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Solution to Problem Set 5

Question 1

1) d **2)** d: A contract is an option on 100 shares.

Question 2

a. Top left diagram: Purchase a call with a given exercise price and sell a call with a higher exercise price; borrow the difference necessary. (This is known as a "Bull Spread.")

b. Top right diagram: Sell a put and sell a call with the same exercise price. (This is known as a "Short Straddle.")

c. Bottom right diagram: Buy one call with a given exercise price, sell two calls with a higher exercise price, and buy one call with a higher still exercise price. (This is known as a "Butterfly Spread." It can also be achieved using put options.)

d. Bottom left diagram: Buy >1 call options at the same strike.

Question 3

We have $u = 2$, $d = \frac{1}{2}$ and $R = 1.1$ so that the risk neutral probability that the stock price will go up is

$$q = \frac{R-d}{u-d} = 0.4$$

a. If the stock price rises, the call is worth $C_u = \$160$; if it falls, it is worth $C_d = \$10$. Thus, if not exercised today, the call is worth:

$$C = \frac{1}{R}\left[qC_u + (1-q)C_d\right] = \frac{0.4 \times 160 + 0.6 \times 10}{1.10} = \$63.64$$

If exercised now, the call is worth $\$60$. Thus, the call is worth more if it is not exercised today.

b. If the stock price rises, the put is worth $P_u = \$50$; if it falls, it is worth $P_d = \$200$. Thus, if not exercised today, the put is worth:

$$P = \frac{1}{R}\left[qP_u + (1-q)P_d\right] = \frac{(.4)\,50 + (.6)\,200}{1.10} = \$127.27$$

If exercised now, the put is worth $\$150$. Thus, the put is worth more if it is exercised today.

Question 4

At the end of two months, the derivative is worth $X_d = 529$ if the stock price is 23, or $X_u = 729$ if the stock price 27. We have $u = 1.08$, $d = 0.92$. Moreover, with continuous compounding, the spot rate for maturity T years is $r = e^{10\% \times T} - 1$. Hence, here, we have $R = 1 + r = e^{10\% \times \frac{1}{6}}$ so that the risk neutral probability of "up" is:

$$q = \frac{e^{0.10 \times \frac{1}{6}} - 0.92}{1.08 - 0.92} = 0.6050$$

and so the option value is:

$$X = \frac{1}{R}\left[qX_u + (1-q)X_d\right] = e^{-0.10 \times \frac{1}{6}}\left[0.6050 \times 729 + 0.3950 \times 529\right] = \$639.3$$

Question 5

a) The debt is risk-free as the asset value is greater than the debt's face value in all states of the world. Hence, the bond's price is $p = \frac{1{,}000}{1.07^2} = \$\mathbf{873.44}$

b) In the three final states, debtholders receive $D_{uu} = \$100M$, $D_{ud} = \$84M$ and $D_{dd} = \$49M$.

Hence firm X's debt value at $t = 0$ is $D = \frac{1}{(1+r)^2}\left(p^2 D_{uu} + 2p(1-p)D_{ud} + (1-p)^2 D_{dd}\right)$ with $p = \frac{1+r-d}{u-d} = \frac{1.07-.7}{1.2-.7} = .74$ Hence, $D = \$78.96M$ so that your bond's price is $p = \frac{1{,}000}{100{,}000{,}000} \times D = \$\mathbf{789.55}$. The price is less than in a) because it accounts for the risk of default.

c) Now we have to check for each possible state at $t = 1$ whether equityholders want to call the bonds at $t = 1$. When deciding whether to call the bonds or not, equityholders will compare their value, D_u and D_d, to the cost of calling the bonds, $\frac{100{,}000{,}000}{1{,}000} \times c = \$85M$. Clearly, $D_d = \$70M < \$85M$ so the bond will not be called in the low state at $t = 1$. We have $D_u = \frac{1}{1+r}\left(pD_{uu} + (1-p)D_{ud}\right) = \$89.57M$ Hence, the bond will be called. Firm X's debt value is thus $D = \frac{1}{1+r}\left(p \times 85 + (1-p) \times 70\right) = \$75.79M$ and the price of one bond is $p = \frac{1{,}000}{100{,}000{,}000} \times D = \$\mathbf{757.94}$. The price is less than in b) as it accounts for the possibility that their bond will be called.

d) A-bonds are risk free. Hence $p_A = \$\mathbf{873.44}$. B-bonds receive a payment only after A-bondholders have been fully repaid. Hence, B-bondholders receive $B_{uu} = 70$, $B_{ud} = 84 - 30 = 54$ and $B_{dd} = 49 - 30 = 19$. The value of B-bonds at $t = 0$ is thus

$B = \frac{1}{(1+r)^2}\left(p^2 B_{uu} + 2p(1-p)B_{ud} + (1-p)^2 B_{dd}\right) = \$52.75M$ and the price of one B-bond is $\mathbf{p}_B = \frac{1{,}000}{70{,}000{,}000} \times 52.75M = \$\mathbf{753.60}$. The price is less than in b) as it accounts for the possibility that A-bondholders have priority.

Other method: The total value of firm X's debt (A+B bonds) is the same as in b).

Hence, $30\% p_A + 70\% p_B = \$789.55$. Since $p_A = \$\mathbf{873.44}$, we have $p_B = \$\mathbf{753.60}$.