14.03 Exam 2 Fall 2000

SOLUTIONS

Part I: 5 points each

True, False, or Uncertain AND WHY. You must explain your answer with one or two sentences and/or graphs. Answers without justification receive zero points. 5 points each.

1. Although the First Welfare Theorem demonstrates that a free market in equilibrium is Pareto efficient, the Second Welfare Theorem reveals that there is normally a tradeoff between equity and efficiency.

   False. The Second Welfare Theorem states that (under the conditions on preferences and production technologies that we discussed in class) every Pareto efficient allocation can be sustained as a competitive equilibrium outcome. Therefore efficiency and equity can be treated as separate issues. Equity can be achieved via lump sum transfers that modify the distribution of the initial endowments, and efficiency can be achieved by simply letting the market reach the equilibrium.

2. Consider an expected utility maximizer who is globally risk seeking (meaning, she is risk seeking at all wealth levels) with $U(\$0) = 0$. For this person, $2 \cdot U(\$500) < U(\$1{,}000)$. [A diagram is required.]

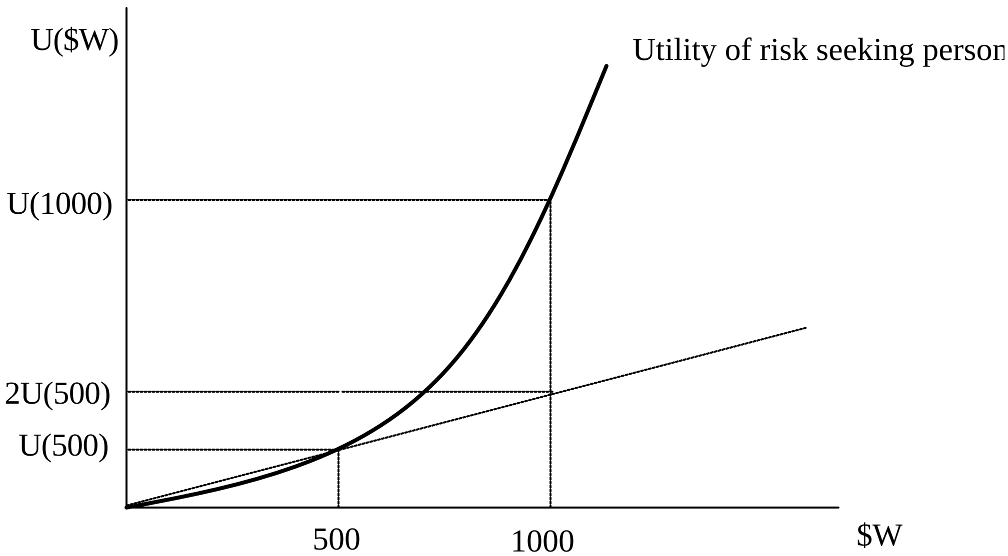
   True. By Jensen's Inequality, applied to a risk seeking (i.e. convex) utility function, we know that the expected utility of a lottery is larger than the utility of the expected value of the lottery. Consider a lottery that has probability ½ of winning \$0 and probability ½ of winning \$1000, the above mentioned inequality translates into

   $$1/2 \cdot U(\$1{,}000) + 1/2 \cdot U(\$0) > U(1/2 \cdot \$0 + 1/2 \cdot \$1000)$$

   Using the assumption that $U(\$0) = 0$ *and multiplying by two, we can rewrite the inequality as*

   $$U(\$1{,}000) > 2 \cdot U(1/2 \cdot \$0 + 1/2 \cdot \$1000) = 2 \cdot U(\$500)$$

   which proves the result. (See picture.)



3. Consider an economy that produces two goods, A and B, using two inputs K and L. In the production of these goods, $MRTS_A = MRTS_B$. Among consumers in this economy, $MRS_i = MRS_j$. The operation of this economy is Pareto efficient.

   False (or Uncertain). For Pareto Efficiency in a General equilibrium model, three conditions have to be satisfied. The condition that is not mentioned in the question is Product Mix, i.e. $MRS = RPT$. In other words, the indifference curve of the representative consumer has to be tangent to the Production Possibility Frontier. The two equilibrium conditions given in the problem – Productive and Allocative efficiency – are <u>*necessary but not sufficient*</u> *for Pareto efficiency.*

4. Kane and Staiger found that small decreases in abortion availability reduced both abortions *and* births. Based on their analysis is, we should expect that banning abortion altogether (for example, by overturning Roe v. Wade) would reduce births even further.

   False (or Uncertain). Kane and Staiger point out in their paper that the results they found are applicable when small changes in the cost of abortion occur. By contrast, banning abortion can be considered as forcing the parameter A go to infinity. In this case two opposite effects occur on births. First, women who use abortion as insurance against out of wedlock births will decide not to get pregnant (shift from region 3A to region 1 in the graph presented in class), thus reducing the number of births. But the second effect is that women who always get pregnant but abort out of wedlock births when the psychic cost is low will decide to bear out of wedlock children instead (shift from region 3B to region 2). This second effect may outweigh the first, leading to more total births.

   [In a 1999 paper in the <u>*American Journal of Public Health*</u>*, Philip Levine, Douglas Staiger, Thomas Kane and David Zimmerman find that the* <u>*legalization*</u> *of abortion in 1973 (i.e., passage of Roe v. Wade) reduced total birth rates by 8 percent, and by twice as much for out of wedlock births. This suggests that overturning Roe v. Wade would* <u>*increase*</u> *birth rates substantially.]*

Part II: 15 points each

1. (This is not a math problem. Answer using principles of expected utility maximization.)

Consider a "nation" of two risk averse expected utility maximizers with identical utility functions. Each person has $10,000. One person is perfectly healthy. The other person will contract a disease and must pay $10,000 for a cure (there are no other psychic or monetary costs). Neither knows in advance who will stay healthy and who will get the disease. There is no insurance.

a) Someone proposes a voluntary national health fund. Before finding out who gets sick, each person can choose to pay $5,000 into the fund. Afterwards, the person who gets sick receives the money from the health fund, provided he paid into the fund beforehand. What is a person's expected utility if he does not pay into the fund? What is a person's expected utility if he does pay into the fund, assuming the other person pays in as well?

(3 points) Since there are only two people in the population, each individual has a probability ½ of getting the disease. The expected utility in case a person does not pay into the fund is

$$EU(not\ \ pay) = 1/2 \cdot U(1{,}0000) + 1/2 \cdot U(0)$$

The expected utility in case the person pays is

$$EU(pay) = 1/2 \cdot U(1{,}0000 - 5000) + 1/2 \cdot U(10000 - 5000 - 10000 + 10000) = U(5000)$$

that is, with probability ½ he does not get the disease, he does not have to pay for the cure, but still pays $5000 into the fund. And with probability ½ he gets the disease, he has to pay $10000 for the cure, he has to pay $5000 into the fund, but then he gets $10000 back as insurance.

b) Will people join the national health fund? What is the impact of the health fund on social welfare, i.e., the sum of individual utilities? What insurance principle is at work?

(4 points) Yes the individuals will join the fund because they are risk averse and for them it is the case that

$$EU(not\ \ pay) = 1/2 \cdot U(1{,}0000) + 1/2 \cdot U(0) < U(5000) = EU(pay)$$

The social welfare increases with respect to the situation where no fund exists. The social welfare is defined as the sum of the individual expected utilities

$$SW(pay) = 2 \cdot U(5000) > SW(not\ \ pay) = U(0) + U(10000)$$

The insurance principle at work is Risk Pooling. Because one person will get the disease and the other not, their risks offset one another. By placing these risks in a common pool, the citizens reduce their total variance of outcomes to zero. Each person must spend $5,000 with certainty, and both are better off than if they faced the same risk without the pool.

c) A new genetic test is invented. At birth, each person finds out for free whether or not he will get sick. How will the genetic test impact the insurance fund from part (A)? What is the impact of the genetic test on social welfare? Explain.

(4 points) Since it's free, each individual will take the test. The person that finds out that he's healthy will not pay into the fund. The other person would like to pay, but the fund is useless now because it does not receive the contribution of the healthy person, so it can only pay back $5000. In this case the social welfare is reduced to the level when the insurance fund did not exist.

d) Someone proposes that the government *mandate* that every citizen must pay $5,000 into the fund, regardless of his or her genetic test. What is the impact of the mandate on social welfare relative to part (b)? What insurance principle is at work? Explain.

(4 points) In this case the social welfare is the same as part (b), when both individuals paid into the fund. So this situation represents an improvement in social welfare over part (c). The insurance principle at work here is Risk Spreading. Although in this case there is no uncertainty to insure against (because both parties know their health outcome), it is still the case that the total social welfare loss from each party paying $5,000 is lower than the welfare loss of one party paying $10,000. This follows from the concavity of their risk averse utility functions. In a world with perfect genetic testing with no uncertainly about health costs, there is still a reasonable economic case to be made that mandatory 'insurance' requiring everyone to pay into a shared pool for health insurance costs raises total social welfare via risk spreading. But note that unlike the case of risk pooling with uncertainty in part (b), this type of mandatory system does not represent a Pareto improvement because healthy people are made worse off by having to pay into the fund.

2. (15 points)

The domestic demand for professional soccer balls is given by

$$Q = 5000 - 100P$$

where the price (P) is measured in dollars and quantity (Q) in units of professional soccer balls per year. The domestic supply curve for professional soccer balls is

$$Q = 100P - 1000.$$

a) What is the domestic equilibrium in the professional soccer balls market?

(3 points) The domestic equilibrium is found by equating domestic demand to domestic supply. The equilibrium price is 30 and the equilibrium quantity is 2000.

b) Now suppose that this market is opened to international competition, and professional soccer balls can be imported at a world price of $20 per ball. What is the free trade equilibrium in this market? How many balls are imported per year? Draw a diagram describing the gain of total surplus from free trade relative to part (a).

(3 points) The free trade equilibrium is found by replacing the international price of $20 into the domestic demand function. The quantity demanded at that price is 3000. Moreover, since the domestic supply at the price $20 is 1000, the quantity of balls that have to be imported is 2000. (See picture at the end of solutions set.)

c) If the Government decided to impose a quota of 1500 professional soccer balls per year, what would the equilibrium price in the market be? What would the quantity of balls produced domestically be? Show the deadweight loss relative to free trade in a diagram.

Hint: the total supply of soccer balls, including imports, for this market is:

$$Q = \begin{cases} 2500, & \text{if } P < 20 \\ 100P + 500, & \text{if } P \geq 20 \end{cases}$$

(3 points) The equilibrium price will be higher than the free trade equilibrium price $20, since the quantity of imports is restricted below the free trade quantity of imports. Therefore, we use the total supply (domestic supply plus imports) that is valid for a price above $20. Equating domestic demand to total supply we find that the equilibrium price in the domestic market is $22.5. For this price the quantity produced domestically is obtained by replacing 22.5 into the domestic supply curve. The quantity produced domestically is 1250. (See picture at the end of solutions set.)

d) Find the tariff per unit of imports that would give the same level of domestic demand for soccer balls as the quota in part (c). What would the quantity of balls imported and the quantity of balls produced domestically be?

The tariff is obtained simply by equating the world price ($20) plus tariff to the price with quota ($22.5). The resulting tariff is $22.5. The quantities demanded and imported are the same as in part (c), 2750 and 1500 balls, respectively.

e) Give the ranking of the trade arrangements in parts (b), (c) and (d) in terms of the total domestic surplus. Remember to account for the domestic consumer surplus, the domestic producer surplus, and the Government revenue (No math is necessary to answer this question.)

(3 points) Free trade gives the maximum surplus. Since with the tariff the government gets the revenue and with the quota the surplus goes to foreign producers, the tariff is preferred to the quota.

Part III: 30 points

Suppose that a country (Home) has identical consumers with utility functions $U(X,Y) = X^{1/3}Y^{2/3}$. The production possibility frontier (PPF) is given by $Y^2 + 4X^2 = 48$. Another country (Foreign) has also identical consumers with utility functions $U(X,Y) = X^{2/3}Y^{1/3}$. The PPF in Foreign is the same as in Home: $Y^2 + 4X^2 = 48$.

a) Solve for the competitive equilibrium when each of the two countries is considered as a closed economy. That is find the consumption levels of X and Y and the price ratios prevailing in each of the two countries. Draw a qualitative diagram of the equilibrium in each of the two countries.

   (8 points) For each country, the domestic equilibrium is found by equating the MRS to the RPT and imposing that the quantities of X and Y have to be on the PPF. The price ratios are equal to the MRS's once that the equilibrium quantities have been substituted in.

   In Home: MRS=(1/2)(Y/X) and RPT=4X/Y. The equilibrium quantities are X=2 and $Y = 4\sqrt{2}$. *The price ratio is* $\sqrt{2}$.

   In Foreign: MRS=2Y/X and RPT=4X/Y. The equilibrium quantities are X=2$\sqrt{2}$ *and Y=4. The price ratio is 2*$\sqrt{2}$.

   Notice that since the countries are closed to trade, the quantity produced and consumed have to be the same. (See pictures.)

b) In each of the two countries there is public debate on whether to start a trade relationship between Home and Foreign. The opponents of free trade argue that since the two economies are identical in terms of production technology, there will be no gains from trade. Do you agree with this argument? Briefly motivate your answer (you can refer to the diagram you drew for the previous answer).

   (8 points) No, free trade is still beneficial. Although the countries have the same production technology, they have different preferences. This implies that the price ratios are different and each country has a different valuation of one good in terms of the other good. Therefore there are gains from trade. If preferences and production technology were identical, there would be no gains from trade.

c) Now suppose that the free trade lobbies prevail, and the two economies are opened to international trade with each other. Suppose further that the price ratio P_x/P_y is set at a level *R* such that is intermediate between the closed-economy equilibrium price ratios in each country, which you have already determined. Find the quantities of X and Y that are produced in each country as a function of *R*. (Hint: the production technology is the same in the two countries and so is the price ratio.) Then find the quantities of X and Y that are consumed in each country as a function of R and the quantities produced. Draw a qualitative diagram of the trade equilibrium in each country.

   (8 points) Now for each country, you have to distinguish between the quantity produced and consumed, since there can be imports.

In order to determine the quantities produced, first notice that the technology is the same in the two countries, so that for a given R the quantities produced of X and Y will be the same in the two countries. To obtain these quantities equate the RPT to the price ratio R and impose the condition that X and Y produced have to be on the PPF. This gives

$$X^P = 2\sqrt{\frac{3R^2}{4+R^2}} \text{ .and } Y^P = \frac{8}{R}\sqrt{\frac{3R^2}{4+R^2}} .$$

To obtain the quantities consumed in each country use the following conditions:

$$MRS(X^C, Y^C) = R$$
$$R(X^C - X^P) = Y^P - Y^C$$

where these conditions have to be satisfied by the quantities produced in each of the two countries. Therefore we obtain that in Home

$$X_H^C = \tfrac{1}{3}(X^P + \frac{1}{R}Y^P)$$
$$Y_H^C = \tfrac{2}{3}(RX^P + Y^P)$$

and in Foreign

$$X_F^C = \tfrac{2}{3}(X^P + \frac{1}{R}Y^P)$$
$$Y_F^C = \tfrac{1}{3}(RX^P + Y^P)$$

[Notice that these results could have been easily obtained using the formula for the demand functions for Cobb-Douglas preferences, where in this case the budget constraint is $p_X X^C + p_Y Y^C = p_X X^P + p_Y Y^P$*, and then expressing the demands in terms of the price ratio R=* p_X / p_Y *.]*

d) Now realize that since this is a general equilibrium model, the price ratio P_x/P_y is endogenously determined. Solve for the equilibrium level of *R*. (Hint: the total demand of a good has to be equal to the total supply of that good. By Walras' Law it is enough that you impose this condition for just one of the two goods.)

(6 points) Impose the market clearing condition

$$X_H^C + X_F^C = 2 \cdot X^P$$

and use the formulas for the quantities produced and consumed that are given above.

Notice that by Walras' Law, only one market clearing condition (e.g. the one for X) is enough to determine R.

The value of R that satisfies the market clearing condition is 2.