Expandability in Chengdu 2010. Red: buildable area of the hypothetical 100% expansion; Blue: unbuildable. Expandability in 2000 74.1% and in 2010 70.1%

(Below) Expandability in Chengdu 2010. Red: buildable area of the hypothetical 100% expansion; Blue: unbuildable. Expandability in 2000 88.1% and in 2010 87.5%. Source: (Schneider, Friedl et al., 2009, 2010).

2.6 Discontiguity

The expansion of metropolitan areas does not only happen on the edge. Leapfrog growth is a universal phenomenon, happening almost in all contemporary metropolitan areas. Climatic conditions, availability of affordable land outside the main developed cluster, flexible zoning regulations, ground-water distribution, and low transportation costs are often the key drivers of discontinuous growth (Burcfield, Overman et al. 2006).

While the number of urban clusters and their size provide a general description of the discontiguity of a metropolitan area (see Figure 1), they do not tell us much about the structure of the city's fragmentation. In order to gain a deeper quantitative description of the discontiguity of metropolitan form, we need to look at the rank order and relative size difference between discontinuous urban clusters.

Efforts to quantify the contiguity of urban form or its reverse quality – discontiguity – have been rare. The most popular description of contiguity, developed by Angel and his colleagues (2005), describes contiguity as the ratio between the main (largest) built-up area of the city and the sum total built-up area of the city. The more built-up area is concentrated into the single largest cluster, the more contiguous the city is. This measure is easy to compute and is useful as long as the main built-up area constitutes a large portion of the total built extent of the city. But the metric is not well suited to distinguish forms of discontiguity when a city is made of multiple larger or smaller built-up clusters, with a large portion of the total built-up area located outside of the biggest cluster. The metric does not account for rank-size relationships between individual discontinuous areas beyond the largest cluster.

Similar to Angel et al. (2005), we assume that the fewer the total number of discontinuous developments, the more contiguous a metropolitan area is (Figure 14). Although our proposed description is a measure of discontiguity rather than contiguity, it is in essence similar to the measure developed by Angel and his colleagues (2005). While based on the relative size of urbanized clusters, the metric additionally accounts for the areas of all clusters that are smaller than the largest cluster (Figure 15). We define discontiguity as follows:

$$DC = \sum_{n=1}^N \left(\frac{\sum_{i=n+1}^N A_i}{A_n} \right) \left(\frac{\sum_{i=n}^N A_i}{A_{total}} \right)$$

where DC is the discontiguity of the built-up area, N the number of urbanized clusters, A_n the area of cluster n , and A_{total} the joint area of the urban extent. Note that $A_n \geq A_{n+1}$, so that the denominator in the first part of the index always compares other areas to the largest continuous area.

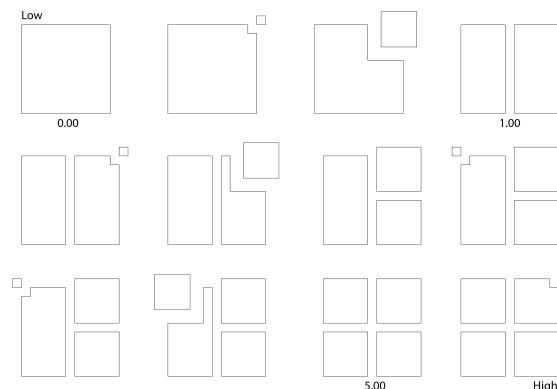


Figure 14: Interpretation of the discontiguity metric; the top left configuration has the lowest discontiguity results, the bottom right one the highest. The numbers indicate the actual computed results for selected configurations.

The key improvement of the proposed index is that it accounts for the size relationships between all the clusters in the city by calculating the same ratio – the area of each cluster that is smaller than the largest cluster to the area of the largest cluster – and summing up these ratios, weighted by their share of the total area.



Figure 15: (Above) Discontiguity in Chengdu 2010: 210; number of cluster in 2010: 164. Discontiguity in Chengdu 2000: 419; number of cluster in 2000: 153.

(Below) Discontiguity in Guangzhou 2010: 4533; number of clusters in 2010: 734. Discontiguity in Guangzhou 2000: 6220, number of clusters in 2000: 757. Source: (Schneider, Friedl et al., 2009, 2010).

2.7 Land-Use Mix

Land-use Mix constitutes another important characteristic of the built environment that we include in the Metropolitan Form Analysis toolbox. Land-use distribution impacts on traffic congestion, transportation energy consumption, real estate values and crime rates are extensively discussed in the planning literature. While mixed-use developments are widely promoted by planners, supporting quantitative evidence of their effects is underdeveloped and even contradictory. This is partly attributable to a lack of intuitive and commonly accepted metrics that can capture how mixed or segregated the land-uses of an urban area are. Perhaps more importantly, there has been little discussion on what levels and configurations of mixing are actually desirable.

There are two popular types of metrics that capture land-use mixing. The first focuses on the number of different uses that are found in a given area, allowing comparative areas to be ranked according to the number of land-use types they accommodate (1-n). The second focuses on the relative balance between uses; it tells us how heterogeneous or homogenous the land-use pattern of an area is based on how equally the area is occupied by different uses.

The former may not be meaningful if different uses occupy notably different amounts of land. This shortcoming is addressed in the latter, but heterogeneity indices, too, have important shortcomings. Most of such indices weigh all uses equally and assume that an equal distribution of each type of use is the benchmark to compare an observed pattern against. Cities do not have an equal share of all land-uses – a much larger share of land is typically used for residential purposes than commercial purposes. Industrial and transportation lands often top commercial land, as the former tend to accommodate rather land-intensive activities. Second, not all types of land-uses tend to mix with each other at equal likelihoods (Hess, Moudon, & Logsdon. 2001). A mixture between commerce and housing is far more likely in most cities than a mixture between industry and housing. Military land and agricultural land tend to stay apart from other land uses for logistical, security, and economic reasons. Instead of an equal weighting and mixing benchmark, a land-use mix metric could use a “expected weighting” that is based on the observed citywide balance of land-use types, and an “expected mixing” ratio that is based on realistic examples.

Proposed measure of land-use mix

For each location, we propose a land-use mix metric that illustrates how closely the distribution of observed uses in a given neighborhood around that location corresponds to an expected distribution. There is no consensus on how large the evaluation neighborhood ought to be, but we propose to use a standard of one square kilometer (1 km²) that is large enough to detect use mixing and convenient to derive in many cities (Burchfield et al., 2006). In order to avoid making the index vulnerable to small shifts in the boundaries of analysis units, we overlap the square kilometer units of analysis with each other, as shown in Figure 17 below.

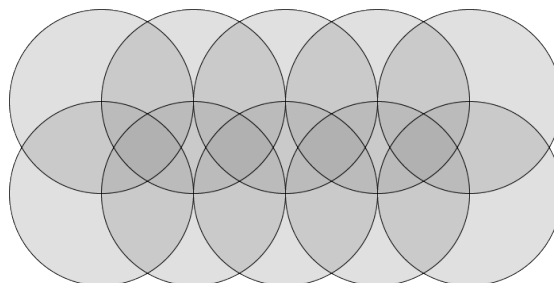


Figure 16: Square kilometer size estimation areas overlapped with each other, such that the orthogonal distances between centers equals the radius of a circle.

Let us define the square kilometer neighborhood of a given cell as i and a particular land use of interest n as LU_n .

We can give a weight w to each of the land uses in i , based on how much area of the neighborhood they occupy. $w[LU_{n,i}]$ thus denotes the weight of land use n within the square kilometer neighborhood of point i . We specify $S_{n,i}$ as the share of LU_n among all land uses of interest within the area i (including n itself) which ranges between 0 and 1.

$$S_{n,i} = \frac{w[LU_{n,i}]}{\sum_{n=1}^N w[LU_{n,i}]}$$

If the expected share of land use n in all land-uses of interest in the square kilometer around i is defined as $ES_{n,i}$, we can estimate a “matching index” $M_{n,i}$ that shows how closely the observed coverage of land use n matches the expected distribution of that same land use n within the area of i :

$$M_{n,i} = 1 - |S_{n,i} - ES_{n,i}|$$

$ES_{n,i}$ can be determined in a number of ways depending on the intentions of the analyst; we propose to base its specification on the following criteria:

- 1) The expected distribution $ES_{n,i}$ of land use n in area i should depend on the city-wide total balance of all land-uses of interest (Figure 17). Given the presence of land use n in the whole area of the city, we can determine the likelihood of its presence in any smaller sub-area of the city.
- 2) The expected distribution should also depend on the likelihood of co-location between any pair of land uses. A mixture between commerce and housing is more likely in most cities than industry and housing. The “expected mixing” ratio should, however, not reflect the city-wide average, but instead a desirable scenario based on typical examples of urban tissue.

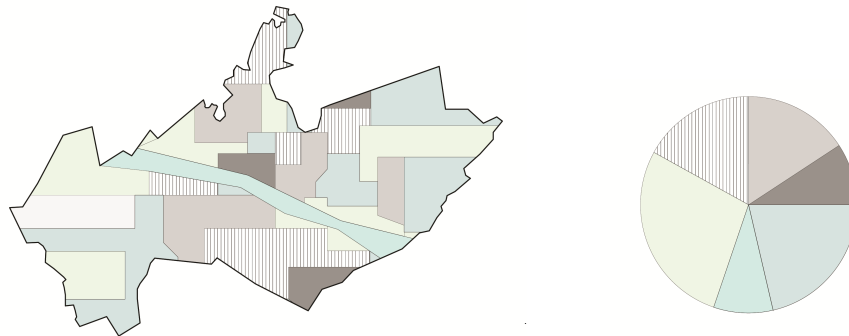


Figure 17: The expected land-use mix should be based on the city-wide distribution. Different cities include different types and balances of land-uses.

The absolute value alone would tell us how much the observed value deviates from the expected value, and subtracting this deviation from one tells us how closely, in terms of percentage, each observed land use area matches its expected target.

Finally, we need one more ingredient before finding the total land use mix index around i – the total observed share of all land uses of interest around i , which we call S_i . Using S_i allows us to focus our analysis on only a selected set of land uses and ignoring others without compromising the validity of the index. If we are only interested in the mixing between commercial and residential land, then we specify S_i to only include these two uses.

The final land use mix index MX_i around location i is given by multiplying the observed share of all land uses of interest around i (S_i) with the product of all individual matching indices $M_{n:i}$ around i :

$$MX_i = S_i \cdot \left(\prod_{n=1}^N M_{n:i} \right)$$

This land use metric tells us how closely the distribution of all land uses of interest around location i correspond to their expected distribution (Figure 18). MX_i always ranges between 0 and 1. MX_i is at its maximum when the land uses in the immediate square kilometer around i perfectly match the expected distribution. MX_i is zero when none of the expected uses are found in the area of i .

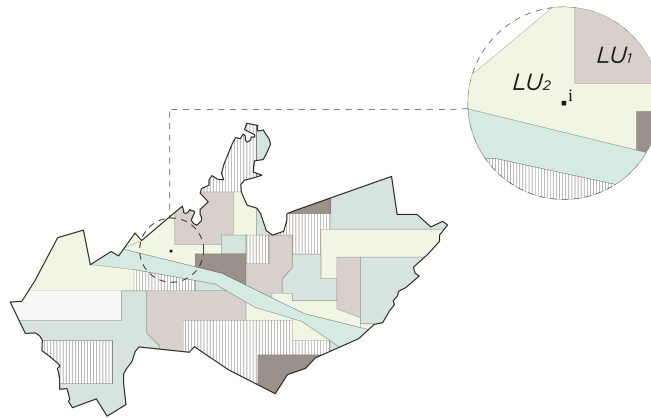


Figure 18: This land use metric tells us how closely the distribution of all land uses of interest around location i correspond to their expected distribution.

In order to obtain a combined index for the entire city, we can simply take the average of all individual MX_i indices:

$$MX = \frac{\sum_{i=1}^N MX_i}{N}$$

The combined city-wide land-use mix metric MX tells us how closely the average distribution of land uses across all analysis areas in the city corresponds to the expected distribution.

The key advantage of this proposed land-use mix metric lies in its flexibility in working with different combinations of uses, as well as the fact that it can be calibrated for different expected distributions. It can be used to evaluate land use mixing for only a narrow set of uses or all uses found in a city. A similar metric can also be used for evaluating other types of spatial mixing or segregation, such as the spatial mixing of different demographic, income, or racial groups.

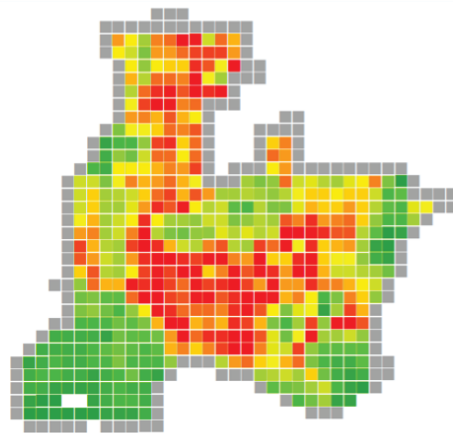
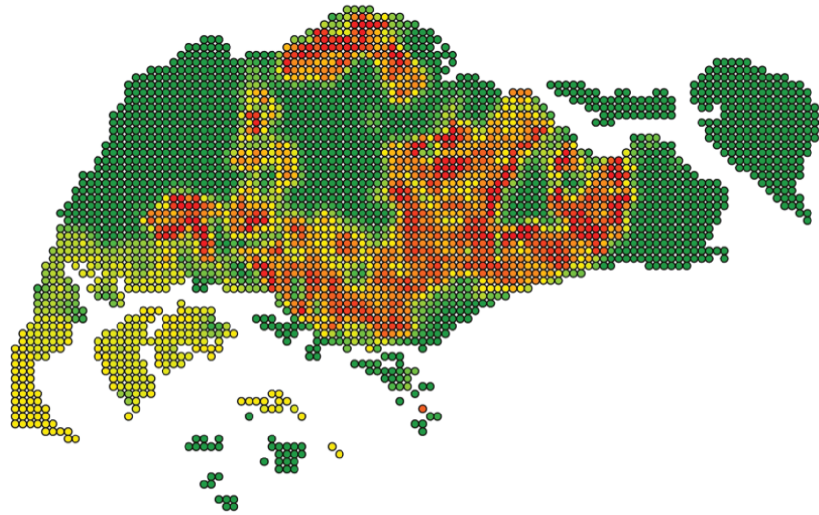


Figure 19: (Above) Land use mix in Singapore 2010. Max: 0.42, Mean: 0.09.

(Below) Land use mix in Long Beach, LA county 2010. Max: 0.58, Mean: 0.21 (controlled for edge effect. The gray cells on edge are not taken into account, for remaining cells the 1 km² search area is fully within the boundary). Source: LA County and Singapore employment data.

3 Discussion

We have implemented the seven metropolitan form metrics described above in ArcGIS, and are currently working on developing an open-source ArcGIS toolbox that will allow the metrics to be implemented on any input datasets. The toolbox is expected to be completed later this year and made publicly available through the City Form Lab website.

There are a number of important challenges in the current data collection and measurement techniques that need to be addressed in future research. Distinguishing the extent of a metropolitan area is probably the most fundamental challenge at present. It initially requires distinguishing the urban land coverage from non-urban land, which depends on our definition of urban land. Current remote sensing approaches mainly search for places dominated by “non-vegetative human-constructed elements” rather than examining the actual functionality of land. Vegetation coverage of different size and granularity, could be detected as non-urban land even if it functionally operates with urban uses.

Filtering out green spaces within a city from the total urban land extent profoundly impacts density metrics, and their related measures including coverage and polycentricity. Polycentricity measures rely on employment densities in detected centers. Different green spaces used as part of land areas for measuring employment densities can result in confusion when it comes to comparisons across metropolitan areas. Expandability, however, benefits from the separation of green areas, as it distinguishes land that is actually developed from land that is available for development.

Another important challenge for fragmented metropolitan areas, is to distinguish the extent to which individual urbanized clusters around the main built-up area should still be considered as a part of the same metropolitan area. The question is even more difficult if a distinct ‘main’ built-up cluster does not exist. What proportion of people need to routinely commute from a developed cluster to other clusters for them to be considered as part of the same metropolitan area? How far can such clusters lie from each other? Current remote-sensing technologies are not able to detect functional relationships between built-up clusters.

Challenges can also arise from using inconsistent data aggregation in different cities. One of the primary objectives of developing metropolitan form analysis toolbox is to allow one to compare profiles of selected cities in a consistent way. This consistency, however, only partially depends on the specifications of the metrics. Inconsistency in the resolution of data and its aggregation can make results across cities incomparable. A land-use mix value derived from a parcel-level dataset is not comparable to a land-use mix value derived from a tract-level dataset, as the second will return a lower value due to the modifiable areal unit problem (Openshaw 1984; Swift and Liu 2008).

While the metrics alone only capture, but do not explain the causes behind the observed growth and change patterns in metropolitan form, they do address the first crucial step of describing the observed trends by measurable means. Their implementation across a number of comparative cities offers an empirical basis to advance the analysis of the social, economic and environmental drivers that shape the observed patterns. Spatial economics offers a number of theories that predict urban expansion based on population demand, transportation costs, land availability, institutional structures, and other contextual factors (Isard 1956; Alonso 1964; Mills 1967). Neoclassical economics additionally factors in spillover effects and externalities between different land users of the city (Porter 1998, Fujita and Ogawa 1982; Krugman 1995). It has so far been relatively difficult to test the accuracy of the urban economic models empirically. The increasing availability of remote sensing data, combined with empirical measurements of metropolitan form open up a new resource for testing and refining economic models that explain metropolitan structures that we observe, especially in the rapidly expanding cities of the Global South.

For individual cities, a longitudinal capture of metropolitan form measures over time can be used to detect development trends and informed forecasts for future expansion and restructuring. Knowing the current trends in the distribution of employment centers in the polycentricity measurements, for instance, is valuable for predicting future land values. Knowing whether existing centers are shrinking or growing, and where new centers are emerging, can inform public investment and infrastructure plans. In a similar vein, detecting mono-functional areas of the metropolis as opposed to areas with a diverse combination of land uses can inform future planning and infrastructure investment decisions. Most growth in the developing world today is absorbed in a piecemeal manner, with little advance preparation in infrastructure, land-readjustment and resource allocation. Empirical evidence of trends and development directions from metropolitan form measures could be used to prioritize resource allocation and land preparation for large-scale growth, as witnessed in a number of Western cities in the 19th century (Schuyler, 1988; Sola-Morales 2000, Abercombe 1945).

In cross-city comparison, metropolitan form metrics can help highlight how much and how cities are growing on average. A large statistical sample allows trends to be further segmented by region or type of city. Comparing trends in compactness or polycentricity, for instance, can illuminate whether there are dominant trends in metropolitan growth patterns, whether cities of a particular size are becoming more polycentric, or less compact. Examining all measures combined may also reveal certain classes of cities that are experiencing similar trends in all aspects of their form. Having an empirical basis for quantifying metropolitan growth can thus lead to exciting new research on relating observed growth and change patterns with planning and development theory, and policy impact analysis across a universe of cases, and move us a step closer to a science of cities.

Acknowledgements

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