

14.384 Problem Set 2

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1 Modeling Trend (continued)

Consider the following time series model

$$\begin{aligned}x_t &= \beta_0 + \beta_1 t + \epsilon_t \\ \epsilon_t &\sim \text{White Noise } (0, \sigma^2)\end{aligned}$$

- Use least squares to detrend x_t (Assume β_0 and β_1 in the population are known and determine the predicted \hat{x}_t) What is the autocovariance function of the residual component?
- Use first differences to detrend x_t . What is the autocovariance function of the residual component?
- Use first differences to detrend x_t assuming that $\epsilon_t = \rho\epsilon_{t-1} + u_t$. What is the autocovariance function of the residual component?

Now assume that the true model is given by a random walk with drift

$$\begin{aligned}x_t &= \mu + x_{t-1} + \epsilon_t, \quad t = 1, \dots, T \\ \epsilon_t &\sim \text{White Noise } (0, \sigma^2)\end{aligned}$$

- Use first differences to detrend x_t . What is the autocovariance function of the residual component?
- Use a linear trend to detrend x_t (write the process introduced in part (d) in a similar way as the one used in part (a)) What is the autocovariance function of the residual component? (Hint: denote $x_0 = \beta_0$ and write the process in terms of μ , β_0 and the ϵ_t 's. You can do this using an iterative method starting from $t = 1$).
- Simulate the results obtained for $n = 100$, $\sigma^2 = 1$, and plot the autocorrelation function for the 2 models considered using the two detrending methods. Comment on the results.

2 Forecasting

Assume that ϵ_t is *iid* with $E\epsilon_t^4 < \infty$, $E\epsilon_t = 0$ and $E\epsilon_t^2 = 1$. Let ϕ_1 and ϕ_2 be such that $\phi(L) = 1 - \phi_1L - \phi_2L^2$ has all its roots outside the unit circle. Assume that x_t is the stationary solution to

$$\phi(L)X_t = \epsilon_t$$

- (a) You are given a sample X_1, \dots, X_T of observations. Find the best linear predictor in the mean square sense of X_{T+1} and X_{T+2} .
- (b) Calculate the MSE of your forecast conditional on X_1, \dots, X_T , i.e. calculate $E((X_{T+h} - \hat{X}_{T+h})^2 | X_T, X_{T-1}, \dots)$ where \hat{X}_{T+h} is the h -step ahead forecast.
- (c) Find the unconditional MSE of the forecast.

3 Simulation

Consider the following time series model

$$x_{t+1} = 0.5x_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{iid } N(0, 1).$$

- (a) Compute the theoretical ACF for the process.
- (b) Simulate a series of length 1000 from this process. Calculate the sample autocovariance. Does the theoretical autocovariance look close to the sample one?
- (c) Simulate 100 different paths of size 1000 from this process. Estimate the first order autocovariance. Plot an histogram of the sample estimated first order autocovariance from this process.
- (d) Find the theoretical asymptotic distribution of the sample first order autocovariance.