UNIFIED LECTURE #2:
THE BREGUET RANGE EQUATION

I. Learning Goals

At the end of this lecture you will:

A. Be able to answer the question "How far can an airplane fly, *and why*?";

B. Be able to answer the question "How do the disciplines of structures & materials, aerodynamics and propulsion jointly set the performance of aircraft, and what are the important performance parameters?";

C. Be able to use empirical evidence to estimate the performance of aircraft and thus begin to develop intuition regarding important aerodynamic, structural and propulsion system performance parameters;

D. Have had your first exposure to active learning in Unified Engineering

II. Question: How far can an airplane (or a duck, for that matter) fly?

OR:

What is the farthest that an airplane can fly on earth, *and why*?

We will begin by developing a mathematical model of the physical system. Like most models, this one will have many approximations and assumptions that underlie it. It is important for you to understand these approximations and assumptions so that you understand the limits of applicability of the model and the estimates derived from it.

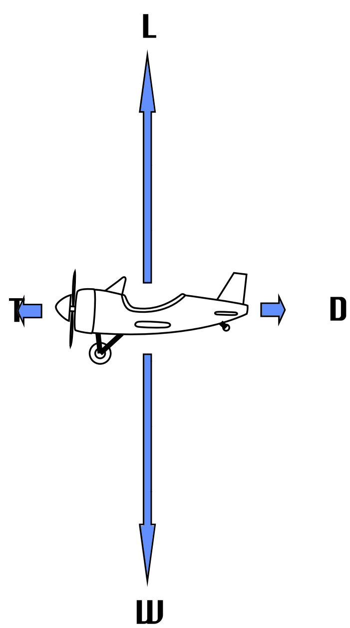


Figure 1.1 Force balance for an aircraft in steady level flight.

For steady, level flight,

$$T = D, \quad L = W \qquad \text{or} \qquad W = L = D\frac{L}{D} = T\left(\frac{L}{D}\right)$$

The weight of the aircraft changes in response to the fuel that is burned (rate at which weight changes equals negative fuel mass flow rate times gravitational constant)

$$\frac{dW}{dt} = -\dot{m}_f \cdot g$$

Now we will define an overall propulsion system efficiency:

$$\textit{overall efficiency} = \frac{\text{what you get}}{\text{what you pay for}} = \frac{\text{propulsive power}}{\text{fuel power}}$$

$$\textit{propulsive power} = \text{thrust} \cdot \text{flight velocity} = Tu_o \quad \text{(J/s)}$$

$$\textit{fuel power} = \text{fuel mass flow rate} \cdot \text{fuel energy per unit mass} = \dot{m}_f h \quad \text{(J/s)}$$

Thus

$$\boxed{\eta_{overall} = \frac{Tu_o}{\dot{m}_f h}}$$

We can now write the expression for the change in weight of the vehicle in terms of important aerodynamic (L/D) and propulsion system ($\eta_{overall}$) parameters:

$$\frac{dW}{dt} = -\dot{m}_f \cdot g = \frac{-W}{\left(\frac{L}{D}\right)\frac{T}{\dot{m}_f \cdot g}} = \frac{-Wu_0}{\frac{h}{g}\left(\frac{L}{D}\right)\frac{Tu_0}{\dot{m}_f \cdot h}} = \frac{-Wu_0}{\frac{h}{g}\left(\frac{L}{D}\right)\eta_{overall}}$$

We can rewrite and integrate

$$\frac{dW}{W} = \frac{-u_0 dt}{\frac{h}{g}\left(\frac{L}{D}\right)\eta_{overall}} \qquad \Rightarrow \qquad \ln W = \text{constant} - \frac{tu_0}{\frac{h}{g}\left(\frac{L}{D}\right)\eta_{overall}}$$

applying the initial conditions, at $t = 0$ $W = W_{initial}$ $\therefore$ const. = $\ln W_{initial}$

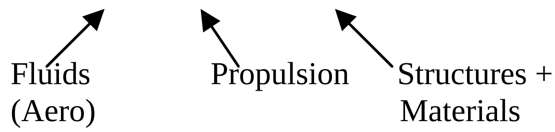
$$\therefore t = \frac{-L}{D}\,\eta_{overall}\,\frac{h}{gu_0}\,\ln\frac{W}{W_{initial}}$$

the time the aircraft has flown corresponds to the amount of fuel burned, therefore

$$t_{final} = \frac{-L}{D}\,\eta_{overall}\,\frac{h}{gu_0}\,\ln\frac{W_{final}}{W_{initial}}$$

then multiplying by the flight velocity we arrive at the **Breguet Range Equation** which applies for situations where overall efficiency, L/D, and flight velocity are constant over the flight.

$$\boxed{\text{Range} = \frac{h}{g}\,\frac{L}{D}\,\eta_{overall}\,\ln\frac{W_{initial}}{W_{final}}}$$



Note that this expression is sometimes rewritten in terms of an alternate measure of efficiency, the *specific fuel consumption* or SFC. SFC is defined as the mass flow rate of fuel per unit of thrust (lbm/s/lbf or kg/s/N). In the following expression, V is the flight velocity and g is the acceleration of gravity.

$$\text{Range} = \frac{V(L/D)}{g \cdot SFC}\ln\left(\frac{W_{initial}}{W_{final}}\right)$$

Thus we see that the answer to the question "How far can an airplane fly?" depends on:

1. How much energy is contained in the fuel it carries;

2. How aerodynamically efficient it is (the ratio of the production of lift to the production of drag). During the **fluids lectures** you will learn how to develop and use models to estimate lift and drag.

3. How efficiently energy from the fuel/oxidizer is turned into useful work (thrust times distance traveled) which is used to oppose the drag force. **Thermodynamics** helps us describe and estimate the efficiency of various energy conversion processes, and **propulsion** lets us describe how to use this energy to propel a vehicle;

4. How light weight the structure is relative to the amount of fuel and payload it can carry. The **materials and structures lectures** you will teach you how to estimate the performance of aerospace structures.

Below are some data to allow you to make estimates for various aircraft and birds.

Table 3 The heat of combustion or metabolic equivalent for various foodstuffs and fuels. The prices are based on a "snapshot" in 1994; large fluctuations may, of course, occur over time.

	MJ/kg[a]	$/kg	$/MJ	Comments
Prime beef	4.0	20	5	
Beef	4.0	8	2	
Whole milk	2.8	0.90	0.32	600 cal/quart
Honey	14	4	0.29	
Sugar	15	1	0.07	100 cal/ounce
Cheese	15	6	0.40	
Bacon	29	4	0.14	
Corn flakes	15	3.50	0.23	100 cal/ounce
Peanut butter	27	4	0.15	180 cal/ounce
Butter	32	4.50	0.14	
Vegetable oil	36	2	0.06	240 cal/ounce
Kerosene	42	0.40	0.010	0.82 kg/liter
Diesel oil	42	0.40	0.010	0.85 kg/liter
Gasoline	42	0.40	0.010	0.75 kg/liter
Natural gas	45	0.24	0.005	0.8 kg/m^3

a. megajoules per kilogram

Figure 1.2 Heating values for various fuels (from The Simple Science of Flight, by H. Tennekes)

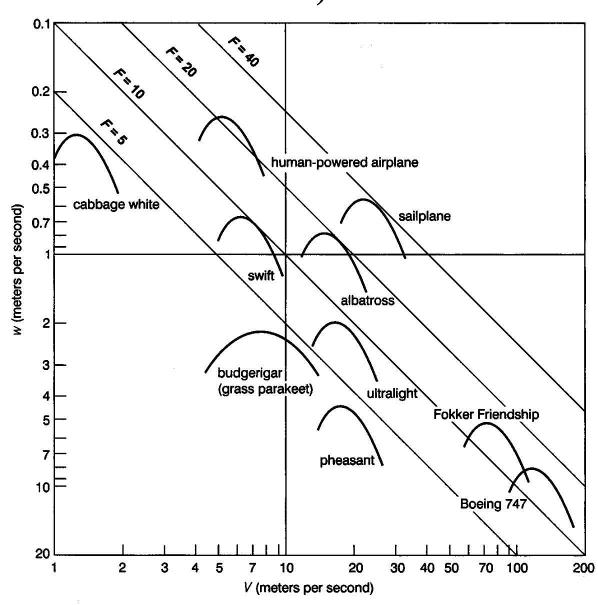


Figure 15 The Great Gliding Diagram. Airspeed is plotted on the horizontal axis. Rate of descent is plotted along the vertical axis, downward. The diagonals running from top left to bottom right are lines of constant finesse. The practical soaring limit, 1 meter per second, is indicated by the horizontal line.

Figure 1.3 Gliding performance as a function of L/D (where L/D=F, from The Simple Science of Flight, by H. Tennekes)

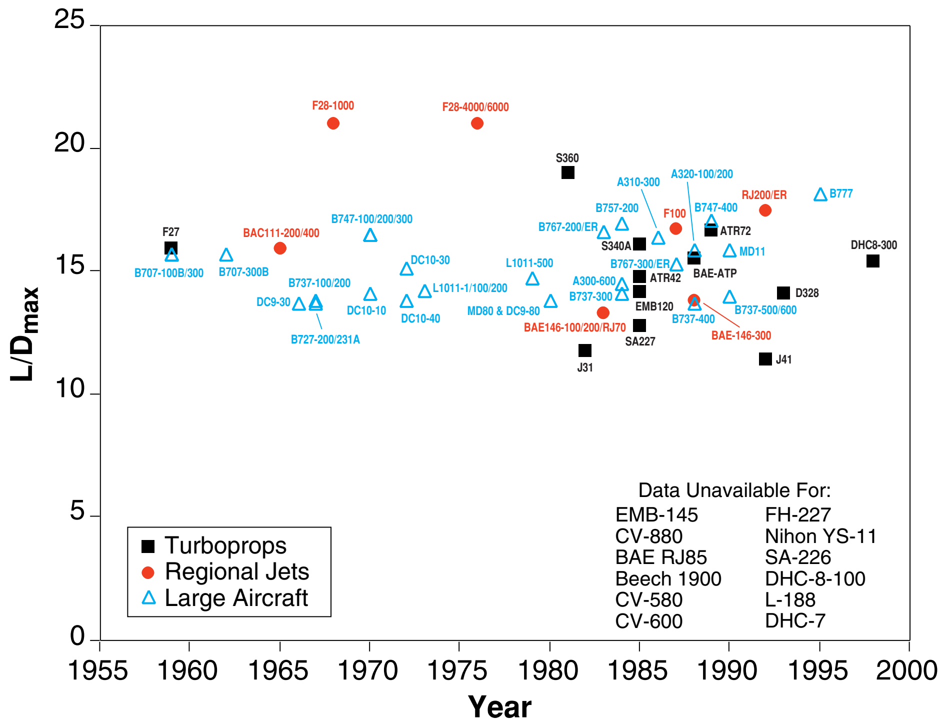


Figure 1.4 Aerodynamic data for commercial aircraft: L/D for cruise (Babikian, R., *The Historical Fuel Efficiency Characteristics of Regional Aircraft From Technological, Operational, and Cost Perspectives*, SM Thesis, MIT, June 2001)

Table 5 Aspect ratio *A* and finesse *F* for various birds and airplanes. The values of *A* have been calculated from $A = b^2/S$; the values of *F* have been measured or estimated.

	W (N)	*S* (m^2)	*b* (m)	*A*	*F*
House sparrow	0.28	0.009	0.23	6	4
Swift	0.36	0.016	0.42	11	10
Common tern	1.2	0.056	0.83	12	12
Kestrel (sparrow hawk)	1.8	0.06	0.74	9	9
Carrion crow	5.5	0.12	0.78	5	5
Common buzzard	8.0	0.22	1.25	7	10
Peregrine falcon	8.1	0.13	1.06	9	10
Herring gull	12	0.21	1.43	10	11
Heron	14	0.36	1.73	8	9
White stork	34	0.50	2.00	8	10
Wandering albatross	85	0.62	3.40	19	20
Hang glider	1000	15	10	7	8
Parawing	1000	25	8	2.6	4
Powered parawing	1700	35	10	2.7	4
Ultralight (microlight)	2000	15	10	7	8
Sailplanes					
standard class	3500	10.5	15	21	40
open class	5500	16.3	25	38	60
Fokker F-50	19×10^4	70	29	12	16
Boeing 747	36×10^5	511	60	7	15

Figure 1.5 Weight and geometry for aircraft and birds (where L/D=F, from The Simple Science of Flight, by H. Tennekes)

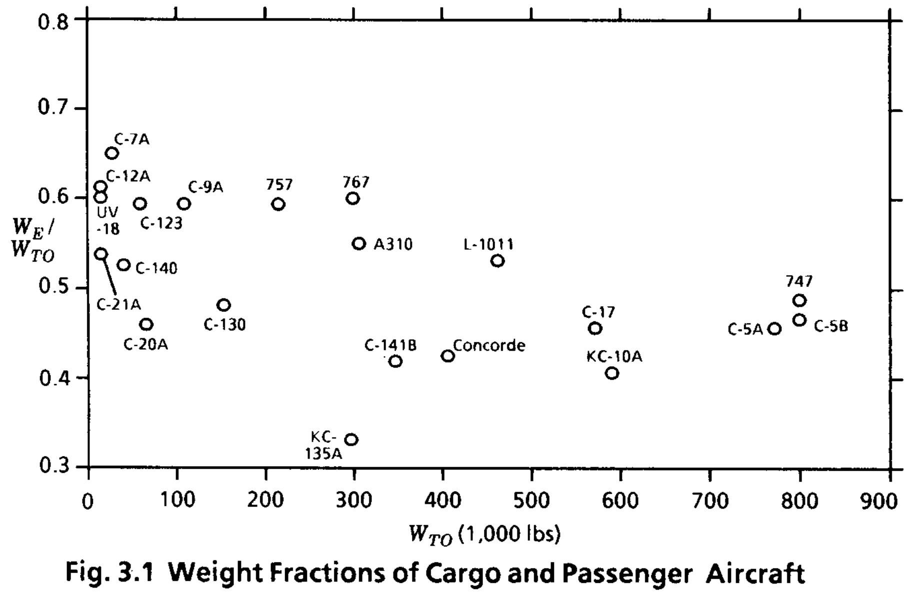


Figure 1.5 Weight fractions for transport aircraft in terms of empty weight over max take-off weight (Mattingly, Heiser & Daley, Aircraft Engine Design, 1987)

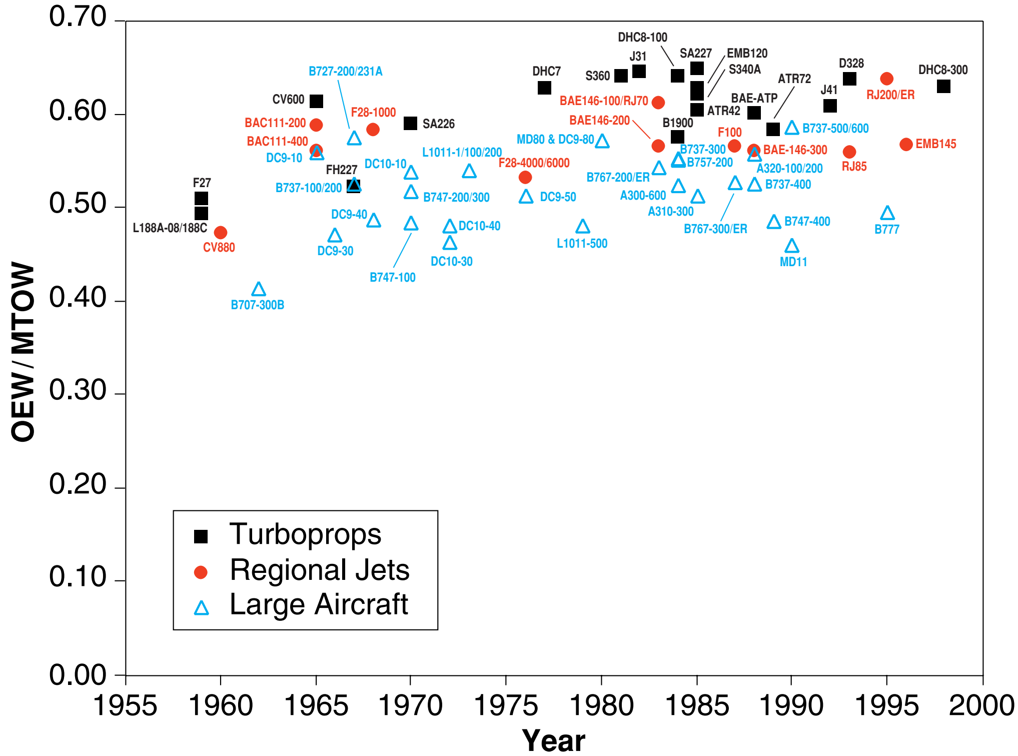


Figure 1.6 Structural efficiency data for commercial aircraft: Operating empty weight over maximum take-off weight (Babikian, R., *The Historical Fuel Efficiency Characteristics of Regional Aircraft From Technological, Operational, and Cost Perspectives*, SM Thesis, MIT, June 2001)

Table 7 Data on popular airliners. Sources: *Jane's All the World's Aircraft*, *KLM Holland Herald*, *Martinair Magazine*, *Transavia Inflight Magazine*.

	Takeoff weight *W* (tons)	*S* (m^2)	*b* (m)	Sea-level thrust *T* (tons)	Fuel consumption (liters/hour)	Cruising speed *V* (km/hour)	Range (km)	Seats
Boeing 747-400	395	530	65	4 × 25.7	12300	900	12200	421
Boeing 747-300	378	511	60	4 × 23.8	13600	900	10500	400
Boeing 747-200	352	511	60	4 × 21.3	13900	900	9500	387
Douglas DC-10-30	256	368	50	3 × 23.1	10400	900	9900	248
Airbus A310	139	219	44	2 × 22.7	5500	860	6400	200
Boeing 737-300	57	105	29	2 × 9.1	2700	800	4200	124
Fokker F-100	43	94	28	2 × 6.7	2400	720	1800	101
Fokker F-28	33	79	25	2 × 4.5	2500	680	1700	80

Figure 1.7 Aircraft performance (from The Simple Science of Flight, by H. Tennekes)

For aircraft engines it is often convenient to break the overall efficiency into two parts: thermal efficiency and propulsive efficiency where the subscripts e and o refer to exit and inlet:

$$\eta_{\text{thermal}} = \frac{\text{rate of production of propellant k.e.}}{\text{fuel power}} = \frac{\left(\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_o u_o^2}{2}\right)}{\dot{m}_f h}$$

$$\eta_{\text{prop}} = \frac{\text{propulsive power}}{\text{rate of production of propellant k.e.}} = \frac{T u_o}{\left(\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_o u_o^2}{2}\right)}$$

such that

$$\eta_{\text{overall}} = \eta_{\text{thermal}} \cdot \eta_{\text{prop}}$$

During the first semester thermodynamics lectures we will focus largely on thermal efficiency. In next semester's propulsion lectures we will combine thermodynamics with fluid mechanics to obtain estimates for propulsive and thus overall efficiency. The data shown in Figure 1.6 will give you a rough idea for the conversion efficiencies of various modern aircraft engines.

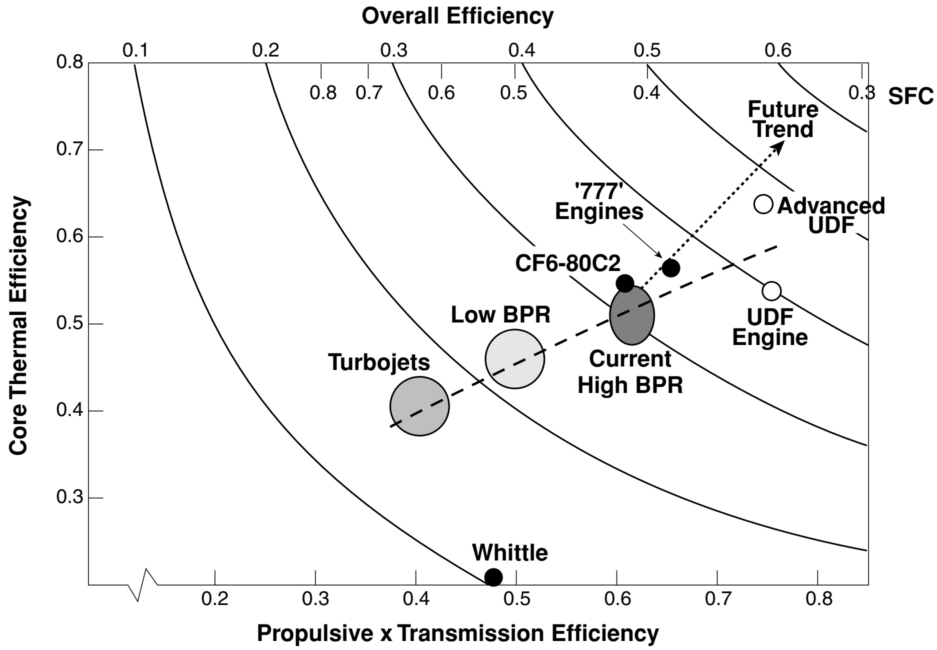


Figure 1.8 Trends in aircraft engine efficiency (after Pratt & Whitney)

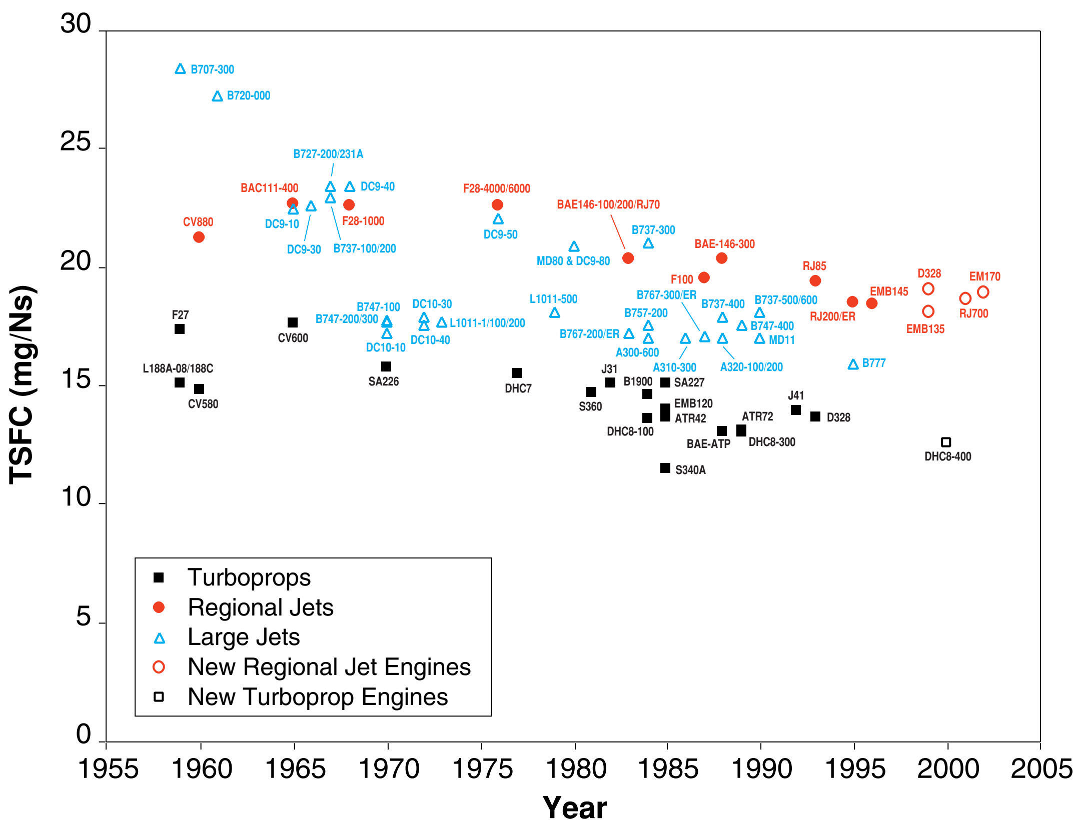


Figure 1.9 Engine efficiency for commercial aircraft: specific fuel consumption (Babikian, R., *The Historical Fuel Efficiency Characteristics of Regional Aircraft From Technological, Operational, and Cost Perspectives*, SM Thesis, MIT, June 2001)

The **accuracy of the range equation** in predicting performance for commercial transport aircraft is quite good. The Department of Transportation collects and reports a variety of operational and financial data for the U.S. fleet in something called DOT Form 41. Operational data for fuel burned and payload (passengers and cargo) carried was extracted from Form 41 and combined with the technological data shown in Figures 1.4, 1.6 and 1.9 to estimate range. In Figure 1.10 these estimates are compared to the actual stage length flown (range) as reported in Form 41. The difference between the actual stage length flown and the estimated stage length is shown in Figure 1.11. Figure 1.11 shows that the percent deviation between the Breguet range equation estimates and the actual stage lengths flown is a function of the stage length. For long-haul flights, the assumptions of constant velocity, L/D, and SFC are good. However, for short-haul flights, taxiing, climbing, descending, etc. are a relatively large fraction of the overall flight time, so the steady-state cruise assumptions of the range equation are less valid.

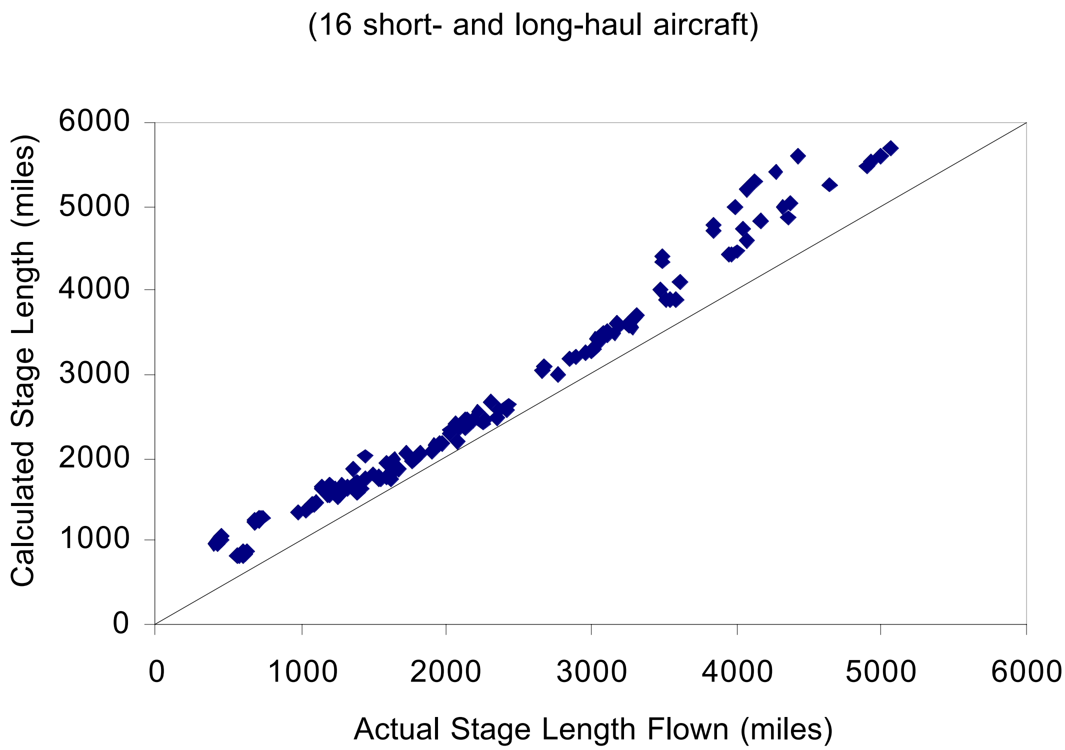


Figure 1.10 Performance of Breguet range equation for estimating commercial aircraft operations (J. J. Lee, MIT Masters Thesis, 2000)

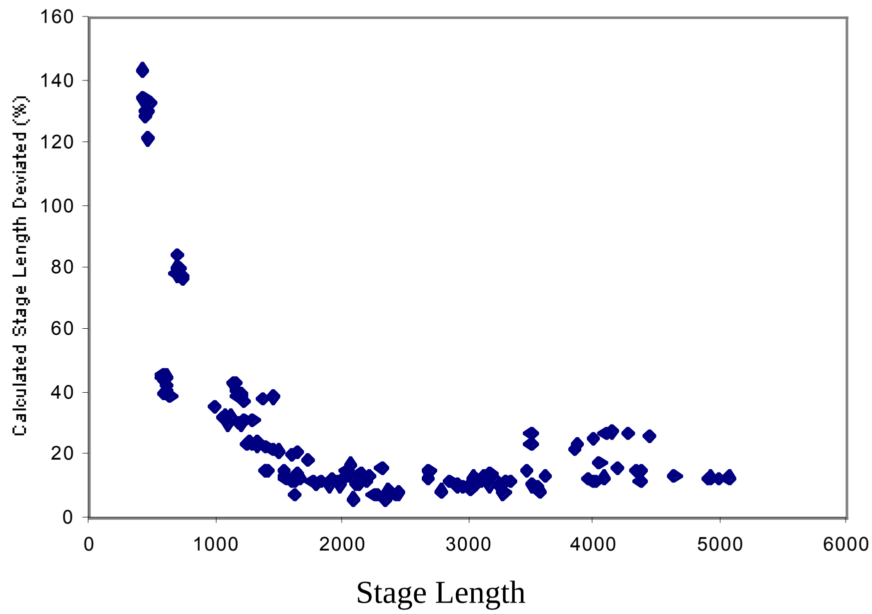


Figure 1.11 Deviation (%) of Breguet range equation estimates from actual stage length flown is a function of the stage length (J. J. Lee, MIT Masters Thesis, 2000)