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Solution to Problem Set 1

Question 1

- a. $PV = \$100 (.905) = \90.50
- b. $PV = \$100 (.295) = \29.50
- c. $PV = \$100 (.035) = \3.50
- d. $PV = \$100 (2.402) = \240.20

Question 2

- a. $PV = \$100 (4.279) = \427.90
- b. $PV = \$100 (4.580) = \458.00
- c. **Method 1:** We can think of the cash flows in this problem as being the sum of two separate streams of cash flows. The first stream is \$100 a year received in years 1 through 12; the second is \$100 a year paid in years 1 through 2.

The PV of \$100 received in years 1 to 12 is:

$$PV = \frac{100}{.09} \left[1 - \frac{1}{1.09^{12}} \right] = \$716.10$$

The PV of \$100 paid in years 1 to 2 is:

$$PV = \frac{100}{.09} \left[1 - \frac{1}{1.09^2} \right] = \$175.90$$

Therefore the present value of \$100 received in years 3 through 12 is $\$716.10 - \$175.90 = \$540.2$.

Method 2 (absolutely equivalent to method 1): We can think of this as a 10-year annuity starting in year 3. Be careful: The Present Value **IN YEAR 2** of a 10-year annuity starting in year 3 is:

$$PV = \frac{100}{.09} \left[1 - \frac{1}{1.09^{10}} \right]$$

To obtain the PV in year 0, you need to discount this by $\frac{1}{(1.09)^2}$. (A common mistake is to use $\frac{1}{(1.09)^3}$). Hence, the PV of the cash flows is

$$PV = \frac{1}{1.09^2} \times \frac{100}{.09} \left[1 - \frac{1}{1.09^{10}} \right] = \$540.2$$

Question 3

a. $DF_1 = \frac{1}{1+r_1} = .88$; thus, $r = .136$

b. $DF_2 = \frac{1}{(1+r_2)^2} = \frac{1}{1.105^2} = .82$

c. $AF_2 = DF_1 + DF_2 = .88 + .82 = 1.70$

d. PV of an annuity = C [Annuity factor at r for t years]

Here, $\$24.49 = \$10 [AF_3]$ and so $AF_3 = 2.45$.

e. $AF_3 = DF_1 + DF_2 + DF_3 = AF_2 + DF_3$

$$2.45 = 1.70 + DF_3$$

$$DF_3 = .75$$

Question 4: IRA Accounts

a) The yearly contribution is $2000 \times (1 - 28\%) = 1440$. Since you pay tax on the interest income, the relevant interest rate is $6\% \times (1 - 28\%) = 4.32\%$.

To compute the money that you have at your retirement, you can use the present value formula with 30 cash flows. You can do this on a spreadsheet. However, there is a simpler way. You can compute the future value of the cash flows, using the annuity formula, and then compute the future value of this present value, multiplying by $(1 + 4.32\%)^{30}$. The FV is

$$\frac{1440}{4.32\%} \left(1 - \frac{1}{1.0432^{30}}\right) \times 1.0432^{30} = 85218$$

b) The yearly contribution is the same as in part a). However, the interest rate is 6%. The before tax money that you have at retirement is

$$\frac{1440}{6\%} \left(1 - \frac{1}{1.06^{30}}\right) \times 1.06^{30} = 113844$$

You pay taxes on the interest income. The interest income is the difference between the 113844 and the money you would have had if the interest was 0%. Therefore, the interest income is $113844 - 30 \times 1440 = 70644$ and the tax is $70644 \times 28\% = 19780$. Your retirement money is thus $113844 - 19780 = 94064$.

c) The yearly contribution is 2000 and the interest rate is 6%. The before-tax money at retirement is

$$\frac{2000}{6\%} \times \left(1 - \frac{1}{1.06^{30}}\right) \times 1.06^{30} = 158116$$

Your retirement money is thus $158116 \times (1 - 28\%) = 113844$

d) The benefit should increase because deferred tax on which interest accrues is greater.

Question 5: Project Decisions

a) The cash flow table is

Year	Cost	Revenue	Net Cash Flow
0	4.6	2.52	-2.08
1	0.7	0.88	0.18
2	2.8	3.44	0.64
3	3.1	3.84	0.74
4	2.5	3.12	0.62
5	1.7	2.58	0.88
6	0	0.42	0.42

The revenues are computed as follows

- Year 0: $16.8 \times 15\% = 2.52$
- Year 1: $1.1 \times (1 - 0.2) = 0.88$
- Year 2: $4.3 \times (1 - 0.2) = 3.44$
- Year 3: $4.8 \times (1 - 0.2) = 3.84$
- Year 4: $3.9 \times (1 - 0.2) = 3.12$
- Year 5: $2.7 \times (1 - 0.2) + 16.8 \times 5\% \times 0.5 = 2.58$
- Year 6: $16.8 \times 5\% \times 0.5 = 0.42$

b) The NPV is 223700. Since this is a positive NPV project, the company should undertake it.

c) The IRR is 15.33% (using a calculator or a spreadsheet). Notice that all the conditions for IRR to give you the correct answer are satisfied: You are considering only one project (i.e. you are not comparing mutually exclusive projects); All future cash flows are positive; The appropriate discount rate is constant (at 12%). Hence by setting the hurdle rate at 12%, the IRR rule should give you the correct recommendation. Since the IRR is greater than 12%, you should go ahead with the project.

The payback (period) is 4 years and the discounted payback (period) is 5 years.

d) The cash flows with the new payment schedule are

Year	Cost	Revenue	Net Cash Flow
0	3.6	2.52	-1.08
1	1.83	0.88	-0.95
2	2.8	3.44	0.64
3	3.1	3.84	0.74
4	2.5	3.12	0.62
5	1.7	2.58	0.88
6	0	0.42	0.42

The Year 0 cost is now 3.6M and the Year 1 cost is $0.7 + 1.13 = 1.83M$. The NPV is 214800, so the original payment plan should be taken. The IRR is 15.71. If we based our decision on the IRR, we would take the new payment plan. However, this is the wrong decision.