M.I.T.
Sloan School of Management

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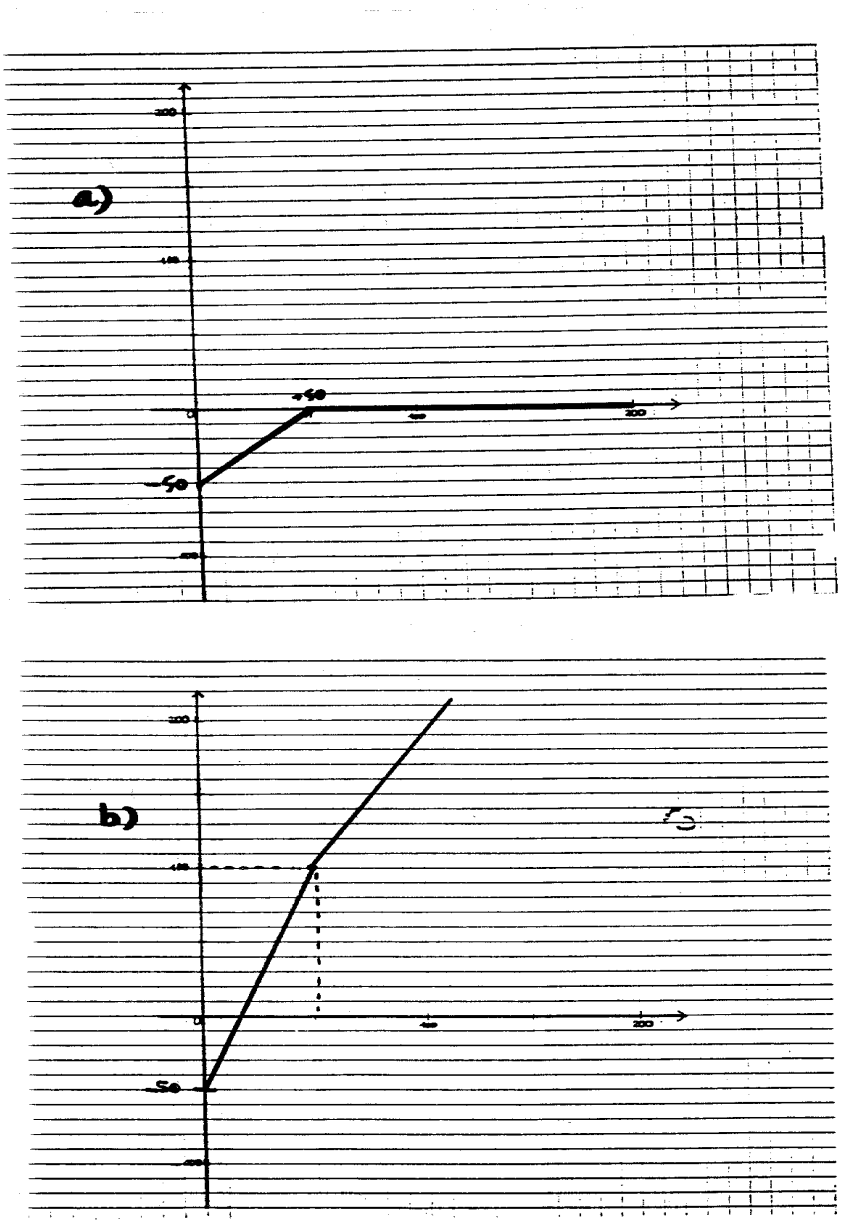
Professor Denis Gromb

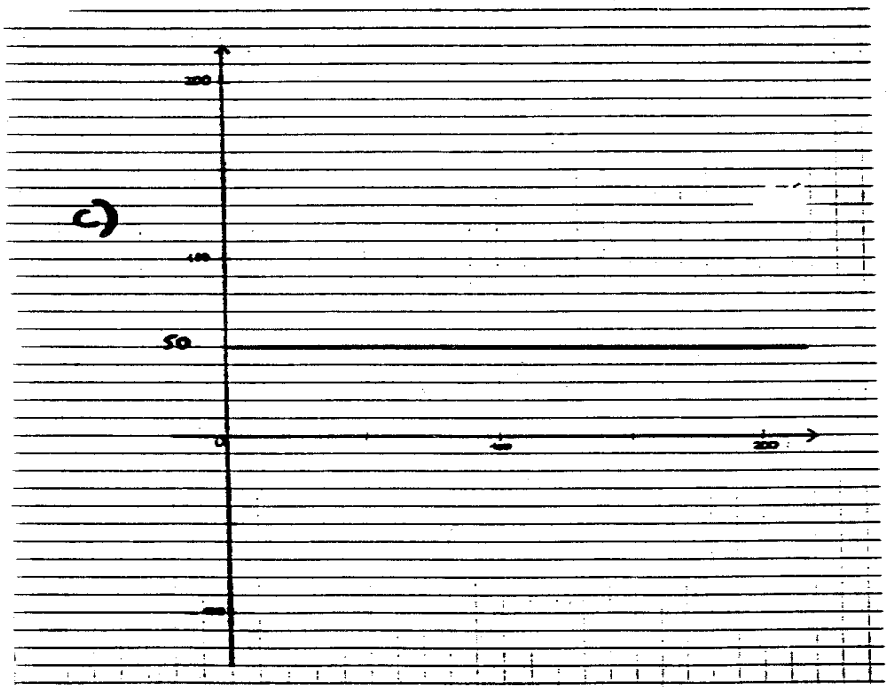
Solution to Problem Set 6

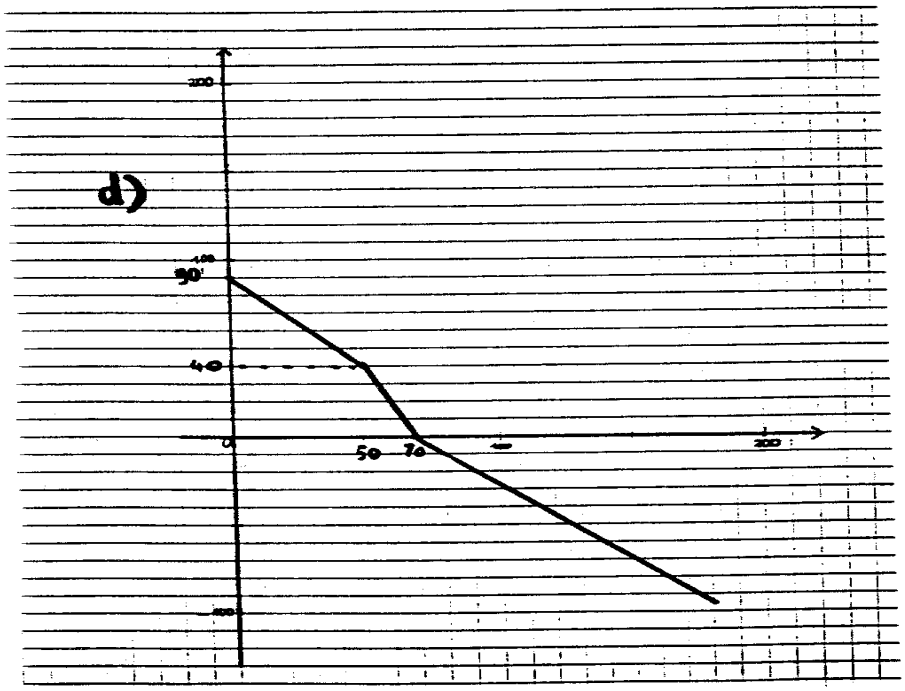
Question 1

1) c and d; **2)** a; **3)** c; **4)** d

Question 2




c)
50
0


d)
30
40
0
50
70

Question 3

a) We have $C_{uu} = \$70$, $C_{ud} = \$5$ and $C_{dd} = 0$. Hence

$$\begin{aligned} C &= \frac{1}{R^2} \times \left[q^2 \times C_{uu} + 2q(1-q) \times C_{ud} + (1-q)^2 C_{dd} \right] \\ &= \frac{1}{1.1^2} \times \left[0.6^2 \times 70 + 2 \times 0.6 \times 0.4 \times 5 \right] \\ &= \$22.81 \end{aligned}$$

b) By put call parity

$$\begin{aligned} P &= C + \frac{K}{R^2} - S \\ &= 22.81 + \frac{99}{1.1^2} - 100 \\ &= \$4.63 \end{aligned}$$

c) $p_{uu} = p_{ud} = 0$ and $p_{dd} = \$35$

If the stock price moves up to $S_u = \$130$, then exercising the put is not valuable. Moreover, the put will not be exercised in the future either ($p_{uu} = p_{ud} = 0$). Hence, $p_u = 0$.

If the stock price moves down to $S_d = \$80$ then exercising would bring a $\$19$ payoff. If not exercised, the put is worth

$$\begin{aligned} \frac{1}{R} \times (1-q) \times p_{dd} &= \frac{1}{1.1} \times \left(\frac{1.3 - 1.1}{1.3 - 0.8} \right) \times 35 \\ &= \$12.73 \end{aligned}$$

Hence, it is optimal to exercise the put and so $p_d = \$19$.

Finally, the put should not be exercised currently because $\$100 > \$99$. Hence, the value of the put today is

$$\begin{aligned} p &= \frac{1}{R} \times (1-q) \times p_d \\ &= \$6.91 \end{aligned}$$

d) The value of an American call on a non-dividend paying stock is equal to that of an otherwise identical European call. Hence

$$c = C = \$22.81$$

Thus the portfolio price is

$$\begin{aligned} S - c &= 100 - 22.81 \\ &= \$77.19 \end{aligned}$$

Question 4

a) We have

$$\begin{aligned}\beta_1 &= \frac{\sigma_{1m}}{\sigma_m^2} = \frac{0.064}{0.04} = 1.6\\ \beta_2 &= \frac{\sigma_{2m}}{\sigma_m^2} = \frac{0.032}{0.04} = 0.8\end{aligned}$$

b) We can now compute the beta of the portfolio:

$$\begin{aligned}\beta_p &= \frac{50,000}{60,000} \times 1.6 + \frac{10,000}{60,000} \times 0.8\\ &= 1.47\end{aligned}$$

The portfolio's systematic risk is thus

$$\beta_p^2 \times \sigma_m^2 = 1.47 \times 0.04 = 0.086$$

Question 5

a) The investment returns -20% if the economy is the good state, and $+10\%$ in the bad state. Hence an expected return of -5% and a standard deviation of 15%.

b)

$$\begin{aligned}\text{Cov}[r, r_m] &= \left[\frac{1}{2} \times (-20\%) \times (18\%) + \frac{1}{2} \times (10\%) \times (2\%)\right] - (-5\%) \times 10\%\\ &= -0.012\end{aligned}$$

$$\beta = \frac{-0.012}{0.08^2} = -1.875$$

Note that you cannot compute the β by solving the CAPM equation for β.

$$E(\tilde{r}_i) = r_f + \beta\left[E(\tilde{r}_m - r_f)\right]$$

The CAPM is the relationship that must hold in *equilibrium* if all assets are fairly priced. We have given you a situation where you are the sole participant in the project. It is quite possible that the project's return is above or below the Security market line. This is why there are positive NPV projects!

c) Invest some money because the fair return in CAPM would be

$$2.5\% - 1.875 \times [10\% - 2.5\%] = -11.56\%$$

But you should not invest all your money because you would be earning a negative return on your entire portfolio! Just because a project or security has a negative β, does not guarantee that you want to hold it. Having a negative β is a necessary condition for a security to have an equilibrium return less than the risk free rate, but it is by no means sufficient. As an example, think about

homeowners insurance. If you have a large proportion of your wealth invested in a home, then an insurance policy will have a negative β relative to your overall wealth. But there are still insurance policies that are over-priced and which you will not buy.

d) You should undertake the project at full scale. The shareholders can always diversify their own portfolios. Many people argued that since it has a negative expected return, it must have a negative NPV and therefore the project should be avoided. This is false. The project can have a positive NPV while still having a negative expected return if the β is low enough, i.e. if the equilibrium required return is more negative than the expected return.