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Solution to Problem Set 4

Question 1

a) We have:

$$P = \frac{E_0[D_1]}{r - g} = \frac{pE_0[EPS_1]}{r - g}$$

hence,

$$\frac{P}{E_0[EPS_1]} = \frac{p}{r - g} = \frac{45\%}{15\% - 10\%} = 9$$

b) The dividend growth rate is given by

$$g = ROE \times (1 - p)$$

Hence,

$$\begin{aligned} P &= \frac{1}{(1+r)^2} \times \frac{E_0[D_3]}{r - g} = \frac{1}{(1+r)^2} \times \frac{E_0[D_3]}{r - ROE \times (1 - p)} \\ &= \frac{1}{1.12^2} \times \frac{2}{12\% - 15\% \times 60\%} = \$53.15 \end{aligned}$$

Question 2

a) $P_0 = \$10, E_1 = \$2, b = .5, ROE = .2$

$$r = D_1/P_0 + g$$

$$D_1 = .5 \times \$2 = \$1$$

$$g = b \times ROE = .5 \times .2 = .1$$

Therefore, $r = \$1/\$10 + .1 = .1 + .1 = 20\%$

b) Since $r = ROE$, the NPV of future investment opportunities is zero:

$$PVGO = P_0 - \frac{E_1}{r} = 10 - 10 = 0$$

c) Since $r = ROE$, the stock price would be unaffected by cutting the dividend and investing the additional earnings.

d) Even if the dividend were eliminated altogether, this should have no impact on the stock's price since the NPV of the firm's investments is zero.

Question 3

a) Given 20% plowing back policy, the dividend grows at 7%. We can use the Gordon Model to solve the market capitalization rate.

$$\begin{aligned} P_0 &= \frac{D_1}{r - g} \\ 100 &= \frac{30 \cdot (1 - .2)}{r - .07} \\ r &= 31\% \end{aligned}$$

ROE is found according to the relation between the growth rate and the plowback ratio

$$\begin{aligned} g &= ROE \cdot b \\ .07 &= ROE \cdot .2 \\ ROE &= 35\% \end{aligned}$$

b) We need to use the Gordon Model. We start by first calculating the new growth rate with the new plowback ratio:

$$g = ROE \cdot b = .35 \cdot .8 = 28\%$$

We now use the Gordon Model to calculate the new price of the stock

$$P'_0 = \frac{30(1 - .8)}{.31 - .28} = \$200$$

Question 4

a) We can use Gordon Model to calculate the implied growth rate:

$$94.50 = \frac{2.40}{.12 - g} \text{ so that } g = 9.46\%$$

To calculate ROE, we can use the relationship between the growth rate, the ROE and the plowback ratio:

$$g = ROE \cdot b.$$

b) We start by first calculating the plowback ratio b . Since we have the dividends to be paid out at the end of the year, \$2.40, what is missing is earnings:

$$\frac{P_0}{E} = 8 \text{ so that } E = \frac{94.50}{8} = 11.81$$

Then we can calculate the plowback ratio:

$$b = 1 - \frac{D}{E} = 1 - \frac{2.40}{11.81} = 79.68\%$$

Now we can calculate the ROE as

$$\begin{aligned} g &= ROE \cdot b \\ .0946 &= ROE \cdot .7968 \\ ROE &= 11.87\% \end{aligned}$$

Question 5

a) The dividend to be paid at the end of year 3 is the first installment of a dividend stream that will increase indefinitely at the constant growth rate of 5%. Therefore, we may use the constant growth model as of the end of year 2, and add the present value of the first two dividends plus the present value of the sales price of the stock at the end of year 2.

The expected price 2 years from now is:

$$P_2 = D_3 / (r - g) = \frac{1 \times (1 + 5\%)^3}{r - g} = 1.953 / (.20 - .05) = \$13.02$$

The PV of this expected price is $13.02 / 1.20^2 = \$9.04$

The PV of expected dividends in years 1 and 2 is:

$$\frac{D_1}{1.20} + \frac{D_2}{1.20^2} = \$2.13$$

Thus the current price should be $\$9.04 + \$2.13 = \$11.17$.

b) Expected dividend yield = $D_1 / P_0 = 1.25 / 11.17 = 11.2\%$

c) The expected price one year from now is the PV of P_2 and D_2 :

$$P_1 = (D_2 + P_2) / 1.20 = (1.5625 + 13.02) / 1.20 = \$12.15$$

The implied capital gain is

$$(P_1 - P_0) / P_0 = (12.15 - 11.17) / 11.17 = .088 \text{ or } 8.8\%$$

The implied capital gains rate and the expected dividend yield sum to the market capitalization rate. This is consistent with the DDM.