Notes on Path Planning for 16.410 and 16.413

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?						
Time						
Space						
Optimal?						

Things you should know:

Representation issues:

- Configuration space
- Topological vs. Metric
- Discrete vs. continuous spaces
- Convexity
- Skeletonization

Graph-based representations:

- Visibility graphs
- Voronoi diagrams
- Probabilistic roadmaps
- Rapidly-exploring randomized trees
- Distinctiveness surfaces

Metric representations:

- Potential fields
- Numerical potential fields
- Value functions

Algorithms:

- Breadth-first vs. Depth-first
- Iterative deepening
- Uniform cost
- Dijkstra's algorithm
- Greedy best-first
- A* Search
- Admissible heuristics and how to find them
- Complexities in time and space
- Bi-directional search
- Relationship between dynamic programming and search

Path Planning

(The references given may not be canonical, but should be useful.)

Configuration Space

T. Lozano-Perez. Spatial Planning: A Configuration Space Approach, IEEE Transactions on Computers, Vol C-32, No. 2, February 1983, pp.108–120. Also, IEEE Tutorial on Robotics, IEEE Computer Society, 1986, pp.26–38. Also, AI Memo 605, December 1980.

- Choose a point on the robot
- C-Space is the set of places that the point can be. Is often viewed as "growing" obstacles

Visibility Graph

J.C. Latombe. Robot Motion Planning. Kluwer, 1991.

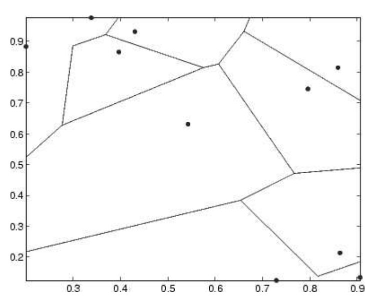
1. Insert all obstacle vertices into VG, plus start and goal positions
2. $\forall$ pairs of nodes $u, v \in VG$
   (a) If $\vec{uv}$ is an obstacle edge, then insert edge $\vec{uv}$ into VG else
   for every obstacle edge e
   - if $\vec{uv}$ intersects e return to step 2
   insert edge $\vec{uv}$ into VG
3. Use Dijkstra's Algorithm or A* to search for a path from start to goal.

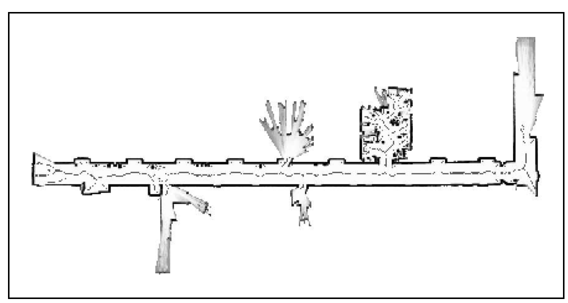
Killer problem: ?

Voronoi Graphs

No single best canonical reference.
H. Choset and J. Burdick. "Sensor-Based Exploration: The Hierarchical Generalized Voronoi Graph". The International Journal of Robotics Research 19, no. 2 (2000): 96-125
The Voronoi Graph is the dual of the Delaunay Triangulation.
A fast way to construct Voronoi graphs in a polygonal world:

1. Construct a polyhedral world by turning each polygon co-ordinate (x, y) into a polyhedral co-ordinate $(x, y, \sqrt{x^2 + y^2})$ ($O(n)$)
2. Compute the convex hull of this world ($\Omega(n^2)$) to get Delaunay triangulation (dual of Voronoi)
3. Project hull to 2-space, and recover dual $O(e^2)$ for e edges





Killer problem: ?

Potential Field Methods

O. Khatib ”Real-Time Obstacle Avoidance for Manipulators and Mobile Robots” The International Journal of Robotics Research, 5(1), 1986.
Define

$$\begin{aligned} U(\vec{x}) &= U_{att}(\vec{x}) + U_{rep}(\vec{x}) && (1)\\ F(\vec{x}) &= -\nabla U_{att}(\vec{x}) - \nabla U_{rep}(\vec{x}) && (2) \end{aligned}$$

Suppose

$$\begin{aligned} U_{att}(\mathbf{x}) &= ||\mathbf{x} - \mathbf{x}_{goal}|| && (3)\\ \Rightarrow \nabla U_{att}(\mathbf{x}) &= \nabla(\sum_i (x_i - x_{goal,i})^2)^{1/2} && (4)\\ &= \frac{1}{2}(\sum_i (x_i - x_{goal,i})^2)^{-1/2}\nabla(\sum_i (x_i - x_{goal,i})^2) && (5)\\ &= \frac{2(\mathbf{x} - \mathbf{x}_{goal})}{2(\sum_i (x_i - x_{goal,i})^2)^{1/2}} && (6)\\ &= \frac{(\mathbf{x} - \mathbf{x}_{goal})}{||\mathbf{x} - \mathbf{x}_{goal}||} && (7)\\ & && (8) \end{aligned}$$

F_{att} is unit vector towards goal, and singular (and unstable) at the goal.
Suppose

$$\begin{aligned} U_{att}(\mathbf{x}) &= \frac{1}{2}||\mathbf{x} - \mathbf{x}_{goal}||^2 && (9)\\ \Rightarrow \nabla U_{att}(\mathbf{x}) &= \frac{1}{2}2||\mathbf{x} - \mathbf{x}_{goal}||\nabla(||\mathbf{x} - \mathbf{x}_{goal}||) && (10)\\ &= ||\mathbf{x} - \mathbf{x}_{goal}||\frac{(\mathbf{x} - \mathbf{x}_{goal})}{||\mathbf{x} - \mathbf{x}_{goal}||} && (11)\\ &= \mathbf{x} - \mathbf{x}_{goal} && (12) \end{aligned}$$

F_{att} is converges linearly to 0 with distance to goal: good for stability, but grows without bound far from goal.
Hybrid:

$$U_{att}(\mathbf{x}) = \begin{cases} \alpha\frac{1}{2}||\mathbf{x} - \mathbf{x}_{goal}||^2 & \text{if } ||\mathbf{x} - {}_{\mathbf{x}goal}|| < d \\ \beta||\mathbf{x} - \mathbf{x}_{goal}|| & \text{if } ||\mathbf{x} - {}_{\mathbf{x}goal}|| \geq d \end{cases} \quad (13)$$

Suppose

$$\begin{aligned} U_{rep}(\mathbf{x}) &= \frac{1}{||\mathbf{x} - \mathbf{x}_c||} && (14)\\ \Rightarrow \nabla U_{rep}(\mathbf{x}) &= \frac{1}{||\mathbf{x} - \mathbf{x}_c||}\frac{(\mathbf{x} - \mathbf{x}_c)}{||\mathbf{x} - \mathbf{x}_c||} && (15) \end{aligned}$$

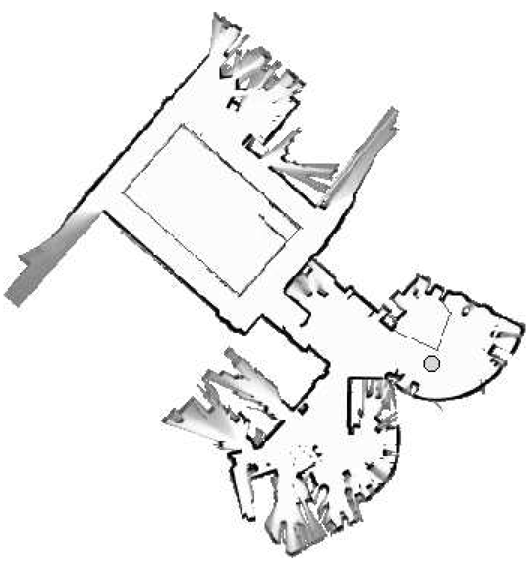
where $\mathbf{x}_c$ is the closest point of the obstacle.
For multiple obstacles: add a separate U_{rep} for each obstacle.

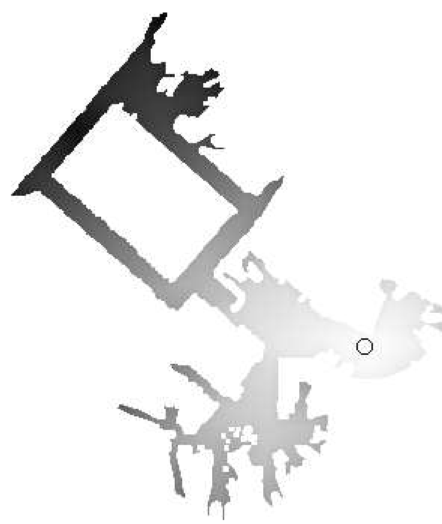
Problem: non-convex obstacles.
Solution: triangulate single obstacle into multiple convex obstacles.
Problem: summed repulsive fields could be greater than single convex object of same area.
Solution: weight separate fields from same object.
Killer problem: ?

Numerical Potential Field Methods

E. Rimon and D. E. Koditschek. Exact Robot Navigation Using Artificial Potential Functions. IEEE Transactions on Robotics and Automation, 8(5):501518, October 1992.
Numerical Potential Field Techniques for Robot Path Planning. J. Barraquand, B. Langlois, and J.C. Latombe. IEEE Transactions on Systems, Man, and Cybernetics, 22(2):224-241, 1992
Essentially Dijkstra's Algorithm:

1. $\forall \mathbf{x} : U(\mathbf{x}) = \infty, F(\mathbf{x}) = F_{rep}(\mathbf{x} + \alpha$
2. $U(\mathbf{x}_{goal}) = 0$
3. push Q, $\mathbf{x}_{goal}$ using 0
4. while Q != empty
   (a) $\mathbf{x}_i$ = pop_min Q
   (b) $\forall$ neighbours $\mathbf{x}_j$ of $\mathbf{x}_i$
   if $U(\mathbf{x_j}) > U(\mathbf{x_i}) + \beta F(\mathbf{x}_j)$ then
      i. $U(\mathbf{x_j}) = U(\mathbf{x_i}) + \beta F(\mathbf{x}_j)$
      ii. push Q, $\mathbf{x}_j$ using $U(\mathbf{x_j})$
5. while $\mathbf{x_i} \neq \mathbf{x_{goal}}$
   (a) $\mathbf{x_i} = \mathrm{argmin}_{\mathbf{x_j} \in \text{neighbours}(\mathbf{x_i})} U(\mathbf{x_j})$



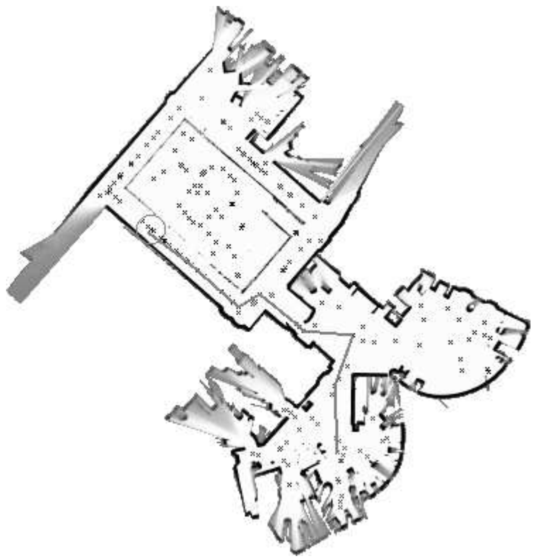


Killer problem: ?

Randomized Techniques

Probabilistic Roadmap

Kavraki, L. E., Svestka, P., Latombe, J.-C., and Overmars, M.. Probabilistic Roadmaps for Path Planning in High Dimensional Configuration Spaces. IEEE Transactions on Robotics and Automation, 12(4):566-580, 1996.



S.M. LaValle and J.J. Kuffner. Randomized kinodynamic planning. International Journal of Robotics Research, 20(5):378-400, May 2001.