

14.06 Problem Set 3 2005

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Due: Thursday, March 10

Question 1. Decentralized Ramsey Model with Labor and Capital Tax

This model is a variation on the Taxation and Redistribution model discussed in the class notes (Chapter 3.5.2).

As usual, households maximize the following utility function

$$\int_{t=0}^{\infty} e^{-\rho t} \frac{c^j(t)^{1-\gamma}}{1-\gamma} dt$$

Note that this specification of the utility function assumes that labor supply is exogenous. More specifically, we assume that every household supplies one unit of labor. There is no population growth and no technological progress.

We now introduce a government into the economy, where the government imposes a tax on households' labor income and a separate tax on their capital income. The government then redistributes the tax revenue uniformly across households in the form of a lump sum transfer, $T(t)$. The household budget constraint is

$$\dot{k}^j(t) = (1 - \tau^l)w(t) + (1 - \tau^k)r(t)k^j(t) - c^j(t) - \delta k^j(t) + T(t)$$

The budget constraint of the government is

$$T(t) = \int_j [\tau^l w(t) + \tau^k r(t)k^j(t)] dj$$

- What is the resource constraint of the economy?
- Solve the household's maximization problem and give the system of differential equations that characterizes the optimal solution.
- Draw the Phase diagram. How does it compare to the Phase diagram in the model without distortive taxation?
- Suppose that the government decides to increase the tax on capital permanently (this is unanticipated). How does this affect the $\dot{c} = 0$ locus, the $\dot{k} = 0$ locus, and the steady state? What happens to c , k , and y ?
- Now suppose that instead the government decides to increase the tax on labor income. Does this have the same effect as increasing the tax on capital? Why?
- Finally, suppose that labor supply is endogenous in the maximization problem. Would this change your answer to e) and why? [You do not have to do the math, just give the intuition in one or two sentences.]

Question 2. AK Model

The discrete time version of this model is discussed in the class notes (Chapter 6.1). Consider the social planner's problem where utility is given by

$$\int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

The resource constraint is

$$\dot{k}(t) = f(k(t)) - c(t) - \delta k(t)$$

and the production function is of the AK form

$$y(t) = Ak(t)$$

where $A > 0$.

- a) Solve the social planner's maximization problem and give an expression for optimal consumption growth. Which assumption do we need to impose to ensure perpetual growth?
- b) As in the lecture notes, use the guess that consumption is a linear function of capital

$$c(t) = (1 - s)Ak(t).$$

What does this imply about the growth rates of capital and output?

- c) Do k and c converge to a steady state? Is the economy on a balanced growth path?
- d) Find the optimal savings rate, s .

Question 3. Knowledge Spillovers

This question is based on the Learning by Doing and Knowledge Spillovers model discussed in the class notes (Chapter 6.4) and Romer's textbook (page 120). The economy is described as follows

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\gamma}}{1-\gamma} dt$$

$$\dot{k}(t) = F(k(t), h(t)) - c(t) - \delta k(t)$$

$$Y(t) = K(t)^\alpha (h(t)L(t))^{1-\alpha}, \text{ where } h(t) = bk(t) \text{ with } b > 0 \text{ and } L(t) = L$$

- a) Describe how human capital, as given by h , accumulates in this economy. Do firms or individuals directly invest in improving the level of human capital? Or, is it simply a side effect of physical capital accumulation? Given your answer, do you believe that the social planner and decentralized competitive equilibrium will coincide in this model?
- b) Solve the social planner's problem in this economy. What is the growth rate of consumption?
- c) Solve for the competitive equilibrium in this economy. [For simplicity, assume that the labor supply of individuals is exogenous and equal to 1.]
 1. Use the FOCs of the household to find the Euler condition.
 2. Using the firm's profit maximizing behavior, what is the equilibrium interest rate for physical capital and the equilibrium wage rate? (Remember, the firm takes human capital as exogenous).
 3. What is the growth rate of consumption?
- d) Compare the growth rate of the social planner's equilibrium from b) to that of the decentralized equilibrium in c). How are they different, and why?
- e) Suppose we introduce a government into the competitive equilibrium problem. What could the government do to ensure that the competitive equilibrium growth rate of consumption coincides with the social planner's outcome?