M.I.T. **15.415-Fall 1998**
Sloan School of Management **Professor Denis Gromb**

Solution to Problem Set 2

Question 1

1) c and d; **2)** e; **3)** e; **4)** c

Comments: A calculator was not necessary.

2) We know $D_A < 19$, $D_B = 19$, $D_C < 20$ and $D_D = 20$. Moreover, $D_A < D_C$. Indeed, the Yield Curve being flat, all bonds have the same YTM, $y = 8\%$. Moreover, Bonds A and C are premium bonds ($c > y$). Hence, maturity is increasing in maturity (which is almost always the case in general but strictly always the case for premium bonds). The portfolio'ssss duration is between those of bonds B and C. Hence, Bond D ranks first and Bond A ranks last and the portfolio ranks third.

3) The Yield Curve being flat, all bonds have the same YTM, $y = 8\%$. Bonds a and c trade below par and d trades at par while b and e trade above par. These are two premium bonds with same YTM, coupon rate and par value. Hence, bond e's price is lower because its maturity is shorter. Note: The actual ranking is: b, e, d, e, a.

Question 2

a) $DPP_B = 1$ because (amounts being expressed in thousands of dollars): $80 < \frac{100}{1.1}$

Clearly, $DPP_A > 1$ because: $520 > \frac{10}{1.1}$

Hence, the DPP rule would suggest project B

b) The NPV criterion suggests project A because

$$NPV_A = -520 + \frac{10}{1.1} + \frac{30}{1.1^2} + \frac{30}{1.1^3} + \frac{850}{1.1^4} = 117.0$$

$$NPV_B = -80 + \frac{100}{1.1} + \frac{10}{1.1^2} + \frac{1}{1.1^3} + \frac{1}{1.1^4} = 20.6$$

c) Using a calculator, you would find:

$$IRR_A = 16\% \text{ and } IRR_B = 35\%$$

Hence, with both thresholds, the IRR rule recommends project B.

However, you did not have to compute the projects' IRR.

$$NPV_A(20\%) = -520 + \frac{10}{1.2} + \frac{30}{1.2^2} + \frac{30}{1.2^3} + \frac{850}{1.2^4} = -63.56 < 0$$

$$NPV_B(20\%) = -80 + \frac{100}{1.2} + \frac{10}{1.2^2} + \frac{1}{1.2^3} + \frac{1}{1.2^4} = 11.34 > 0$$

Hence,

$$IRR_A < 20\% < IRR_B$$

Since $IRR_B > IRR_A$, project B will always be recommended over project A whatever the threshold. Moreover, $IRR_B > 20\%$ and $IRR_B > 10\%$ so that for both thresholds, project B is recommended over "doing nothing".

d) The PI criterion would suggest project B because $PI_B > PI_A$ and $PI_B > 1$. Indeed:

$$PI_A = \frac{\frac{10}{1.1} + \frac{30}{1.1^2} + \frac{30}{1.1^3} + \frac{850}{1.1^4}}{520} = 1.22$$

$$PI_A = \frac{\frac{100}{1.1} + \frac{10}{1.1^2} + \frac{1}{1.1^3} + \frac{1}{1.1^4}}{80} = 1.26$$

e) The NPV rule is better than all the others, one should undertake project A.

Note: In this example, the failure of the alternatives to NPV comes from their not accounting for the projects' difference in scale.

Question 3

A quick look should convince you that SI should definitely open the park. Consider the Argentina project for instance. Even in the worst case scenario, the park will generate a revenue net of costs of more than $\$8M - \$5 = \$3M$ annually in perpetuity starting January 2000. The PV of this revenue stream computed in January 1999 is thus $\frac{3M}{10\%} = 30M$ which is greater than the initial investment of $\$20M$.

Where to invest? The two locations generate the same revenues. Hence we only have to compare the PV of their costs. Argentina is a better location because:

$$PV_{Argentina} = 20 + \frac{5}{10\%} = \$70M$$

$$PV_{Brazil} = 27 + \frac{4.5}{10\%} = \$72M$$

Note: Again, a quick look might have been enough. The difference in operating costs is a $\$0.5M$ perpetuity, hence worth $\$5M$ which does not compensate for the $\$7M$ increase in initial investment.

Question 4

Today is January 1, 1998. The yield curve is flat at 7%. As the manager of a pension fund, you are forecasting that you will need to pay $\$2M$ to retirees every year on January 1 in perpetuity. The next payment is due one year from today.

a) We need to form a portfolio combining both bonds with the same PV and duration as the liability.

$$PV(\text{liability}) = \frac{2}{7\%} = \$28.57M$$

$$D(\text{liability}) = \frac{1+7\%}{7\%} = 15.29 \text{ yrs}$$

We thus need to invest $\$28.57M$ in a portfolio of both bonds. Let w be the fraction of that amount that we invest in the 20-year zero. Since the Yield Curve is flat, both zeros have the same YTM and so the portfolio's duration is

$$D = 20w + 5(1-w)$$

Hence

$$w = \frac{15.29-5}{15} = 0.686$$

We should thus invest $0.686 \times 28.57 = \$19.60M$ in the 20-year STRIP and $\$8.97M$ in the 5-year STRIP.

b) One basis point = 0.01%. Thus, the new interest rate = 6.99%.

$$\begin{aligned} PV(\text{liability}) &= \frac{2}{6.99\%} = \$28.61M \\ PV(\text{port. of STRIPS}) &= 8.97 \times \frac{(1+0.07)^5}{(1+0.0699)^5} + 19.60 \times \frac{(1+0.07)^{20}}{(1+0.0699)^{20}} = \$28.61M \end{aligned}$$

Both asset and liability increase by \$0.04m. This is to be expected since the portfolio was formed to immunize the liability against small intertst rate change. Another way to calculate the changes in value is to use modified duration. Both portfolio of STRIPS and liability have $MD = \frac{D}{1+y} = 14.3$. One basis point decrease in yield curve corresponds to 14.3 basis point increase in PV, or in dollar terms 14.3*$10^{-4} * 28.57$ = 0.04m.

To rebalance the portfolio, you need to go through the same calculation as in part a) for r=6.99%. With PV(liability) = 28.61m and D(liability) = 15.306, the new portfolio weight is 31.3% in 5-year and 68.7% in 20-year. So, you will need to invest \$8.956m in the 5-year STRIP and \$19.657m in the 20-year STRIP.

Question 5

a) The prices B_1 and B_2 of the 1-year and 2-year zeros with face value $\$1$ solve

$$\begin{cases} 60B_1 + 1060B_2 = B_A \\ 120B_1 + 1120B_2 = B_B \end{cases} \text{ which gives } \begin{cases} B_2 = \frac{2B_A - B_B}{1000} = 0.87344 \\ B_1 = \frac{B_B - B_A}{60} - B_2 = 0.93456 \end{cases}$$

We thus get spot rates and the forward rate:

$$\begin{aligned} r_1 &= \frac{1}{B_1} - 1 = 7\% \\ r_2 &= \frac{1}{(B_2)^{1/2}} - 1 = 7\% \\ f_2 &= \frac{(1+r_2)^2}{1+r_1} - 1 = 7\% \end{aligned}$$

b) The Yield Curve being flat (at least up to bond C's maturity), its YTM is 7%. Hence, bond C trades at par, i.e. its price is $\$1,000$.