16.31 Homework 2

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Issued: September 22, 2006

Due: September 29, 2006

1. (Dominant Pole Locations) [FPE 3.36 (a),(c),(d), page 161]. Consider the second order system

$$H(s) = \frac{\omega_n^2}{(s/p + 1)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

The unit step response is

$$y(t) = 1 + Ae^{-pt} + Be^{-\sigma t}\sin(\omega_d t - \theta)$$

where

$$A = \frac{-\omega_n^2}{\omega_n^2 - 2\zeta\omega_n p + p^2} \qquad B = \frac{p}{\sqrt{(p^2 - 2\zeta\omega_n p + \omega_n^2)(1 - \zeta^2)}}$$

$$\theta = \tan^{-1}\frac{\sqrt{1 - \zeta^2}}{-\zeta} + \tan^{-1}\frac{\sqrt{1 - \zeta^2}}{p - \zeta\omega_n}$$

   (a) State the steps that you would follow to show that this is the step response. Which inverse transforms would you use from the Tables?

   (b) Which term dominates $y(t)$ as p gets large?

   (c) Which term dominates as p gets small? (small with respect to what?)

   (d) Using the explicit expression for $y(t)$ above and the *step* command in Matlab (assume $\omega_n = 1$ and $\zeta = 0.7$), plot the step response for various values of p ranging from very small to very large. At what point does the extra pole cease to have an impact on the system response?

2. (Basic Root Locus Plotting) [FPE problem 5.4 (a), (e), and problem 5.7 (e)] Sketch the root locus for the following systems. As we did in class, concentrate on the real-axis, and the asymptotes/centroids.

   (a) $G_c G(s) = \dfrac{K}{s(s^2 + 2s + 10)}$

   (b) $G_c G(s) = \dfrac{K(s + 2)}{s^4}$

   (c) $G_c G(s) = \dfrac{K(s + 1)(s - 0.2)}{s(s + 1)(s + 3)(s^2 + 5)}$

   (d) Once you have completed the three sketches, verify the results using Matlab. How closely do your sketches resemble the actual plots?

3. The attitude-control system of a space booster is shown in Figure 2. The attitude angle θ is controlled by commanding the engine angle δ, which is then the angle of the applied thrust, F_T. The vehicle velocity is denoted by v. These control systems are sometimes open-loop unstable, which occurs if the center of aerodynamic pressure is forward of the booster center of gravity. For example, the rigid-body transfer function of the Saturn V booster was

$$G_p(s) = \frac{0.9407}{s^2 - 0.0297}$$

This transfer function does not include vehicle bending dynamics, liquid fuel slosh dynamics, and the dynamics of the hydraulic motor that positioned the engine. These dynamics added 25 orders to the transfer function! The rigid-body vehicle was stabilized by the addition of rate feedback, as shown in the Figure 2b (Rate feedback, in addition to other types of compensation, was used on the actual vehicle.)
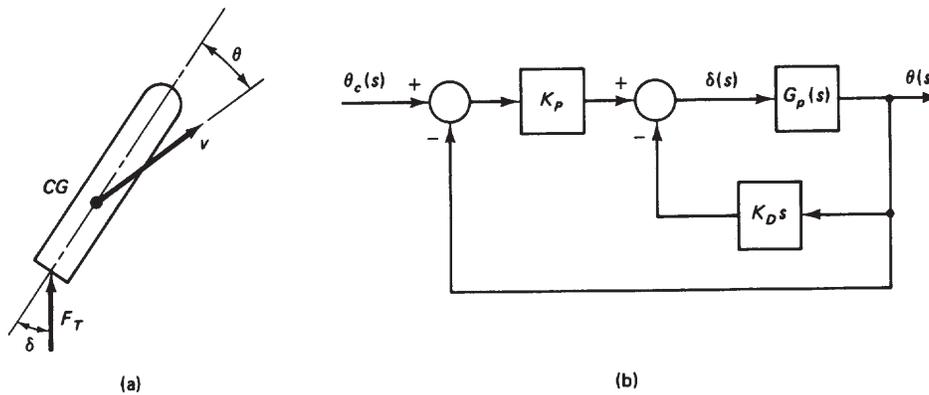

Figure 1: Booster control system

(a) With $K_D = 0$ (the rate feedback removed), plot the root locus and state the different types of (unstable) responses possible (relate the response with the possible pole locations)

(b) Design the compensator shown (which is PD) to place a closed-loop pole at $s = -0.25 + j0.25$. Note that the time constant of the pole is 4 sec, which is not unreasonable for a large space booster.

(c) Plot the root locus of the compensated system, with K_p variable and K_D set to the value found in (b).

(d) Use Matlab to compute the closed-loop response to an impulse for θ_c.