14.384: Time Series Econometrics December, 2003

Final Examination

Instructions: This is an open book exam. You have 180 minutes to answer the questions. Write the answers directly on the exam itself. If you need more space, use an exam booklet. Answer 4 out of 5 questions. Good luck!

NAME:_____________________________________

1. **Linear Trends**

   Let $Y_t = \frac{\delta}{\sqrt{t}} + Y_{t-1} + \varepsilon_t, t = 1, ..., T$ where $\varepsilon_t \sim$ iid $N(0, \sigma^2),\ Y_0 = 0,\ \delta > 0$.

   a. Derive $P(Y_t \leq 0)$

      As an approximation, you can use $\sum_{r=1}^{t} \left(\frac{1}{\sqrt{r}}\right) \approx \sqrt{t}$.

   b. Consider regressing Y_t on Y_{t-1}. What is the probability limit of

   $$\hat{\rho} = \frac{\sum_t Y_{t-1} Y_t}{\sum_t Y_{t-1}^2}$$

   (An informal derivation is fine.)

2. **Unit Root?**

Let the DGP be as follows,

$$\begin{aligned} Y_t &= 0.5Y_{t-1} + 0.5Y_{t-2} + \varepsilon_t \qquad (1) \\ \text{where } \varepsilon_t &\sim \text{iid } N(0, \sigma^2) \\ \text{and } Y_0 &= 0,\ Y_{-1} = 0. \end{aligned}$$

a. Consider regressing Y_t on Y_{t-1}. What is the probability limit of

$$\hat{\rho} = \frac{\sum_t Y_{t-1}Y_t}{\sum_t Y_{t-1}^2} \quad ?$$

b. We transform by adding and subtracting $0.5\ \Delta Y_{t-1}$ on the right hand side of equation (1). This gives

$$Y_t = Y_{t-1} - 0.5\Delta Y_{t-1} + \varepsilon_t$$

Regressing Y_t on Y_{t-1} and ΔY_{t-1} yields

$$\begin{aligned} Y_t &= 0.98Y_{t-1} - 0.02\Delta Y_{t-1} + \widehat{\varepsilon}_t \\ & \quad \text{where} \\ \frac{\Sigma \hat{u}_t^2}{T-2} &= 1.5 \end{aligned}$$

Test for a unit root in the DGP. Motivate which test and which table you choose.

3. **Consumption and Income**

Suppose we have quarterly data on aggregated consumption on GDP of Canada from 1993-2002 (200 data points). Regressing the logarithm of consumption and the logarithm of GDP and a constant yields

$$\begin{aligned}
\ln\ \text{Consumption}_t &= 0.78 \\
&\quad + \ln\ \text{GDP}_t + \hat{u}_t \\
\text{where } \frac{\sum_{t=1}^{T} \hat{u}_t^2}{198} &= 1.03 \\
\frac{\sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1}}{\sum_{t=2}^{T} \hat{u}_{t-1}^2} &= 0.11.
\end{aligned}$$

- Are the logarithm of consumption and the logarithm of GDP cointegrated? Motivate which test and which table you choose.

4. **ACD**

Consider the following Autoregressive conditional duration model,

$$T_i \mid T_{i-1}, T_{i-2}, ... \sim \text{ exponential } \left(T_{i-1}^{\gamma}\right)$$

$$\text{where } i = 1, ..., N \text{ and } T_0 = 1.$$

That is,

$$\begin{aligned} p(T_i \quad &| \quad T_{i-1}, T_{i-2}, ...) = p(T_i \mid T_{i-1}) = T_{i-1}^{\gamma} e^{-T_{i-1}^{\gamma} T_i} \\ E(T_i \quad &| \quad T_{i-1}) = \frac{1}{T_{i-1}^{\gamma}}, \\ Var(T_i \quad &| \quad T_{i-1}) = \left(\frac{1}{T_{i-1}^{\gamma}}\right)^2. \end{aligned}$$

a. Derive conditions on γ and the DGP under which the conditional maximum likelihood estimator is consistent and prove consistency of the conditional maximum likelihood estimator under your conditions.
b. Derive conditions on γ and the DGP under which the conditional maximum likelihood estimator is asymptotically normally distributed.

5. **Multivariate Time Series**

Consider the following bivariate system

$$\begin{aligned} x_t &= x_{t-1} + \nu_t \\ y_t &= \alpha_1 y_{t-1} + \alpha_2 x_t + u_t \qquad |\alpha_1| < 1 \end{aligned}$$

where u_t and ν_t are *iid* $\sim (0, 1)$ and mutually independent.

(a) Are x_t and y_t stationary?

(b) Find the matrix $\phi(L)$ in the representation

$$\begin{bmatrix} \Delta x_t \\ \Delta y_t \end{bmatrix} = \phi(L) \begin{bmatrix} u_t \\ \nu_t \end{bmatrix}$$

(c) Find the multivariate error correction representation of the model. i.e.

$$A_0 \begin{bmatrix} \Delta x_t \\ \Delta y_t \end{bmatrix} = \Pi \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ \nu_t \end{bmatrix}.$$

What is the rank of the matrix Π?