

14.06 Problem Set 5

Spring 2005

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Due: Tuesday, April 26

Question 1 *Consumption Smoothing with Certainty*

Consider the following utility function and lifetime budget constraint:

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \quad \gamma > 0$$
$$\sum_{t=0}^T q_t c_t = q_0 (1 + R_0) a_0 + \sum_{t=0}^T q_t y_t, \quad \text{where } q_t = \frac{q_{t-1}}{1 + R_t}.$$

1. Assume that the agent's discount factor β is such that $1 + R_t = 1/\beta$ for all t .
 - (a) Prove that the optimal consumption path is constant;
 - (b) Solve for the optimal consumption level in each period of life;
 - (c) Why was it necessary for us to assume that $1 + R_t = 1/\beta$ for all t ?

To simplify the math, assume $R = 0$ and $q_0 = 1$ for the remainder of the question.

2. Suppose the individual's income is constant in each period of life such that $y_t = \bar{y} \quad \forall t$.
 - (a) What is the optimal consumption level in each period of life now?
 - (b) How much does the individual save in period t ?
 - (c) Are the savings positive or negative? Explain your answer.
3. Now suppose the government decides to tax individuals. The government implements a tax on income in each period of life such that an individual's income is now given by $(1 - \tau)\bar{y}$, where $\tau = 1/(1 + T)$. The government takes the revenue from the tax and throws it in the ocean.
 - (a) How much did the tax reduce the individual's per-period (contemporaneous) income?
 - (b) How much did the tax reduce the individual's lifetime income?
 - (c) How much does the optimal consumption at time $t = 0$ fall?
4. Now suppose the government taxes individual's income only in the last period of life when $t = D$. Specifically, assume that $\tau = 1$, in the last period of life, and $\tau = 0$ in all earlier periods.
 - (a) How much did the tax reduce the individual's per period (contemporaneous) income in each period?
 - (b) How much did the tax reduce the individual's lifetime income?
 - (c) How much does the optimal consumption at time $t = 0$ fall?

5. In a few sentences, explain the intuition of why the government taxes in parts (3) and (4) have the same effect on consumption at time $t = 0$. Be sure to discuss how changes in contemporaneous income and lifetime (permanent) income each affect consumption.

Question 2 *Consumption Smoothing with Uncertainty*

Consider the following utility function and lifetime budget constraint.

$$u(c(S^t)) = c(S^t) - b \frac{c(S^t)^2}{2}, \quad b > 0$$

$$\sum_{t=0}^{\infty} \sum_{S^t} q(S^t) c(S^t) \leq \sum_{t=0}^{\infty} \sum_{S^t} q(S^t) y(S^t).$$

In this model of uncertainty, S^t represents the ‘event’ S , at time t , and all prices and consumption level will be a function of S^t . There are two possible states of the world, $S^t \in \{\text{war, peace}\}$. Individuals have a discount factor β . For this question you should assume that the individual’s wealth and income stream are such that his or her consumption is always in the range where marginal utility is positive.

1. Please describe (in a few sentences) what a ‘good’ is in this model of uncertainty. (i.e. what will the individual in this economy be optimizing over?) How does this differ from the first question of this problem set, which dealt with a model of certainty? How do prices, $q(\cdot)$, in this economy differ?
2. Interpret the lifetime budget constrain I have given you.
 - (a) What was assumed about the initial assets an individual begins with?
 - (b) Can an individual’s consumption in any one state and time exceed his or her income in that state and time? If so, how is this possible?
3. Solve for the FOCs of the individual’s optimization problem. Remember to account for the probability, $\pi(S^t)$, of each event occurring when you write out the individual’s total *expected* lifetime utility.
4. Now consider two possible states of the world at time t . In the first state, which we will now call W^t , the world has been at war for all periods of time up until time t . In the second state, which we will call P^t , the world has been at peace for all periods of time up until time t . Use your FOCs from 3 to prove the following condition is true. Explain the intuition behind this condition. (No intuition, no points!)

$$\frac{\pi(P^t)(1 - bc(P^t))}{\pi(W^t)(1 - bc(W^t))} = \frac{q(P^t)}{q(W^t)}.$$

5. If we assume that the interest rate is constant and equal to the individual’s discount rate, it can be shown that $q(S^t) = \pi(S^t)\beta^t$ for all S^t . Assume this is true. Using your FOCs from 3, what can you say about how consumption varies across different states of the world? How is consumption smoothing different in this model of ‘uncertainty’ compared to consumption smoothing we find in the model of ‘certainty’?