Homework#3

Given: October 23, 2002 Due: November 6, 2002

1. **Weld design**: Solve Problem 7.2-11, page 541, Gere. Establish the Tresca plastic yield condition and brittle rupture condition for the weld. If the weld material has Young's modulus E and Poisson's ratio ν, calculate the strains acting on the weld.

2. **Thick-walled cylindrical pressure vessel design**: A *closed-end* cylindrical pressure vessel of inner radius *a* and outer radius *b* is subject to internal pressure *p*. The stresses are

$$\sigma_{rr} = -\frac{p}{(b/a)^2 - 1}\left(\frac{b^2}{r^2} - 1\right)$$

$$\sigma_{\theta\theta} = \frac{p}{(b/a)^2 - 1}\left(\frac{b^2}{r^2} - 1\right)$$

$$\sigma_{zz} = \frac{p}{b^2/a^2 - 1}$$

Find the Mises yield and brittle rupture conditions for the vessel.

3. **Thin-walled spherical and cylindrical pressure vessel design:** Let $b = a + t$ where t is the thickness of the spherical pressure vessel/cylindrical pressure vessel.

Substitute the above relation in stress equations for the spherical pressure vessel (Eq. 6.1 Lecture#8 with $p_e = 0, p_i = p$) and cylindrical pressure vessel (equations in Problem 2 above).

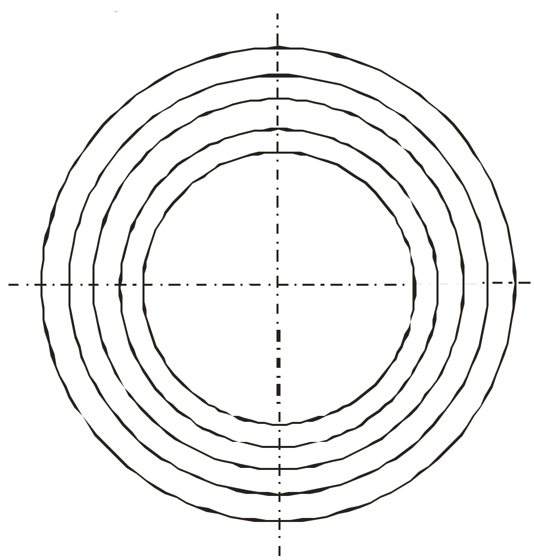
Then assume that the pressure vessels are *thin-walled*, or $a >> t$, so that higher order terms of t are negligible. Simplify the stress relations for this case and compare your results with those of Gere (pg. 557-571). Establish the brittle rupture conditions for thin-walled spherical and cylindrical pressure vessels.

4. **Weight optimization design of a sandwich beam:** Suppose you are provided with many thin steel ($E_{steel} = 240$ GPa, $\rho_{steel} = 7.8\times10^3$ kg/m^3) and aluminum ($E_{aluminum} = 70$ GPa, $\rho_{aluminum} = 2.7\times10^3$ kg/m^3) sheets of thickness *t* and width *b*, and many thick aluminum foam slabs ($E_{foam} = 7$ GPa, $\rho_{foam} = 0.27\times10^3$ kg/m^3) of thickness *h* and width

b. Also given is that $t<<h$. There is only enough adhesive (with shear strength τ_c) to bond only four surfaces so that some of the material will not be used. You are required to design a sandwich beam which strictly includes all the three materials! Design the sandwich beam for the combination of all the following three requirements:

- -maximum bending resistance
- -maximum shear resistance
- -minimum weight

5. **Brittle rupture design of a multi-layered composite beam**:



Consider a cylindrical multi-layered composite beam made up of hollow cylindrical beams of different *brittle* materials with Young's moduli; $E_1 = 4E, E_2 = 3E, E_3 = 2E, E_1 = E$. E=400 GPa. The internal radius of cylinder 1 is a=25 mm, while all hollow cylinders are of thickness t=2.5 mm. Observe $a>>t$. The fracture strengths are; $\sigma_f^1 = 2\sigma_f, \sigma_f^2 = \sigma_f, \sigma_f^3 = 0.5\sigma_f, \sigma_f^4 = 0.25\sigma_f$ σ_f =25 MPa

- -find the neutral axis for the composite beam
- -what is the elastic bending stiffness of the beam?
- -what is the critical bending moment M_y^f for brittle rupture?
- -find the corresponding deflection w_o^f at brittle rupture if the beam is under 4-point bending (Let beam length be L).