**M.I.T.** **Fall 1998**
**Sloan School of Management** **15.415**

Prof. Denis Gromb

## MIDTERM 2

### Section A

Tuesday, November 10, 1998

The exam is between **08:10am** and **09:50am**. Books and lecture notes are not allowed. One double-sided sheet of personal notes written reasonably large is allowed. There are 5 independent questions. Do not line up equations without explanation: **No Explanations, No Points** (unless otherwise specified).

- Please, write your name at the top of **each** page **NOW!**
- Answer each question in the corresponding space
- Keep the package stapled together
- **To avoid chaos and ensure equity among the students:**
  - If you finish before 09:35am, you can hand in your exam and leave the room
  - **If you finish after 09:35am, please STAY SEATED UNTIL YOUR EXAM IS COLLECTED AT YOUR DESK**
  - **Once your exam is collected, please leave the room in silence**

**GOOD LUCK!**

| | |
|---|---|
| Question 1 (20 pts) | |
| Question 2 (20 pts) | |
| Question 3 (20 pts) | |
| Question 4 (20 pts) | |
| Question 5 (20 pts) | |
| | |
| Total (100 pts) | |

**Question 1** [20 points]

No explanations needed. Circle **one or several** answers per question. You will get credit for a question only if you circle **all** the correct answers.

1. Which of the following statements about a portfolio of assets are correct?

   (a) The variance of a portfolio's return is always below that of each of the assets
   (b) The variance of a portfolio's return is always above that of each of the assets
   (c) A portfolio's expected return is always below that of each of the assets
   (d) A portfolio's expected return is always above that of each of the assets
   (e) None of the above

2. Which of the following statements are correct?

   (a) Options on XYZ's stock are generally protected against stock dividends
   (b) Options on XYZ's stock are generally protected against cash dividends
   (c) Options on XYZ's stock are generally protected against stock splits
   (d) Options on XYZ's stock are protected against bankruptcy of XYZ
   (e) None of the above

3. Suppose that investors can form portfolios with only two risky assets, A and B, the returns of which are not perfectly positively or negatively correlated. There is no riskless asset. Which of the following statements are correct according to Optimal Portfolio Theory?

   (a) The relative weights on A and B are the same for all investors
   (b) The relative weights put by an investor on A and B depend on her risk aversion
   (c) Any portfolio of A and B is on the Portfolio Frontier
   (d) Any portfolio of A and B is on the Efficient Portfolio Frontier
   (e) None of the above

4. Suppose that investors can form portfolios with the riskless asset and only two risky assets, A and B, the returns of which are not perfectly positively or negatively correlated. Which of the following statements are correct according to Optimal Portfolio Theory?

   (a) The relative weights on A and B are the same for all investors
   (b) The relative weights put by an investor on A and B depend on her risk aversion
   (c) Any portfolio of A and B is on the Portfolio Frontier
   (d) Any portfolio of A and B is on the Efficient Portfolio Frontier
   (e) None of the above

**Question 2** [20 points]

XYZ's current stock price is $\$50$ and its price one-year from today can take two values.

| Probability | $0.3$ | $0.7$ |
|---|---|---|
| Price $P_1$ | $25$ | $75$ |

The current premium on the European put option on one share of XYZ with 1-year maturity and strike price $\$40$ is $\$5.45$.

**a)** What are the mean and the variance of XYZ's stock return?

**b)** What are the mean and the variance of the put option's return?

**c)** Intuitively, what should be the sign of the correlation coefficient of XYZ's stock and the put option? Explain briefly. Compute that correlation coefficient.

**d)** Consider a portfolio formed with two shares of XYZ and ten of the above put options. What are the mean and the variance of its return?

**Question 3** [20 points]

Companies in this question are assumed to pay dividends every year, on January 1. Today is January 1, 1999. The Gordon model is assumed to hold.

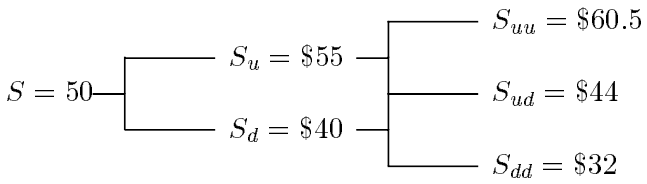
- XYZ has $500,000$ shares and has just paid out a $\$5$ dividend per share which is expected to grow at a constant $2\%$ rate until and including January 1, 2005. It will then remain constant in perpetuity. XYZ's market capitalization rate is $7\%$. Next year's earnings are forecasted to be $\$2M$ (to accrue on January 1, 2000).
- ABC has $200,000$ shares and has just paid out a $\$7$ dividend which is expected to grow at a constant rate $g$ in perpetuity. ABC's current stock price right after the dividend was paid is $\$100$. ABC's market capitalization rate is $9\%$. Next year's earnings are forecasted to be $\$1M$ (to accrue on January 1, 2000).
- HAL has $100,000$ shares. Its assets consists solely of $5\%$ of XYZ's shares and $10\%$ of ABC's shares. All the dividends it receives from XYZ and ABC are immediately paid out to its shareholders.

**a)** What is XYZ's stock price (after paying the dividend of $5 per share)? What is the value of ABC's $g$?

**b)** What are HAL's share price and its PVGO?

**Question 4** [20 points]

Monthly risk-free rate is constant at 2%. XYZ Inc. has just paid out a \$5 quartly dividend, and its current stock price right after the dividend is \$50. The process governing XYZ's stock price over the next two months is represented on the following tree.




**a)** What are the premia on a European and an American put option on one share of XYZ with a 2-year maturity and strike price $\$45$?

[Question 4.a. continued]

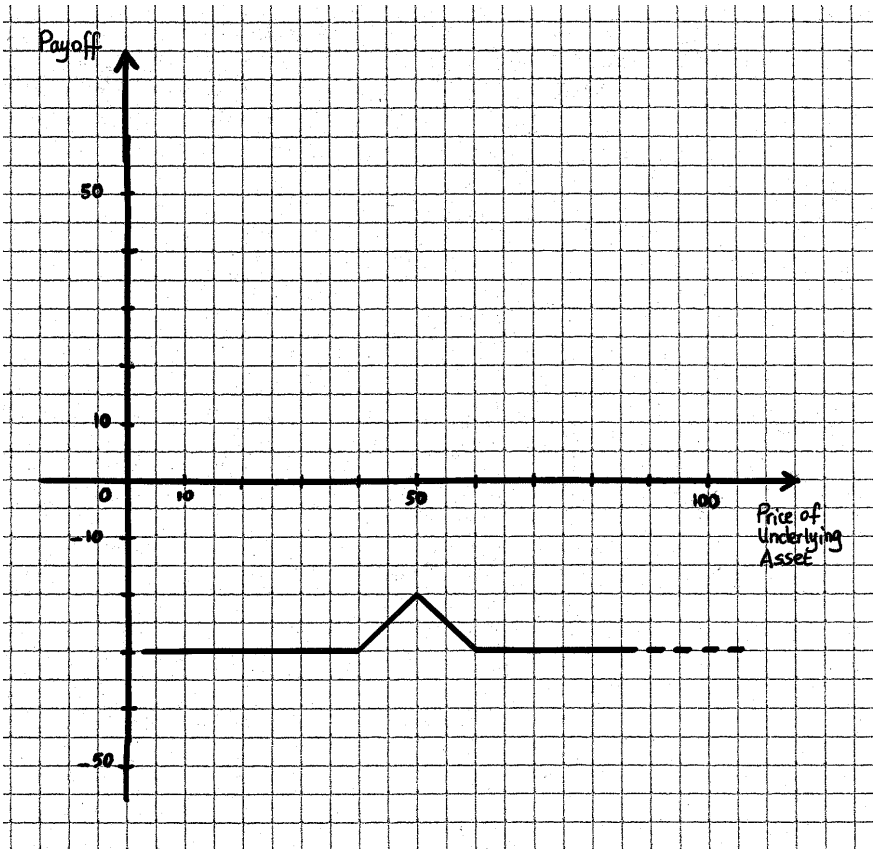
b) What are the premia on a European and an American call option on one share of XYZ with a 2-year maturity and strike price $\$45$?

[Question 4.b. continued]

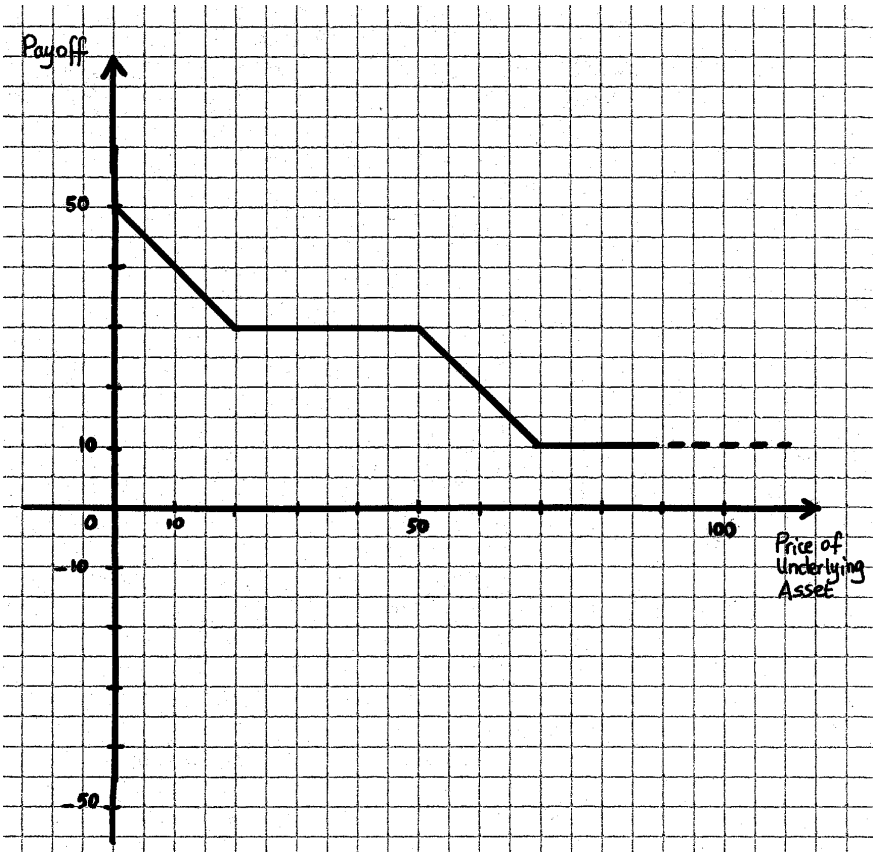
**Question 5** [20 points]

No explanations are needed to answer the following questions. No partial credit will be given. In the following questions, the underlying asset is one share of XYZ Inc.'s stock.

**a)** The following diagram represents a portfolio's payoff 2 months from today depending on XYZ's stock price at that date. Describe as precisely as possible one such portfolio.



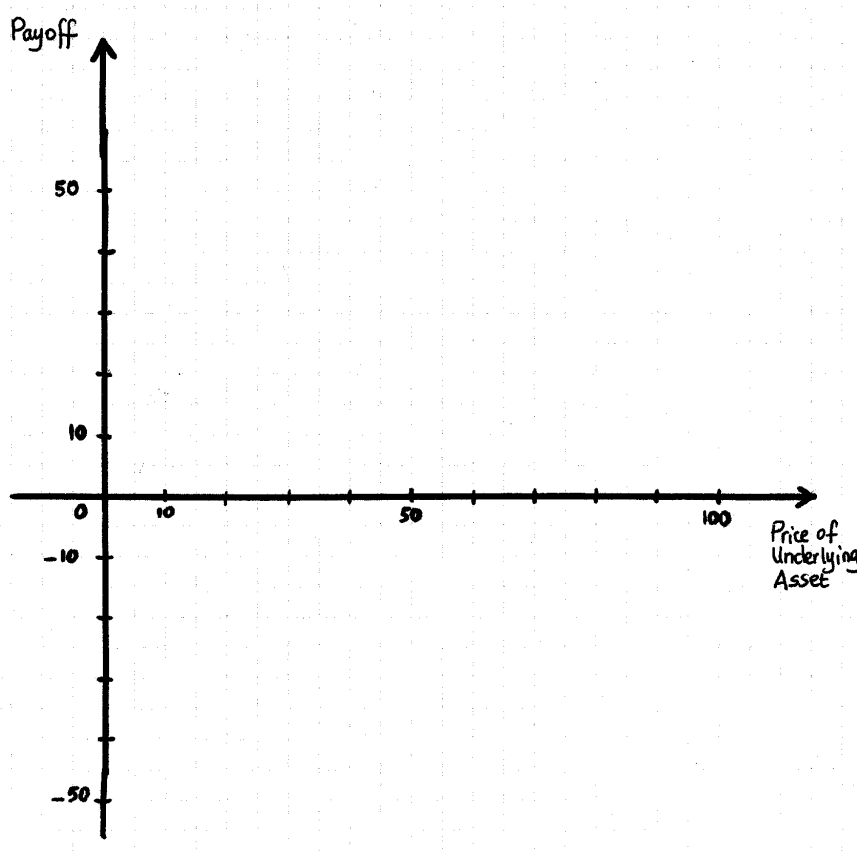
**b)** The following diagram represents a portfolio's payoff 2 months from today depending on XYZ's stock price at that date. Describe as precisely as possible one such portfolio.



**c)** Represent on a graph the payoff 3 months from today of the portfolio combining:

- Two written puts on one share of XYZ with maturity 3 months and strike price $\$30$
- One long call on one share of XYZ with maturity 3 months and strike price $\$60$

Indicate the coordinates of each point at which the graph's slope changes.



**d)** Represent on a graph the payoff 3 months from today of the portfolio combining:

- Two written calls on one share of XYZ with maturity 3 months and strike price $\$30$
- One long put on one share of XYZ with maturity 3 months and strike price $\$60$

Indicate the coordinates of each point at which the graph's slope changes.