M.I.T.
Sloan School of Management

15.415-Fall 1998
Professor Denis Gromb

Problem Set 6

- **Please, make sure to work in a team (3 to 5 students) and hand in only one homework per team**
- **To ease and speed up grading, please:**
  - **Answer one question per page following the questions' order**
  - **Staple all pages of the problem set together (use staples, not paper clips)**
  - **Write the names of all team members very clearly**

Question 1

No explanations needed. Choose **one or several** answers per question. You will get credit for a question only if you circle **all** the correct answers.

1) XYZ Inc. has debt outstanding with face value $K = \$100,000$ maturing in $T = 3$ months. This debt can be seen as:

a. A call on XYZ's assets with strike price K and maturity T

b. A written put on XYZ's assets with strike price K and maturity T

c. Risk-free debt with face value K and a short put on XYZ's assets with strike price K

d. A claim on XYZ's assets and a short call on XYZ's assets with strike price K

e. A put and a written call on XYZ's assets both with strike price K

2) Consider the portfolio formed of:

- A call on one share of ABC's stock with maturity 3 months and strike price $\$45$

- A written put on one share of ABC's stock with maturity 3 months and strike price $\$45$

- A short position in one share of ABC's stock

As ABC's stock price increases, the value of the portfolio:

a. Remains unchanged

b. Decreases

c. Increases

d. Increases and then decreases

e. Changes in a direction that cannot be determined without further information

3) Security A has an expected rate of return of 13% and a beta of 1.2. The risk-free rate is 5% and the expected rate of return on the market portfolio is 10%. According to the CAPM, security A is:

a. Fairly priced

b. Overpriced

c. Underpriced

d. None of the above

4) Suppose that you short the market portfolio. What is the beta of your position?

a. Undefined

b. 1

c. 0

d. -1

e. Infinite

Question 2

For each of the following portfolios, draw the payoff diagram at maturity (ignoring the investment cost). Make sure to indicate the coordinates of each points at which the graph's changes slope. No explanations needed.

a) Portfolio A consists of one written put on one share of XYZ, with strike price $\$50$.

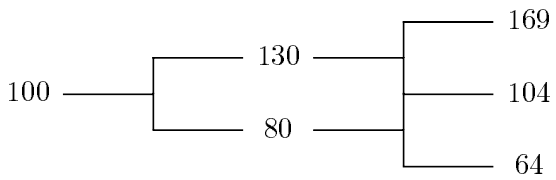
b) Portfolio B consists of: two shares of XYZ and one written put on one share of XYZ, with strike price $\$50$.

c) Portfolio C consists of one share of XYZ, one written call on one share of XYZ, with strike price $\$50$ and one put on one share of XYZ, with strike price $\$50$.

d) Portfolio D consists of one written call on one share of XYZ, with strike price $\$50$, one put on one share of XYZ, with strike price $\$70$ and one risk-free bond with face value $\$20$.

Question 3

The stock of XYZ Inc. trades currently for $\$100$. Each of the next two years, it will either go up by 30% or down by 20% as represented on the tree below. XYZ will not pay any dividend over the next two years. The yield curve is flat at 10%.



To get full credit, make sure you show your work (calculations) and that you explain (in words)

what you are doing.

a) What is the price of an European call on one share of XYZ with a 2-year maturity and a $\$99$ strike price?

b) What is the price of an European put on one share of XYZ with a 2-year maturity and a $\$99$ strike price?

c) What is the price of an American put on one share of XYZ with a 2-year maturity and a $\$99$ strike price?

d) What is the value of a portfolio consisting of one share of XYZ and one written American call on one share of XYZ with a 2-year maturity and a $\$99$ strike price?

Question 4

Consider the covariance matrix of the rates of return of asset 1, asset 2 and the market portfolio:

	r_1	r_2	r_m
r_1	0.16	0.02	0.064
r_2		0.09	0.032
r_m			0.04

Assume CAPM holds.

To get full credit, make sure you show your work (calculations) and that you explain (in words) what you are doing.

a) What are the betas of assets 1 and 2?

b) What is the systematic risk (as measured by variance) of the return on a portfolio with $\$50,000$ invested in asset 1 and $\$10,000$ in asset 2?

Question 5

The one-year risk-free rate is 2.5% and the market rate of return over that same period depends on the state of the economy. If it is in a good state, the return is 18% while it is 2% in a bad state. Both states are equiprobable.

Consider the following investment opportunity. Choose an amount of money $\$X$ less than $\$10M$ to invest in the project. (You can think of X as being the project's scale and of $\$10M$ as its maximum scale). The cash flow generated by the project after one year depends on the state of the economy. If the economy is a good state, the project generates a cash flow of $0.8 \times \$X$. If the economy is in a bad state, it generates $1.1 \times \$X$. For instance, if you choose to invest $\$2M$ in the project today, you will receive next year either $\$1.6M$ if the economy does well or $\$2.2M$ if the economy does poorly.

To get full credit, make sure you show your work (calculations if any) and that you explain (in words) what you are doing.

a) What is this investment's expected return? What is the standard deviation of its return?

b) What is the beta of this investment?

c) Suppose that your personal wealth amounts to $\$10M$ and that investing in the project would mean founding a firm with you as the sole investor. Would you indeed found the firm and undertake the project? If not, explain. If yes, would you undertake the project at full scale (i.e. invest $\$10M$) or less than scale (i.e. invest a positive amount but less than $\$10M$)? Explain.

d) Suppose now that you are the manager of a publicly traded company which is faced with the same investment opportunity. Would your answers change? Explain.