14.06 Problem Set 4 Solutions
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Question 1 *Government Policies and Endogenous Growth*

Consider the model of endogenous growth with expanding product variety covered in class (see Chapter 7 in the Lecture Notes).
Answer. The easiest way to proceed is to introduce the subsidies in the original decentralized model and then see if we can make the solution to this problem be the same as the social planner's allocation (you can follow the notes Miriam prepared for Recitation 7).

Let κ_F, κ_M, and κ_R be the subsidies to final good, intermediate goods and R&D producers.

There are three sectors:

- Final good sector: perfectly competitive and uses labor and intermediate goods in production;
- Intermediate goods sector: monopolistic competition, final good and blueprints are used to produce the intermediate goods;
- R&D sector: perfectly competitive, uses final good to produce new "ideas".

The optimization problem for the final good producers is:

$$\max_{L,X_j} \quad \kappa_F Y - wL - \int_0^N (p_j X_j)\, dj$$

where $Y = AL^{1-\alpha}\left[\int_0^N X_j^\alpha dj\right]$. We shall use the final good as numeraire.

The FOCs for this problem are

$$\begin{aligned} w &= (1-\alpha)\,\kappa_F \frac{Y}{L} \\ p_j &= \alpha A \kappa_F \left(\frac{L}{X_j}\right)^{1-\alpha} \qquad \text{for all } j \in [0, N]\,. \end{aligned}$$

And we know that with constant returns to scale and perfect competition profits will be zero.

We now look at the intermediate goods sector. In this case, each producer maximizes profits subject to demand for the intermediate input

$$\max_{X_j} \quad \Pi_j = \kappa_M p_j X_j - X_j$$
$$\text{s.t.} \quad p_j = \alpha A \kappa_F \left(\frac{L}{X_j}\right)^{1-\alpha}.$$

FOCs:

$$X_j = (\kappa_F \kappa_M)^{\frac{1}{1-\alpha}} xL$$
$$p_j = \frac{1}{\kappa_M \alpha}$$

where

$$x = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

Notice that the price charged to the producers of the final good will be a decreasing function of the subsidy κ_M, while the quantity produced is an increasing function of κ_M, and κ_F; actually the effects of the subsidies on the quantity produced cannot be distinguished. The profits will still be positive because $\kappa_M p_j = 1/\alpha > 1$ in equilibrium. In particular profits are given by

$$\Pi_j = (\kappa_M p_j - 1)(\kappa_F \kappa_M)^{\frac{1}{1-\alpha}} xL = \left(\frac{1-\alpha}{\alpha}\right)(\kappa_F \kappa_M)^{\frac{1}{1-\alpha}} xL. \quad (1)$$

The R&D sector will sell blueprints to the intermediate good producers charging them exactly the present value of the profits it will generate

$$V(t) = \int_t^\infty e^{-r(\tau - t)} \Pi_{\tau j} d\tau = \left(\frac{1-\alpha}{\alpha}\right) \frac{(\kappa_F \kappa_M)^{\frac{1}{1-\alpha}} xL}{r}.$$

If there is perfect competition in the market for blueprints then in equilibrium

$$V = \kappa_R \left(\frac{1-\alpha}{\alpha}\right) \frac{(\kappa_F \kappa_M)^{\frac{1}{1-\alpha}} xL}{r} = \eta$$

where η is the cost of inventing a new blueprint. The equilibrium interest rate is

$$r = \kappa_R \left(\frac{1-\alpha}{\alpha}\right) \frac{(\kappa_F \kappa_M)^{\frac{1}{1-\alpha}} xL}{\eta}.$$

Finally, from the consumer's optimization problem we obtain the standard Euler equation

$$\frac{\dot{c}}{c} = \theta (r - \rho).$$

Total production of final and intermediate goods is

$$X = (A\kappa_F\kappa_M)^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}L \quad (2)$$

$$Y = \left(A\kappa_F^{\alpha}\kappa_M^{\alpha}\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} \; L\; N \quad (3)$$

We can now solve for the social planner's optimal allocation.

We do not need to introduce the subsidies in this solution (again, I will closely follow Miriam's notes in this case). The social planner solves the following problem

$$\max_{c,X,N} \int_0^\infty e^{-\rho t}\frac{c^{1-1/\theta}}{1-1/\theta}dt$$

subject to

$$\dot{N} = (1/\eta)\left[(L/\eta)\,A^{1/(1-\alpha)}\left(\frac{1-\alpha}{\alpha}\right)\alpha^{1/(1-\alpha)} - \rho\right].$$

The solution to this optimal control problem implies the following quantities of final and intermediate goods

$$X = A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}L \quad (4)$$

$$Y = (A\alpha^{\alpha})^{\frac{1}{1-\alpha}} \; L\; N. \quad (5)$$

So, in absence of taxes and/or subsidies the competitive equilibrium does not achieve the first best (social planner's solution) allocation. As you can see the quantities in the competitive equilibrium are smaller than in the first-best by factor of $\alpha^{1/(1-\alpha)}$. ■

1. Show that the government can ensure a first-best equilibrium if it uses a lump-sum tax to finance the appropriate subsidy of the intermediate goods. What rate of subsidy is required? In a richer model, why would it be difficult to carry out the required form of policy?

   Answer. Set κ_F and κ_R equal 0; we want to provide incentives to the intermediate good producers so they set prices equal to marginal cost (= 1). In our simple model this is equivalent to make them choose a production level equal to the one the social planner would choose.

$$(A\kappa_M)^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} = A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}$$

$$(\kappa_M\alpha)^{\frac{1}{1-\alpha}} = 1$$

$$\Rightarrow \kappa_M^* = 1/\alpha.$$

   So the government will pay the intermediate good producer the equivalent to $(1/\alpha)-1$ per unit sold. Then they will produce $X = A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}L$.

   The main problem with this policy is the fact that you need to get the resources to pay the subsidy. One easy way to get around this problem is to assume that you can impose lump-sum taxes on consumers. That assumption is a huge assumption, in general we do not see lump-sum taxes in reality, all the taxes are distortionary, and so introduce some additional distortion in other markets, so you will face a trade off. ■

2. Can the government achieve the first-best solution using a subsidy to the production of the final goods? What assumption of the model is crucial for your answer?

   Answer. Set κ_M and κ_R equal 0; we want to find whether it is possible to subsidize the production of final goods and in that way solve the inefficiency we have in the intermediate goods sector (remember that there is monopolistic competition there so they will not achieve the social optimum). We repeat the exactly the same procedure as in the previous question

$$\begin{aligned}(A\kappa_F)^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} &= A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}} \\ \Rightarrow \kappa_F^* &= 1/\alpha.\end{aligned}$$

   Which is exactly the same subsidy you would give to intermediate good producers. This result is possible because the *final goods market is competitive*, and so without any imperfection on that side, any subsidy given in the production stage will be reflected in the demand for inputs, including intermediate goods. This increases the demand for intermediate inputs and corrects the effect of monopolistic competition, increasing the production of X to the social optimum. The price observed in the market reflect the mark-up still, but the cost for the final good producer drops exactly to 1, which is the marginal cost of producing a unit of any intermediate good, this corrects the distortion in the quantity of X. The increase in the profits will be reflected in an increase in the quantity of blueprints produced and in the interest rate, providing incentives to save. ■

3. Can the government ensure a first-best solution if it relies solely on a subsidy to R&D (financed again by a lump-sum tax)? Explain the answer. What modifications to the model would make it important for the government to subsidize research?

   Answer. You can check equation (2) and see that a subsidy to R&D will have no effect at all here. The reason is that any subsidy to the production of blueprints will be completely absorbed in that sector, production of blueprints will increase. However the production of each variety in the intermediate good sector will still be suboptimal, producers will still charge the markup and the quantity produced will be given by (2), with $\kappa_M = \kappa_F = 0$.

   There will be effects on the interest rate and the growth rate, because of the lower cost of producing blueprints, but the distortion in the intermediate goods market still inhibits the economy from achieving the first best allocation. ■

Question 2 *Scale Effects*

1. Why does the varieties model of technological change with expanding varieties exhibit a scale effect in the sense that the growth rate rises with the aggregate quantity of raw labor, L?

   Answer. In this model, L measures not only the labor force, but also the *"size"* of the economy. The size of the economy is relevant because affects the profits the intermediate good producers obtain. Equation (1) tells us that profits are proportional to the size of the economy, because it determines the "size" of the demand each single producer faces, and so the total revenue it gets.

   The higher profits of the intermediate good producers is then transferred to the R&D sector, inducing a higher production of blueprints, for a given cost η. ∎

2. If you were asked to test this implication of the model, would you use a country's population as the variable to measure L? Explain.

   Answer. A country's population is probably not the best variable to use. The main idea behind the size effect is the market size effect in the intermediate goods sector. Therefore a small country that in fact has access to a huge market through trade will clearly fail to fit in the model if population is used.

   Most of the effect comes from the fact that the innovators who create the blueprints are protected by patents or other intellectual property, and so, the size of the area covered by this patent matters. In fact, if property rights were enforced in the whole world, then world's population would be an adequate measure.[1]

   Choosing the labor force as the relevant measure of size wouldn't work either; the intuition is the same, because it is not the right way to measure the size of the market for the ideas. ∎

3. What types of modifications to the model would eliminate the scale effects? Give one example and explain the intuition behind it.

   Answer. There are several ways to modify the model to eliminate the scale effects. However, introducing perfect competition in the intermediate goods sector does no good to your economy: without (monopoly) rents there is not incentive to create new blueprints.

   A relatively modification would be to introduce a cost of creating new blueprints $\eta(N)$ that is increasing in the total stock of ideas already created in the economy. You can think of this as using the easy ideas first or just the using up of the total number of potential ideas. ∎

[1] If you are interested you can check Jones (1999) and Kremer (1993). Also, check the chapter of the forthcoming Handbook of Economic Growth.

References

Jones, C. (1999); "Growth: With or Without Scale Effects", *American Economic Review*, 89, May, 139-144.

Kremer, M. (1993); "Population Growth and Technological Change: One Million B.C. to 1990", *Quarterly Journal of Economics*, 108, August, 681-716.