

14.06 Problem Set 6

Spring 2005

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Due: Friday, May 13

Question 1

Exercise 10.12 from Romer's textbook.

Question 2

Assume there are a large number of isolated farmers.¹ Each knows the size of his own crop, y_i . The size of the crop on any farm at any date is described by

$$y_i = \alpha + \varepsilon_i \quad (1)$$

where $Cov(\varepsilon_i, \varepsilon_j) = 0 \forall i \neq j$, α and ε_i are independent, $\alpha \sim Normal(\bar{\alpha}, \sigma_\alpha^2)$, and $\varepsilon_i \sim Normal(0, \sigma_\varepsilon^2)$. Thus if $Y \equiv \sum_{i=1}^n y_i$, then $E(Y|y_i) = h_1 + nh_2 y_i$. Assume that there is a linear demand curve for the crop, so

$$Y = a - bP_s \quad (2)$$

where P_s is the spot price next period.

(a) Show that the distribution of P_s for a given y_i is normal with mean $(a - E[Y|y_i])/b$ and variance σ_p^2 , independent of y_i .

Since the individuals differ in their expectations about the spot prices in the next period, there is an incentive to set up a futures market. In this market the farmers trade today rights on the crops tomorrow at a price P_f .

(b) The demand for "futures" by farmer i is given by

$$Y_i^f = \left[\frac{\frac{a - E(Y|y_i)}{b} - P_f}{k\sigma_p^2} \right] + y_i, \quad (3)$$

¹Denote the number of farmers by n , and assume that it is sufficiently large so you can apply the Law of Large Numbers in case you need it.

where k is a preferences parameter equal to all farmers.²

Find an expression for the aggregate demand for futures $\sum_{i=1}^n Y_i^f$. What is the market equilibrium condition?

(c) Show that the equilibrium price P_f is a linear function

$$P_f = h_3 + h_4 P_s \quad (4)$$

What can you say of P_f as a signal?

Hint: use equation (2).

(d) The result you just obtained is different from the one Angeletos and Werning obtain in their model with endogenous information. What is the main fundamental difference between the models that leads to the different results?

How would you modify this model in order to recover the same qualitative result Angeletos and Werning obtain about prices as aggregators of private information? Explain which formula you would modify and provide intuition.

(e) *Optional*

Assume that the timing is as follows. In the first period the farmers trade in the futures market and observe the price P_f , but they have the information about their own crop y_i . In the second period there is an spot market where the crops are sold and the price P_s is observed.

Suppose you are a farmer in this economy, if you can observe the price P_f at no cost, do you have any incentive to trade “futures” in the first period?

What does this tell you about the “futures market”?

²This is the exact demand for agents with constant absolute risk aversion preferences with risk aversion given by k .