

16.410/413 Principles of Autonomy and Decision Making

Problem Set #4, Linear Programming

Due by 5pm, Monday October 11, 2004

Turn in during class or slip under my door (33-315) before 5pm.

Objective

To exercise your ability to formulate problems as linear programs over integer and real-valued decision variables. To encode and solve these problems using the AMPL/CPLEX system.

Problem 1: An Astronaut Task Assignment

Write the mathematical program for each part of the following problem. Then write an AMPL program to solve it. Submit your AMPL program, and the output. Give the assignment of which astronauts will be assigned to what task?

Part 1. NASA has four tasks to be completed on the space station. There are four astronauts available to work on the tasks, but NASA's goal is to finish the entire list in the minimum time (because space walks are expensive). Because of radiation exposure concerns, each astronaut can complete only one task. Assume the tasks will be completed one at a time, that is, the objective is to minimize the total time spent. Write an AMPL program that minimizes this objective. Submit your AMPL program, the AMPL run commands you used, and the list of astronauts assigned to each task.

	John	Edward	Christa	Sally
Adjust mirrors	47.7	42.9	43.8	47.0
Sample dust	53.4	43.1	52.2	44.7
Repair antenna	43.3	38.5	48.9	40.4
Polish fenders	39.2	36.4	39.6	38.5

Part 2. Now write an AMPL program assuming that all will spacewalk at the same time, so that the objective is to minimize the maximum of the task times. Again, submit your AMPL program, the AMPL run commands you used, and the list of astronauts assigned to each task.

Hint: You don't need any additional operators, like a `max` operator within AMPL. But, you might consider an additional variable that represents your objective, and an additional constraint that ties your current solution to the objective.

Bonus points Give brief arguments: why is the linear program guaranteed to give a feasible assignment as the optimum? Why is solving as a linear program preferable to solving as an integer program?

Problem 2: The Simplex Algorithm

(from George Dantzig, Linear Programming and Extensions, Princeton University Press).
Solve using the simplex method the following linear program:

$$\begin{array}{llll} \text{Maximize:} & Z & = & 4y_1 + 3y_2 \\ \text{Subject to:} & y_1 + 2y_2 & \leq & 2 \\ & -2y_1 + y_2 & \leq & 6 \\ & 9y_1 + 3y_2 & \leq & 1 \\ & y_1 & \geq & 0 \\ & y_2 & \geq & 0 \end{array}$$

Interpret each pivot step of the simplex algorithm geometrically in the plane of y_1 and y_2 .

Problem 3: A Mars Rover obstacle avoidance problem

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In this section you will design a receding horizon Rover planner.

Design and a trajectory planner for the Mars Rover, travelling near a line at $x=0$. The rover chooses a new setting for the heading angle relative to the line $x=0$ every 10 seconds. However, the new heading angle must be within 10 degrees of the previous heading angle or the Rover will roll. Use the small angle approximation to model this problem linearly. There are enormous dirt clods in the Rover's path, requiring that the car be displaced from the nominal path $x=0$ at 200 seconds by at least 10 feet, and at 500 seconds by at least 5 feet. The rover's speed is 0.5 feet/second. Use linear programming to design an autopilot for 700 seconds of roving, minimizing the absolute value of the deviation from the lane $x=0$ while avoiding the obstacles, assuming you know about all of the obstacles in advance.

Submit your design either as a standard form LP or in AMPL code, and an explanation for your design.
You will receive 0 for this question if you do not provide an explanation.

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