Massachusetts Institute of Technology

16.410-13 Principles of Autonomy and Decision Making

Due: Monday, 11/22/04

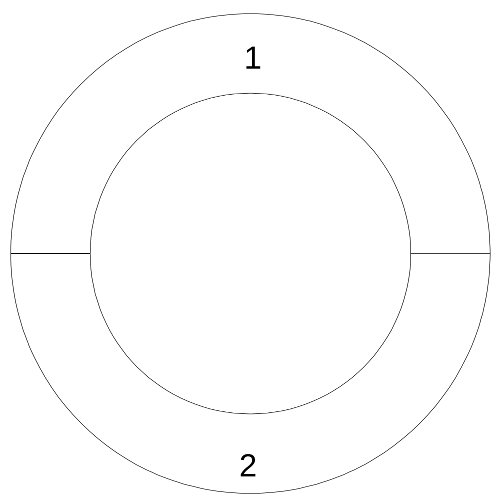
Paper solutions are due no later than 5pm on Monday, 11/22/04. Please give solutions to the course secretary, Brian O' Conaill, at his desk outside of 33-330.

Objectives

The purpose of this problem set is to develop a grounded understanding of Solutions to Markov Decision Processes and Mixed Integer Programs.

Problem 1 – Markov Decision Processes

Consider the circular racetrack shown below.



It has two sections: section 1 is dry, and section 2 is slippery. A race car on this track will crash if it is on section 2 and is going too fast.

Suppose we want to give the race car driver advice about how fast to go in each section. We will model this system using a simple deterministic MDP, where the **state** is simply the section number that the race car is in, and the **action** is one of three speeds: 0, 20, and 40 mph.

The transition function is as follows:

s	a	s+1
1	0	1
1	20	2
1	40	2
2	0	2
2	20	1
2	40	2

The reward function is

s	a	R
1	0	0
1	20	15
1	40	45
2	0	0
2	20	10
2	40	0

Note that if the driver goes 40 mph in section 2, he will stay in section 2, and his reward will be 0 (he will crash).

Assume that the discount rate γ is 0.8

Part A. What is the value function and optimal policy for a 2-step horizon?

Part B. How do these change if the transition function is altered to the table below? Give the value function and optimal policy, and comment on the reason for the changes from Part A.

s	a	s+1
1	0	1
1	20	2
1	40	2
2	0	2
2	20	2
2	40	1

Problem 2 Integer Programming and Branch and Bound

Part A Formulation Using Integer Programming

The Transportation Security Administration needs to maintain an all-night security gate at a busy international airport. All its employees work in **eight hour shifts**. The number of employees needed to run the gate varies according to time of day, because fewer or more passengers travel during those times. From 12pm-4am, 25 people are needed; from 4am-8am, 45 people; from 8am-12am, 85 people, from 12am-4pm, 120; from 4pm-8pm, 55 people; from 8pm-12pm, 33 people.

Part A.1

Write an integer program whose solution gives the minimum-employee solution to the TSA's staffing problem. Include a description in words of what your variables mean. Explain any key modeling decisions in your encoding.

Part A.2

Appearances are important, and the TSA is getting complaints because of the surplus employees that are hanging around Dunkin' Donuts during their shifts. Modify your integer program to minimize the maximum number of unnecessary employees in any one shift. Give your integer program and an explanation for any modifications that you made from part A.1.

Part B Solving Integer Programs using Branch and Bound

Solve the following mixed integer linear program using Branch and Bound.

Minimize $z = 8x_1 + 3x_2 + 18b_1 + 20b_2$

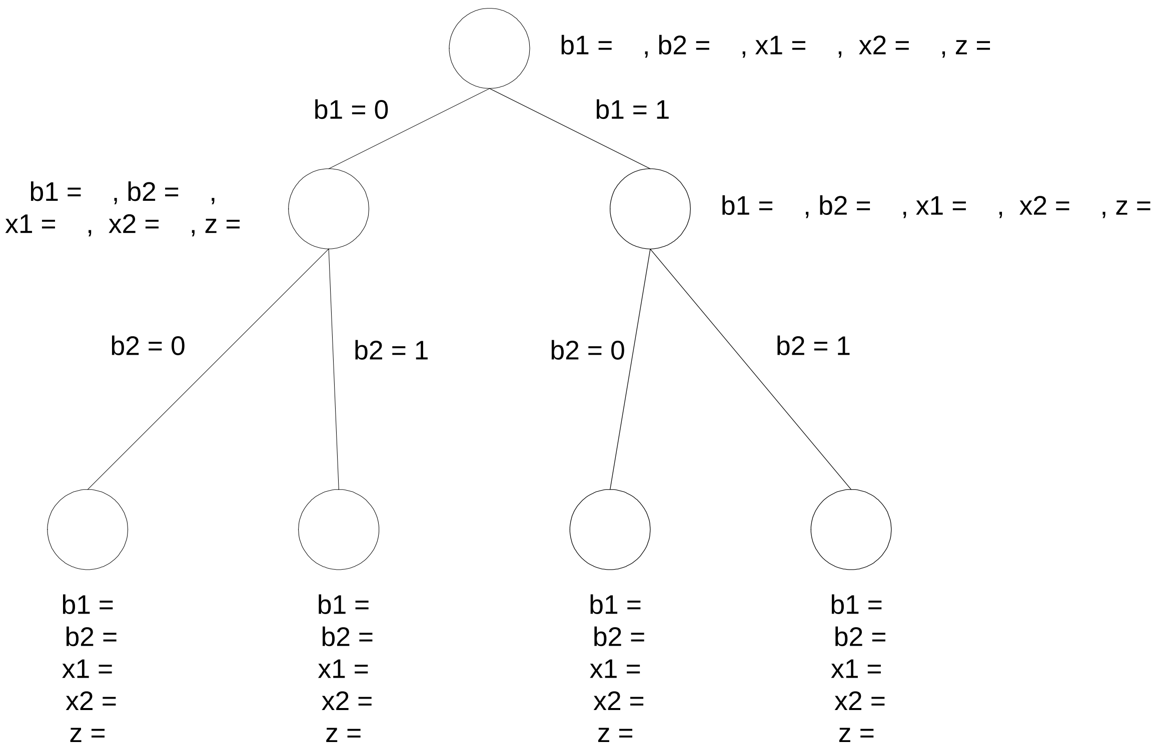
subject to

$$x_1 + x_2 \geq 8$$
$$x_1 \leq 15b_1$$
$$x_2 \leq 10b_2$$

$$b_1 + b_2 \leq 1$$
b_1, b_2 are binary (i.e., $b_i \in \{1,0\}$).

Part B.1 Branch and Bound Search Tree

Construct a branch and bound search tree that augments the template tree given below. Branch on the binary variables in the **following order**: b_1, b_2. Evaluate the **0** branch **before** the **1** branch. Cross off each node that is infeasible or fathomed. For feasible, non-fathomed nodes, give the relaxed solution and the value of Z. For fathomed nodes, give the solution and value of Z.



Part B.2 Minimum Feasible Solution

List your solution, which is the minimum feasible state:

Z = ____, x_1 = ____, x_2 = ____, x_3 = ____, b_1 = ____, b_2 = ____, b_3 = ____