**M.I.T.** **15.415-Fall 1998**
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# Solution to Problem Set 8

**Question 1** (Brealey and Myers 6.10)

**a)** The nominal rate will be

$$r = (1.06 \times 1.05) - 1 = 11.3\%$$

The three-year nominal annuity with present value $\$28.37$ payment $C$ is given by:

$$\frac{C}{11.3\%}\left[1 - \frac{1}{(1.113)^3}\right] = 28.37 \qquad \text{which implies } C = \$11.67$$

Similarly, the two-year annuity with present value $\$21.00$ payment $C$ is given by:

$$\frac{C}{11.3\%}\left[1 - \frac{1}{(1.113)^2}\right] = 21 \qquad \text{which implies } C = \$12.31$$

The nominal annuities are not correct estimates of the equivalent annual costs because they do not take into account the fact that inflation has a different impact on project with different life spans.

**b)** The nominal rate is now

$$r = (1.06 \times 1.25) - 1 = 32.5\%$$

Now the nominal annuities (computed as before) are respectively $\$16.17$ and $\$15.86$.

Machine A had the lower nominal annual cost under a $5\%$ inflation rate, but the higher one under a $25\%$ inflation rate. It is thus clear that inflation has a significant impact on the calculations, and hence the warning in the text to do these calculations in real terms.

**Question 2** (Brealey and Myers 6.11)

Let us calculate the equivalent annual cost of the two alternatives.

(1) *Sell the newer machine:* We receive cashflow from the sale, pay taxes on the gain and must pay the cost associated with keeping the older machine.

$$\begin{aligned} PV_1 &= 50 - 35\% \cdot (50 - 0) - 20 - \frac{30}{12\%} \cdot \left[1 - \frac{1}{1.12^5}\right] \\ &\quad + \frac{5}{1.12^5} - \frac{35\%\,(5-0)}{1.12^5} \\ &= -\$93.80k \end{aligned}$$

Over 5 years, the EAC is:

$$\text{EAC}_1 = \frac{93.8}{\frac{1}{12\%}\left[1 - \frac{1}{1.12^5}\right]} = -\$26,020$$

(2) *Sell the older machine:* We receive cashflow from the sale, pay taxes on the gain and must pay the cost associated with keeping the newer machine.

$$\begin{aligned} PV_2 &= 25 - 35\% \cdot (25-0) - 20 - \frac{20}{12\%} \cdot \left[1 - \frac{1}{1.12^5}\right] - \frac{1}{1.12^5} \cdot \frac{30}{12\%} \cdot \left[1 - \frac{1}{1.12^5}\right] \\ &\quad + \frac{5}{1.12^{10}} - \frac{35\%\,(5-0)}{1.12^{10}} \\ &= -\$127.51k \end{aligned}$$

Over 10 years, the EAC is:

$$\text{EAC}_2 = \frac{127.51}{\frac{1}{12\%}\left[1 - \frac{1}{1.12^{10}}\right]} = -\$22,570$$

It is better to sell the older machine. The key assumption we made is that whenever the machines have to be replaced, they are replaced by a machine like the newer machine.

**Question 3** (Brealey and Myers 6.16)

Assume the following:

(a) The firm will manufacture for at least $10$ years;

(b) There will be no inflation or technological change

(c) The $15\%$ cost of capital is appropriate for all flows and is a real, after-tax rate of return; and

(d) All operating flows occur at the end of the year.

Since purchasing the lids may be considered as a $1$ year "project," the projects have a common chain life of $10$ years. The following figures are in thousands:

$$NPV \text{ (purchase)} = -\sum_{t=1}^{10} \frac{(2)\,(200)\,(1-.35)}{1.15^t} = -1305$$

$$\begin{aligned} NPV \text{ (make)} &= -150 - 30 - \sum_{t=1}^{10} \frac{(1.5)\,(200)\,(1-.35)}{1.15^t} \\ &\quad + (.35)\,(150)\,\left(\frac{.1429}{1.15^1} + \frac{.2449}{1.15^2} + \frac{.1749}{1.15^3} + \frac{.1249}{1.15^4}\right. \\ &\quad \left. + \frac{.0893}{1.15^5} + \frac{.0893}{1.15^6} + \frac{.0893}{1.15^7} + \frac{.0445}{1.15^8}\right) + \frac{30}{1.15^{10}} \\ &= -1,118 \end{aligned}$$

They should make the lids.

**Question 4** (Brealey and Myers 6.18)

| Item | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Sales | | 4200.0 | 4410.0 | 4630.5 | 4862.0 | 5105.1 | 5360.4 | 5628.4 | 5909.8 |
| Manufacturing Costs | | 3780.0 | 3969.0 | 4167.5 | 4375.8 | 4594.6 | 4824.3 | 5035.6 | 5318.8 |
| Depreciation | | 120.0 | 120.0 | 120.0 | 120.0 | 120.0 | 120.0 | 120.0 | 120.0 |
| Earnings before Taxes | | 300.0 | 321.0 | 343.1 | 366.2 | 390.5 | 416.0 | 442.8 | 471.0 |
| Taxes | | 105.0 | 112.4 | 120.1 | 128.2 | 136.7 | 145.6 | 155.0 | 164.8 |
| Cash flow, Operations | | 315.0 | 328.7 | 343.0 | 358.0 | 373.8 | 390.4 | 407.8 | 426.1 |
| Net Working Capital | 350.0 | 420.0 | 441.0 | 463.1 | 486.2 | 510.5 | 536.0 | 562.8 | |
| Incr. NWC | 350.0 | 70.0 | 21.0 | 22.1 | 23.2 | 24.3 | 25.5 | 26.8 | |
| Rent (after tax) | | 65.0 | 67.6 | 70.3 | 73.1 | 76.0 | 79.1 | 82.2 | 85.5 |
| Initial Investment | 1200.0 | | | | | | | | |
| Sales of plant, etc. | | | | | | | | | 400.0 |
| Tax on Sale | | | | | | | | | 56.0 |
| Recovery of NWC | | | | | | | | | 562.8 |
| Net Cash Flow | −1550.0 | 180.0 | 240.1 | 250.6 | 261.8 | 273.5 | 285.8 | 298.8 | 1247.4 |

Present value at $12\%$: $\$85.8$

**Question 5** (Brealey and Myers 6.20)

*Option 1: Remodel with navigation system*

| | 0 | 1 to 5 | 6 to 10 |
|---|---|---|---|
| New Engine | 250 | | |
| Navigation System | 200 | | |
| Repair Hull | 160 | | |
| Depreciation | | 90 | 0 |
| Operating Costs | | 985 | 985 |
| Before Tax Cost Cashflow | 610 | 985 | 985 |
| After Tax Cost Cashflow | 546 | 555 | 591 |

NPV of costs at $10\%$: $\$4,041$

EAC over 10 years: $\$658$

*Option 2: Remodel without navigation system*

| | 0 | 1 to 5 | 6 to 10 |
|---|---|---|---|
| New Engine | 250 | | |
| Navigation System | 0 | | |
| Repair Hull | 160 | | |
| Depreciation | | 50 | 0 |
| Operating Costs | | 1181 | 1181 |
| Before Tax Cost Cashflow | 410 | 1181 | 1181 |
| After Tax Cost Cashflow | 346 | 689 | 709 |

NPV of costs at $10\%$: $\$4,627$

EAC over 10 years: $\$753$

*Option 3: Buy new boat*

| | 0 | 1 to 10 | 11 to 15 |
|---|---|---|---|
| Sale of Vital Spark | 140 | | |
| Tax on Sale | 56 | | |
| Cost of New Boat | 2000 | | |
| Depreciation | | 200 | 0 |
| Operating Costs | | 900 | 900 |
| Before Tax Cost Cashflow | 1860 | 900 | 900 |
| After Tax Cost Cashflow | 1916 | 460 | 540 |

NPV of costs at $10\%$: $\$5,532$

EAC over 10 years: $\$727$

Conclusion: Option 1 has the lowest EAC and so NETCO should remodel the boat. including a new navigation system.

**Question 6** (Brealey and Myers 17.18)

**a)** Because the firms are identical except for financial structure, and there are no taxes or market imperfections, the total value of these companies must be the same. Thus, L's stock is worth $\$500 - \$400 = \$100$.

**b)** $\$20$ of U's common stock represent $4\%$ of all outstanding shares. Hence, you receive per year $4\% \cdot 150 = \$6$ in a boom and $4\% \cdot 50 = \$2$ in a slump.

The equivalent investment is to purchase $4\%$ of L's outstanding stock, which will cost $4\% \cdot 100 = \$4$ and $4\%$ of its outstanding debt, which will cost $4\% \cdot 400 = \$16$. (Note: Since L's debt is risk-free, you need not hold L's debt. Any substitute, such as US Treasury bonds, will do the trick.)

The total invested is the same ($\$20$). Moreover, in a boom you receive per year $4\% \cdot (150 - 40) + 10\% \cdot 16 = \$6$ and in a slump you receive per year $4\% \cdot (50 - 40) + 10\% \cdot 16 = \$2$.

**c)** $\$20$ of L's common stock represent $20\%$ of all outstanding shares. Hence, you receive per year $20\% \cdot (150 - 40) = \$22$ in a boom and $20\% \cdot (50 - 40) = \$2$ in a slump.

The equivalent investment is to purchase $20\%$ of U's stock, which will cost $20\% \cdot 500 = \$100$ and to short $20\%$ of L's debt by which you raise $\$80$ (or, equivalently, borrow $\$80$ at the risk free rate).

The total invested is the same ($\$20$). Moreover, in a boom you receive per year $20\% \cdot 150 - 10\% \cdot 80 = \$22$ and in a slump you receive per year $20\% \cdot 50 - 10\% \cdot 80 = \$2$.

**d)** MM Proposition II as applied to Firm L states that:

$$r_E^L = r_A + \frac{D}{E}(r_A - r_D)$$

Firm L's unlevered cost of capital $r_A$ is the required return on firm L's equity if it were $100\%$ equity financed. hence, it is firm U's required return on equity. The expression is thus

$$r_E^L = r_E^U + \frac{D}{E}\left(r_E^U - r_D\right)$$

Now let us check that this relationship is indeed satisfied in our example.

The expected return on equity for firm U is:

$$\frac{\frac{1}{2}150 + \frac{1}{2}50}{500} = 20\%$$

Firm L's debt-equity ratio is:

$$\frac{D}{E} = \frac{400}{100} = 4$$

Hence, MM Proposition II suggests that firm L's expected return on equity is:

$$r_E^L = 20\% + 4 \times (20\% - 10\%) = 60\%$$

We check that the expected return on equity for firm L is indeed:

$$\frac{\frac{1}{2}(150 - 40) + \frac{1}{2}(50 - 40)}{100} = 60\%$$

**Question 7** (Brealey and Myers 18.18)

**a)** $(1 - \tau) \cdot 20\% = 14\%$ which implies $\tau = 30\%$

**b)** Firm A should borrow to a level such that its marginal rate of corporate tax equals the personal tax rate of the marginal lender. Because the latter is $30\%$, firm A should have a $25\%$ debt ratio.