## 14.03 Exam 1 Fall 1999 – Solutions

## Part I. 5 points each

True, False, or Uncertain AND WHY. You must explain your answer with one or two sentences and/or a diagram. Answers without justification receive zero points. 5 points each.

1. In response to widespread student malnutrition at MIT, President Vest establishes an in-kind food transfer program which gives each student two slices of pizza per day valued at \$1 each (distributed by specially marked "Mozzarella Institute of Technology" trucks). Every day after eating his two free slices, Fred buys a third slice from the MIT truck, also at \$1 per slice. Fred would have been better off if President Vest had given him \$2 per day to spend on whatever he liked.

False. Fred would have spent \$3 on pizza anyway, and is therefore indifferent between the in-kind and cash subsidies.

A few students made an interesting argument along the following lines: Suppose Fred's original income is \$1, and the only other good is soda, which must be consumed in discrete units and costs \$2 per soda. Then Fred would not be able to afford a soda with the in-kind transfer, so if Fred prefers (1 pizza slice, 1 soda) to (3 pizza slices, 0 sodas), then the cash transfer makes Fred better off. To receive full credit for this answer, you must have stated the assumption that goods are indivisible.

2. Orange juice sells for \$2.00 per gallon and gasoline sells for \$1.00 per gallon. Although we don't know how to measure utility, we do know that if a consumer buys both goods, she receives twice as much utility from orange juice as from gasoline.

False. The marginal utility of orange juice is twice that of gasoline. We have no idea whether total utility is greater or less.

You could also get full credit by answering Uncertain to this question if you explained that the reason for the uncertainty was that the question did not specify marginal vs. total utility.

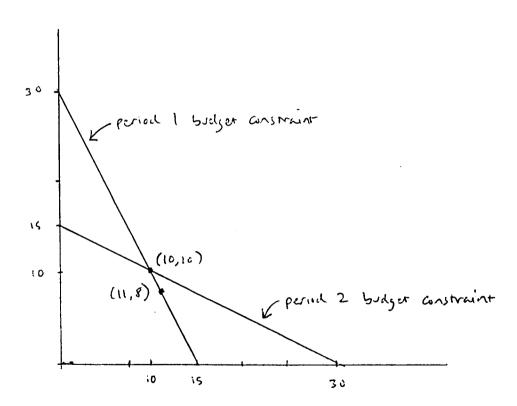
3. When the price of a product rises, the Bureau of Labor Statistics (BLS) estimates how much of the price change is due to quality improvements and how much is due to inflation. A research economist working for the Bureau of Labor Statistics discovers that the price of kitchen stoves in the U.S. did not change at all between 1998 and 1999. Hence, he can conclude that stoves did not contribute to quality change bias in the CPI last year.

False. Quality change might have taken place without any change in price. A common quality change problem for the CPI is that products improve while their prices remain constant (think of computers or any other electronic device). Quality can also decline while price remains constant, as happens at many restaurants in Kendall Square.

4. In the first period,  $P_x=2$  and  $P_y=1$  and the consumer buys 11 units of X and 8 units of Y. In the second period,  $P_x=1$  and  $P_y=2$  and the consumer buys 10 units of X and 10 units of Y. This consumer's preferences violate the Weak Axiom of Revealed Preference.

True. Both bundles are affordable under both price scenarios. Yet the consumer chose (11,8) in the first scenario and (10,10) in the other. If the consumer preferred (11,8) to (10,10) in the first scenario, WARP states that she cannot prefer (10,10) to (11,8) in the second scenario. (See diagram)





## Part II: 10 points each

- 1. A college dean wants to know whether her school's alcohol ban reduces drinking by freshman. All freshmen at her college live on campus in either Dorm A or Dorm B (dorms of equal size) and each is allowed to choose his or her dorm during 'rush' week. The college is located in a 'dry' (alcohol free) town and so there is no off-campus drinking. To estimate the impact of the ban, the dean runs an experiment over two years. In year 1, she hires student informants to monitor the drinking habits of the residents of each dorm (regardless of where they do their drinking). She finds that students from Dorm A drink 3.0 beers per week on average while those from Dorm B drink 2.8 beers per week. In year 2, the dean announces during rush week that drinking will be allowed in Dorm B this year but not in Dorm A. Her informants again monitor drinking and find that Dorm A residents drink 2.1 beers per week on average while Dorm B residents drink 3.7 beers per week. The Dean uses the difference-in-difference methodology to estimate the impact of the drinking ban on the drinking behavior of the average student. She finds a statistically significant difference.
  - A) What is the difference-in-difference estimate of the impact of the dormitory drinking ban on drinking by college freshman? (2 points)
  - -1.8 drinks per week.
  - B) Does the dean's design constitute a valid 'natural experiment? Explain why or why not. (6 points)

Definition of natural experiment should include exogenous assignment of treatment, valid control group.

This experiment is not valid because the choice of dorm is endogenous: students can select their dorms based on the dorms' drinking policies. Because the dean announced that drinking was allowed in Dorm B during rush week, students with a preference for drinking would have chosen this dorm, and vice versa for Dorm A. Hence, both the control and treatment groups were contaminated during the experiment. Another way of stating the same point is that assignment to the treatment (Dorm B) vs. control (Dorm A) group was non-random.

C) The college's trustees are considering lifting the campus drinking ban and ask the dean to report on the impact this will have on overall drinking. What is her best estimate? (2 points)

If you look at the data carefully, you will notice that average drinking on campus did not change at all from one year to another (it is 2.9 in both years). Ironically, this suggests that the drinking ban had no effect on total drinking. Hence, the dean's best estimate would be that lifting the drinking ban should have no effect on overall drinking. It appears that the poorly designed experiment encouraged students to sort themselves by dorm according to their drinking preference without actually impacting total drinking.

Note that there is some uncertainty about the conclusion of "no drinking-ban-effect" since we are relying on a before-after comparison of on-campus drinking. Since we have no control group (as in an ideal experiment), we must assume in order to obtain an estimate that past drinking behavior (in year 1) is a valid control (comparison group) for present drinking behavior (in year 2). This approach is clearly weaker (i.e, relies on stronger assumptions) than using a difference-in-difference (DD) estimator to gauge the treatment effect. Unfortunately, the DD estimator is not available here due to the contamination of the treatment and control groups by self-selection during the dean's experiment.

2. MIT decides to implement an MIT Consumer Price Index (MIT-CPI) to track the well being of its students. It uses a Laspeyres index. MIT finds that all goods and prices remained exactly the same between 1998 and 1999 except for one thing. After the new I-Campus deal was announced between Microsoft and MIT, prices for Macintosh computers rose sharply during 1998 – 1999. (A spokesperson for MS-MIT was unable to provide an explanation for this price increase.) Consequently, some students chose to purchase Windows PCs during 1998 – 99 instead of buying Macintosh computers, even though these students consider Windows PCs to be of lower quality. We conclude that the MIT-CPI understates the true increase in the cost of living since some students are now consuming low quality computers. True, false, or indeterminate and WHY?

False. Students are substituting towards cheaper Windows PCs as the price of Macintosh computers rise. As we know, the Laspeyres index overstates inflation by failing to take into account substitution. Hence, the MIT-CPI will overstate the true increase in the cost of living.

A common mistake was saying that since PCs are of lower quality than Macs, there is a quality change bias in the MIT-CPI. It is important to recognize that there is no quality change bias in this situation. Quality change bias occurs when a price index does not account for the change in a single good's quality across time, not when consumers substitute between goods of varying qualities.

## Part III: 15 points each

1. An administrator for the Food Stamps program calls you up to say, "I heard that you learned in 14.03 that in-kind transfer programs are less efficient (in terms of consumer utility) than cash transfer programs. Given this information, we are considering converting Food Stamps to a cash transfer program. However, we are going to do this in an unusual way. Rather than simply giving cash, we will give recipients 50 cents back for every dollar they spend on food. We call this 'the rebate plan.' Since we are no longer restricting what recipients can buy, we assume that this program is just as efficient (in terms of consumer utility) as writing them an equivalent check each month, right?"

Is the administrator correct? Consider a person with initial budget of \$200 who was originally receiving \$50 in food stamps. On the rebate plan, assume that this person buys \$100 worth of food, meaning that he also receives a \$50 rebate. A pure cash-equivalent transfer program would provide him a check for \$50.

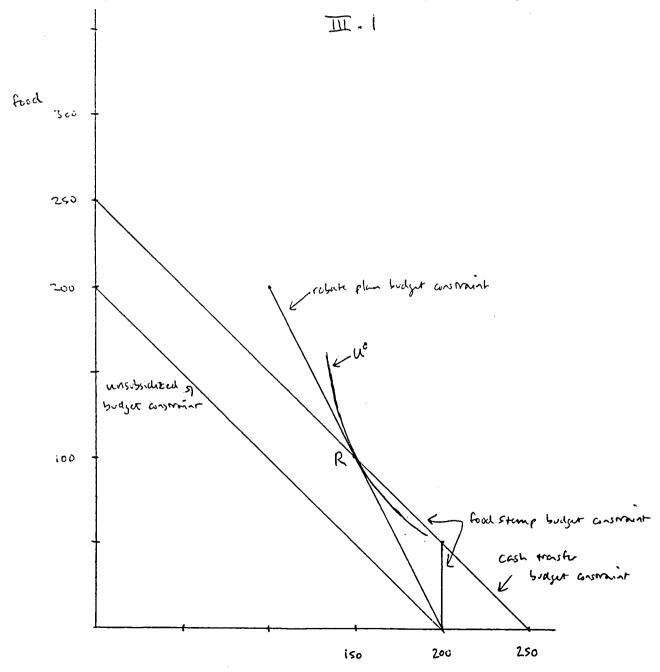
- a) Draw a carefully labeled, appropriately scaled diagram that shows the consumer's budget set for each of the three programs (food stamps, rebate, and cash). Label the Y axis "food" and the X axis "all else." For reference, you may also want to draw the consumer's budget set without any subsidy. Label the consumer's choice under the rebate program as point "R", and draw an indifference curve labeled  $U^0$  that would be consistent with the consumer's choosing this point. (Note that you do not know the consumer's choices for the food stamp program or the cash plan.) (5 points).
- b) Explain whether (and why) this particular consumer is better off, worse off, indifferent, or indeterminate with the rebate plan relative to the pure cash transfer program (5 points).

This consumer is necessarily worse off under the rebate plan that under the pure cash transfer program. This is because he is consuming on the cash transfer budget set (I=\$250) but facing a price ratio that differs from the slope of this budget set. With convex indifference curves, he would never have chosen point "R" on the cash transfer budget set (since the indifference curve that is tangent to the price ratio at "R" cannot also be tangent to the cash transfer budget set at "R"). Hence, by revealed preference, the consumer is worse off at point "R" than he would be if he were facing the cash transfer budget set.

Note that for this consumer, the rebate program is equivalent to a fully rebated sales tax on a single good, an example we discussed many times in class. Even though the consumer is 'made whole' monetarily by the rebate (i.e., still consumes on the I=\$250 cash transfer budget set), he is worse off with the rebate than with the cash transfer because the rebate causes him to consume at a less preferred place on the I=\$250 budget set. Hence, the rebate functions like a distortionary tax. A subtle point you might observe is that the rebate functions not like a tax on food but like a tax on 'everything else.' Whereas without the rebate, the price ratio of food to all else was 1, with the rebate, the price ratio of food to all else is 1/2. Hence, the rebate works like a 50 percent tax on all else.

c) Explain whether (and why) this consumer is better off, worse off, indifferent, or indeterminate with the rebate plan relative to the food stamp program (5 points).

This consumer is also worse off on the rebate plan than on the food stamp plan. The important thing to notice is that "R" was also available under the food stamp program. Hence, in the best case, the consumer is indifferent between the food stamp and rebate programs. However, this could not actually be the case. Consider two cases, first imagine that the consumer on food stamps had chosen some point on the straight (non-kinked) part of the budget set. If so, we know by the argument above that this point could not have been "R." Hence, by revealed preference, this consumer is worse off at "R." Now, consider that the consumer chose the kink point in the budget set, label it "K." In this case, we know that K was preferred to R under the food stamp program. However, K is no longer available under the rebate plan. Here again, the consumer is worse off. Hence, for this consumer, the rebate plan is worse than either food stamps or the cash plan. (See diagram)



- 2. A consumer has the following expenditure function:  $E = 2(P_x^{0.5}P_y^{0.5}V)$ 
  - (A) Write down the Slutsky equation which expresses the effect of a change in the price of X on the uncompensated demand for X and provide an interpretation for each term. You do not need to solve for any of the terms right now. Simply write down the Slutsky equation. (3 points)

$$\frac{\partial d_x}{\partial p_x} = \frac{\partial h_x}{\partial p_x} - x \frac{\partial d_x}{\partial I}$$

The first term is the substitution effect, and the second is the income effect.

(B) Compute Hicksian  $h_x(P_x, P_y, V)$  and Marshallian  $d_x(P_x, P_y, I)$  demand for this consumer for good X. (If you are unable to obtain Hicksian demand from the expenditure function, you should explain in words how you would obtain Marshallian if you *did* know Hicksian demand.) (3 points)

To calculate Hicksian demand, take the partial derivative of the expenditure function with respect to the price of X (Shephard's Lemma) to get  $h_x(P_x, P_y, V) = V(p_y/p_x)^{0.5}$ . To calculate Marshallian demand, first invert the expenditure function to get the indirect utility function,  $V(p_x, p_y, I) = I/2(p_x p_y)^{0.5}$ . Then you can either substitute the indirect utility function into the Hicksian demand function or apply Roy's Identity to get  $d_x(P_x, P_y, I) = I/2p_x$ .

(C) Referring to the Slutsky equation, what is the necessary condition for demand for good X to be Giffen? Be specific about the sign and magnitude of each term. (3 points)

Since the substitution effect is non-positive and  $x \ge 0$ , a necessary condition for good X to be Giffen is  $\partial d_x/\partial I < 0$ . Furthermore, the income effect must outweigh the

substitution effect: 
$$\left| x \frac{\partial d_x}{\partial I} \right| > \left| \frac{\partial h_x}{\partial p_x} \right|$$
.

(D) Now use the Slutsky equation to calculate the effect of a change in the price of X on the uncompensated demand for X (hint: you can check your answer by calculating this effect directly from Marshallian demand). Is good X normal, inferior, or Giffen? (If you were unable to solve (B), you can still provide a partial answer by explaining how you would solve this problem *if* you knew Hicksian and Marshallian demand.) (3 points)

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Take the appropriate derivatives and plug them into the Slutsky equation. The only trick is using the indirect utility function to get  $\partial h_x/\partial p_x$  as a function of I. As can be verified by taking the derivative directly, the answer is  $\partial d_x/\partial p_x = -I/2p_x^2$ . Since this is less than zero, we know the good is not Giffen. To determine whether the good is normal or inferior, look at  $\partial d_x/\partial I = 1/2p_x > 0$ , which means good X is normal.

(E) We said in class that we never observe Hicksian demand, only Marshallian demand. However, looking at the Slutsky equation, you can see that this statement is not quite true. When might a real world situation arise (not necessarily for this particular example) where we could observe the substitution effect directly (i.e., independently of the income effect)? (3 points)

If consumption of X were initially zero, then we could estimate the substitution effect for X (for example, for a fall in the price of X) independently of the income effect. For example, consider a case where the many people consume no margarine so long as it costs the same as or more than butter. If the price of margarine falls below the price of butter, however, however, people may substitute towards margarine. Since these consumers weren't buying any margarine originally, there is no income effect of the price drop — only a substitution effect. Note that this case would not apply to the expenditure function above since the consumer always purchase some of both goods (so long as income is positive and prices are finite). But, for many other goods, the margarine-butter case could readily arise.

Another possible answer to this question is that we could observe the substitution effect independently of the income effect if  $\partial X/\partial I=0$ . This is technically correct, but this is not quite as good an answer as above since we it would be difficult to know in the "real world" that  $\partial X/\partial I=0$ . By contrast, we can in the "real world" observe cases where purchases of X are initially zero. Hence, we can empirically find opportunities to estimate substitution effects independently of income effects.