M.I.T. **15.415-Fall 1998**
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Solution to Problem Set 9

Question 1

1) c; **2)** e; **3)** e; **4)** b; **5)** e; **6)** b and c.

Question 2

a) We have $B_A = \frac{50}{1+8.9745\%} + \frac{1050}{(1+8.9745\%)^2} = \boxed{\$930.060}$

Similarly, we obtain $\boxed{B_B = \$772.183}$ and $\boxed{B_C = \$1,026.164}$

b) First we determine the prices B_1, B_2 and B_3 of the 1-year, 2-year and 3-year zeros with face value $\$1$ by solving: $\begin{cases} B_A = 50B_1 + 1,050B_2 \\ B_B = 1,000B_3 \\ B_C = 100B_1 + 100B_2 + 1,100B_3 \end{cases}$

$$\begin{cases} 1000B_3 = B_B \\ 2,000B_2 = 2B_A - B_c + 1,100B_3 \\ 50B_1 = B_A - 1,050B_2 \end{cases} \quad \text{or} \quad \begin{cases} B_3 = 77.2183\% \\ B_2 = 84.168\% \\ B_1 = 92.592\% \end{cases}$$

Now we can get the spot rates $\boxed{\begin{cases} r_1 = \frac{1}{B_1} - 1 = 8.00\% \\ r_2 = \left(\frac{1}{B_2}\right)^{\frac{1}{2}} - 1 = 9.00\% \\ r_3 = \left(\frac{1}{B_3}\right)^{\frac{1}{3}} - 1 = 9.00\% \end{cases}}$

c) The contract's price is $B = \frac{90}{(1+r_2)^2} + \frac{1090}{(1+r_3)^3} = \boxed{\$917.431}$

d) The contract gives payments at maturities for which the yield curve is flat at 9%. Hence, its YTM is $\boxed{y = 9\%}$. The contract's modified duration is $D = \frac{\frac{90}{(1+y)^2}\times 2 + \frac{1090}{(1+y)^3}\times 3}{\frac{90}{(1+y)^2} + \frac{1090}{(1+y)^3}} = \boxed{2.917}$

Question 3

a)

b) The risk-free probability of an up is $q = \frac{R-d}{u-d} = \frac{1.1-0.8}{1.3-0.8} = 0.6$

The standard call's payoffs are $C_{uu} = \$74$, $C_{ud} = C_{du} = \$9$ and $C_{dd} = 0$. Hence, its premium is:

$$C = \frac{1}{R^2}\left[q^2 C_{uu} + 2q(1-q)C_{ud}\right] = \boxed{\$25.59}$$

c) Ignoring the premium, the payoffs of both calls are identical hence they have the same value. However, the standard call's premium is always paid initially. Instead, the contingent premium call's premium is not always paid and when it is paid, it is paid at maturity. Hence, it has to be greater than the standard call's premium.

d) The premium c is the solution to

$$0 = \frac{1}{R^2}\left[q^2\left(C_{uu} - c\right) + 2q(1-q)\left(C_{ud} - c\right)\right]$$

That is $p = \frac{q^2 C_{uu} + 2q(1-q)C_{ud}}{q^2 + 2q(1-q)} = \boxed{\$36.86}$

Question 4

a) Firm X's equity beta is $\beta_X = \frac{\sigma_{Xm}}{\sigma_m^2} = \frac{0.06}{0.04} = \boxed{1.5}$. The required return on equity is $r_X = r_f + \beta_X \cdot [r_m - r_f] = \boxed{17\%}$. Similarly, we obtain $\boxed{\beta_Y = 0.25 \text{ and } r_Y = 7\%}$ and $\boxed{\beta_Z = 0.75 \text{ and } r_Z = 11\%}$.

b) Holding portfolio P means investing $\$50,000$ in stock X, $\$60,000$ in stock Y and $\$18,000$ in stock Z. Hence, the weight on stock X is $w_X = \frac{50,000}{50,000+60,000+18,000} = 0.39$. Similarly, $w_Y = 0.47$ and $w_Z = 0.14$. Hence,

$$\beta_P = w_X\beta_X + w_Y\beta_Y + w_Z\beta_Z = \boxed{0.8075}$$

The variance of the portfolio's return is:

$$\begin{aligned}\sigma_P^2 &= w_X^2\sigma_X^2 + w_Y^2\sigma_Y^2 + w_Z^2\sigma_Z^2 + 2w_Xw_Y\sigma_{XY} + 2w_Xw_Z\sigma_{XZ} + 2w_Yw_Z\sigma_{YZ} \\ &= 0.101\end{aligned}$$

The systematic risk (as measured by variance) is: $\beta_P^2 \times \sigma_m^2$ and the idiosyncratic risk (as measured by variance) is thus: $\sigma_P^2 - \beta_P^2 \times \sigma_m^2$. Hence,

$$\sigma_P^2 - \beta_P^2 \times \sigma_m^2 = 0.101 - (0.8075)^2 \times 0.04 = \boxed{0.075}$$

Question 5

a) ABC has no debt. Hence, $\beta_A = \beta_E = \boxed{0.92}$

b) WACC$= r_E = r_f + \beta_E \cdot [r_m - r_f] = 5\% + 0.92 \times 8\% = \boxed{12.36\%}$

c) By MM's Theorem, ABC's WACC is independent of its capital structure.

d) We certainly cannot use ABC's WACC to value this project because it does not have the same business risk. (Note: Absent taxes, we do not care about the additional condition that the capital structures should be the same). Instead, we need to estimate the project's asset beta from that of firms in a similar business (firms 3 and 4). Absent taxes we know that:

$$\beta_A = \frac{D}{D+E}\beta_D + \frac{E}{D+E}\beta_E$$

Hence, we have

$$\begin{aligned}\beta_3^A &= \frac{30}{130} \times 0.15 + \frac{100}{130} \times 1.38 = 1.097 \\ \beta_4^A &= \frac{70}{270} \times 0.18 + \frac{200}{270} \times 1.42 = 1.143\end{aligned}$$

Hence, we estimate the project's asset beta to be

$$\beta_A \simeq \frac{\beta_3^A + \beta_4^A}{2} = 1.12$$

The rate appropriate to discount the project's cash flows is thus

$$r_A = r_f + \beta_A [r_m - r_f] = 5\% + 1.12 \times 8\% = 14\%$$

Hence, ABC should not undertake the project because its NPV is

$$-38 + \frac{5M}{14\%} = -\$2.3M < 0$$

Question 7

First note that prospecting costs are sunk and should not enter our reasoning.

a) The field should be shut down just before exploitation costs c exceed revenues from selling the oil. In year t, these revenues are $B \times p \times (1-g)^{t-1}$. Let, t_1 be the solution to:

$$60,000 \times 18 \times (1 - 5\%)^{t_1 - 1} = 2,000,000$$

By trial and error, one finds that the first year for which costs exceeds revenues is $t_1 = 34$ years. (Of course, one could also solve the equation directly and find $t_1 = 1 + \frac{Ln(c_1/(B \times p))}{Ln(1-g)} = 33.9$). This implies that one should shut down the field at the end year $T_1 = 33$. Similarly, one finds $t_2 = 4.55$ years and so $T_2 = \mathbf{4}$ **years**.

After the development costs are sunk and explotation starts, the exploitation costs become known. The value of the field is the difference between a (negatively) growing annuity and of a flat one: $V = \frac{B \times p}{r+g}\left(1 - \frac{(1-g)^T}{(1+r)^T}\right) - c\left(1 - \frac{1}{(1+r)^T}\right)$ that is $V_1 = \$\mathbf{52.29M}$ and $V_2 = \$\mathbf{3.42M}$.

b) In general, we cannot simply compare $\$35M$ and the expectation of V. Indeed, this would be ignoring the fact that at the time we make the investment of $\$35M$, the future cash flows are risky!

What we should do is (1) compute the expected PV of cash flows as of May 1999; (2) determine the discount rate ρ appropriate for the risk associated with the two scenarios; (3) compute the NPV.

(1) In May 1999, PV of future cash flows is equiprobably $(1+r)V_1$ or $(1+r)V_2$. Hence the expected PV at that date is $\frac{(1+r)(V_1+V_2)}{2}$.

(2) The nature of the soil is unrelated to the market. Hence, the appropriate discount rate is the risk-free rate, i.e. $\rho = r = 10\%$.

(3) The project should not be undertaken because its NPV is

$$-35 + \frac{1}{(1+\rho)} \times \frac{(1+r)(V_1+V_2)}{2} = -\$7.145M < 0$$

Note: Step (2) is key. If you miss that step, you will generally get the wrong answer. That is, in general, $\rho \neq r$.