14.03 Fall 2000 Exam 3 Solutions

1.

a) (5 points) In a competitive signaling equilibrium, the wage of GED holders must be equal to their average productivity. When the passing score is reduced, low-productivity people who were not able to pass the GED previously are now able to pass. Hence the average productivity of GED holders goes down, so we would expect the average wages of GED holders to go down as well.

b) (5 points) Since reducing the passing score reduces the wages of GED holders, it makes getting a GED less attractive compared to graduating from high school. Hence we would expect the dropout rate to decrease.

2.

a) (5 points) If MS tries to serve both markets it maximizes $P(100-P)+P(50-2P)=150P-3P^2$. The FOC is $150P-6P=0$, which implies that P = 25. The associated profits are 1875. If MS serves only the MBA market it maximizes $P(100-P)=100P-P^2$. The FOC is $100-2P=0$, which implies that P = 50. The associated profits are 2500. Hence MS will choose to serve only the MBA market.
Profit-maximizing price: 50
MS profits: 2500
CS for MBAs: 1250
CS for PhDs: 0

b) (5 points) Profit-maximizing price of Money: 50
Profit-maximizing price of Money for Economists: 12.5
MS profits: 2812.5
CS for MBAs: 1250
CS for PhDs: 156.25

Since MS and PhDs are better off and MBAs are not worse off, versioning leads to a Pareto improvement.

3.

a) (5 points) Auction sites are subject to network externalities. A site with more buyers is more valuable to sellers (since the sales price goes up with the number of bidders), and a site with more sellers is more valuable to buyers (since buyers are more likely to find the item they are looking for). Hence sellers are willing to pay a premium to reach eBay's much larger pool of buyers.

b) (5 points) From eBay's perspective universal-search auction sites are dangerous because they allow a seller to list items on small auction sites and still reach the same (or even larger) number of buyers. Hence if universal-search auction sites became widespread, eBay would no longer be able to charge a premium over smaller sites.

4.

a) (3 points) The problem is $\max_{p_c, p_p} p_c(24-3p_c+p_p)+p_p(24-3p_p+p_c)$. The FOC w.r.t p_c is $24+2p_p-6p_c=0$. By symmetry, $p_c=p_p=p^*$. Hence both Coke and Pepsi will set price equal to 6.

b) (6 points) The problem for Coke is $\max_{p_c} \pi(p_c, p_p)=p_c(24-3p_c+p_p)$. The FOC is $24-6p_c+p_p=0$. By symmetry, $p_c=p_p=p^{NE}$. Hence both Coke and Pepsi will set price equal to 24/5 = 4.8.

c) (6 points) Trigger strategy: Set price equal to p^* as long as the other firm chooses p^*. If the other firm chooses another price, set price equal to p^{NE} forever. To find the discount rates for which the trigger strategy is an equilibrium, we need to find the optimal deviation for one firm when the other firm plays p^*. Using the Nash FOC, we find that if the other player sets price equal to 6, the best response is to set price equal to 5.

Now compare the payoff from "cheating" to the payoff from "cooperating".

PDV(cheating) $=\pi(5,6)+\dfrac{\delta}{1-\delta}\pi(4.8,4.8)$

PDV(cooperating) $=\dfrac{1}{1-\delta}\pi(6,6)$

So this trigger strategy is subgame perfect if

$$\delta \geq \frac{\pi(5,6)-\pi(6,6)}{\pi(5,6)-\pi(4.8,4.8)}=\frac{75-72}{75-69.12}=\frac{25}{49}\approx 0.51$$

5.

a) (4 points) With observable types, $w_H=\theta_H$ and $w_L=\theta_L$. Since $c<1$, both types accept those offers, and total output is $(1-\lambda)\theta_L+\lambda\theta_H$.

b) (3 points) If only Low workers accept, $w=\theta_L$. Low workers prefer this wage to home production.

c) (5 points) If both types accept, $w=(1-\lambda)\theta_L+\lambda\theta_H$. Since this wage is between θ_L and θ_H, Low workers will accept it. High workers will accept this offer if $(1-\lambda)\theta_L+\lambda\theta_H \geq c\theta_H$.

d) (4 points) With these parameter values, $(1-\lambda)\theta_L+\lambda\theta_H=2.5$ and $c\theta_H=3$. Since High workers would not accept the "pooling" wage, the equilibrium wage is 1 and only Low workers are employed at firms.

e) (4 points) Total output in this equilibrium is $(1-\lambda)\theta_L+\lambda c\theta_H=2$, which is less than output in the full-information equilibrium. Output is lower because asymmetric information has created a "lemons" problem – High workers would rather work less productively at home than work at the firms, where the Low workers would drag their wages down.

6.

a) (4 points) To graph the indifference curves, set the utility function equal to a constant and solve for w (see last page for picture).

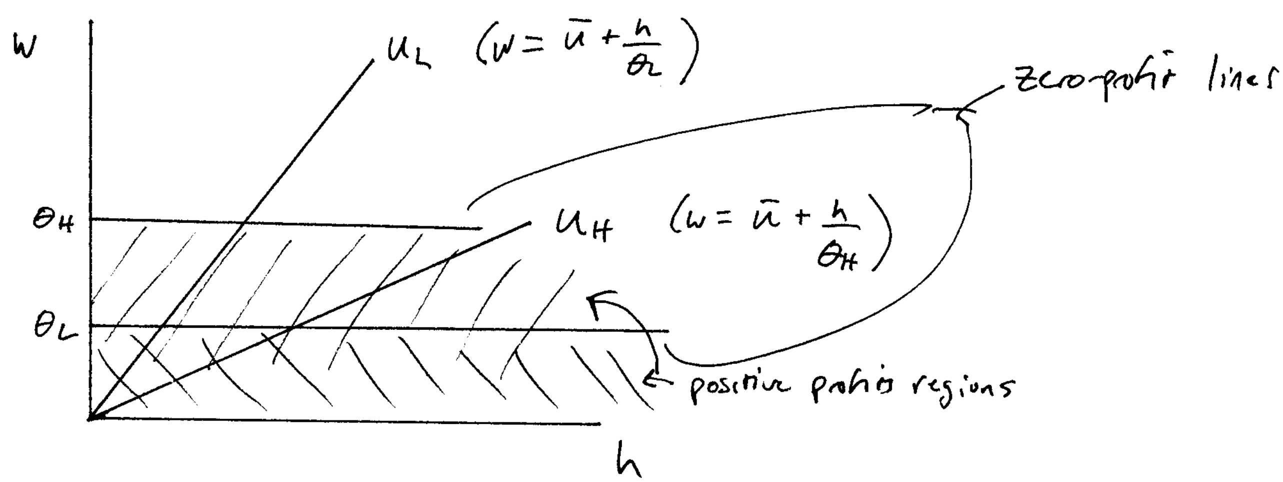
b) (5 points) $w^H = \theta_H, w^L = \theta_L$. See last page for zero-profit lines.

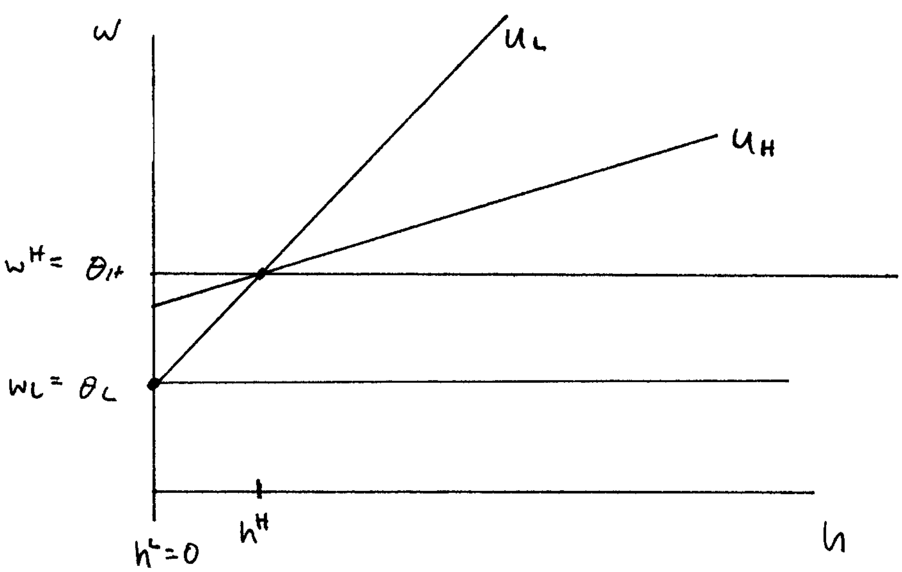
c) (5 points) $h^L = 0$. If h^L were greater than zero, another firm could offer a contract with a lower wage and a smaller number of extra hours. Low workers would accept this contract, and the firm would earn positive profits.

d) (6 points) The minimum number of extra hours that High types should be assigned is h^H such that $U_L(w^H, h^H) = U_L(w^L, 0)$, which yields $h^H = \theta_L(\theta_H - \theta_L)$. The number of extra hours cannot be any higher, because then a firm could earn positive profits by offering a contract with a lower wage and a smaller number of extra hours.

e) (4 points) This mechanism solves the asymmetric information problem because it is less costly for High workers to work extra hours than it is for Low workers.

f) (6 points) No pooling equilibrium exists. Given a candidate pooling equilibrium, there always exists another contract that would attract only High workers and earn positive profits (see last page).


w
u_L $(w = \bar{u} + \frac{h}{\theta_L})$
zero-profit lines
θ_H
u_H $(w = \bar{u} + \frac{h}{\theta_H})$
θ_L
positive profits regions
h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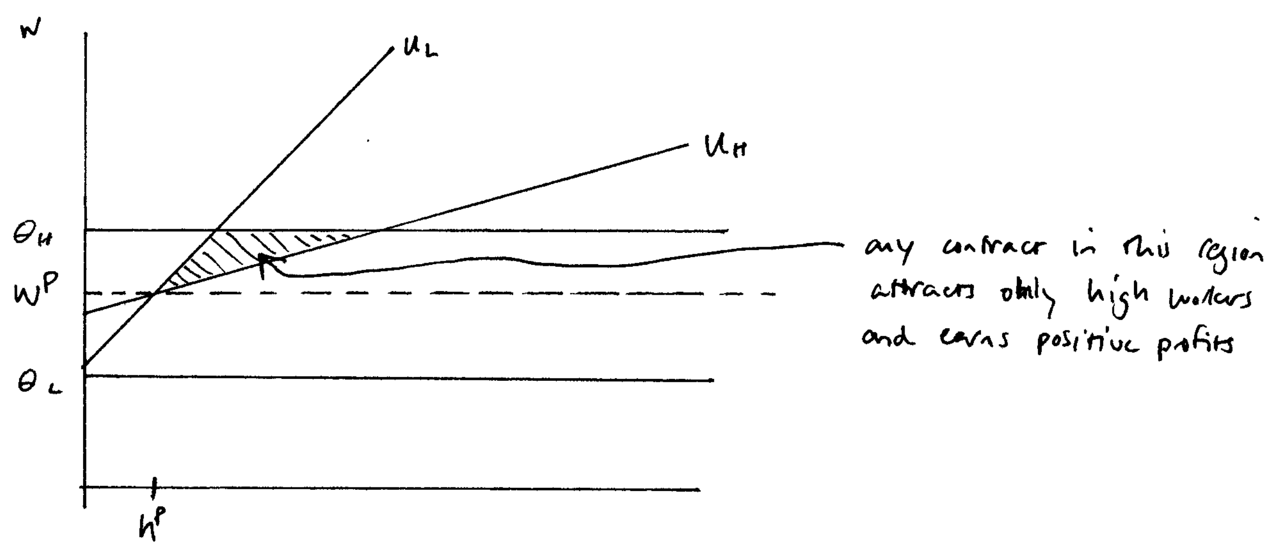
w
u_L
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any contract in this region attracts only high workers and earns positive profits