

Fluids – Lecture 16 Notes

1. Shock Losses

2. Compressible-Flow Pitot Tube

Reading: Anderson 8.6, 8.7

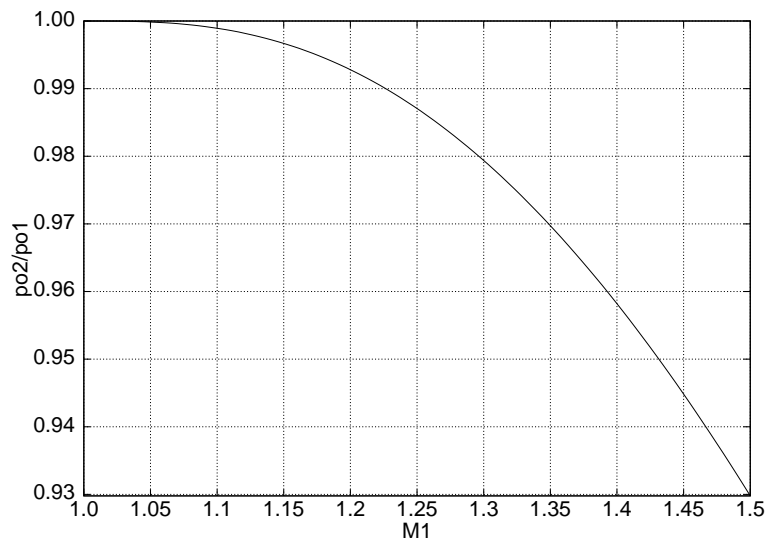
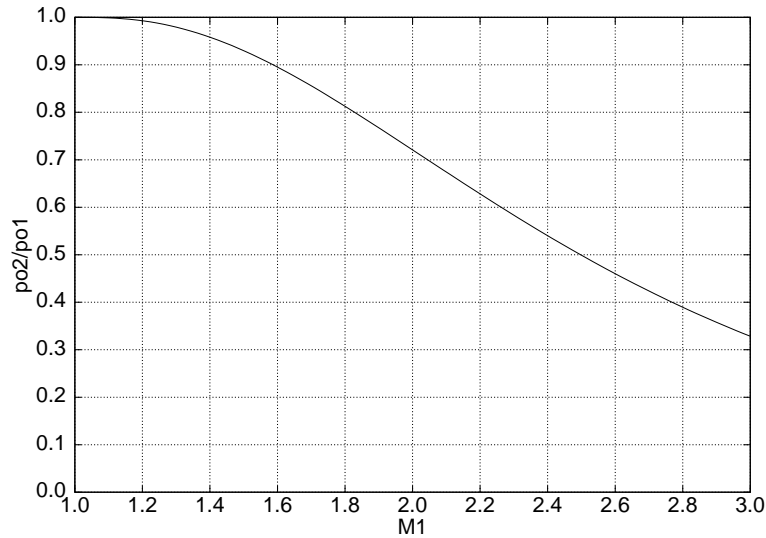
Shock Losses

Stagnation pressure jump relation

The stagnation pressure ratio across the shock is

$$\frac{p_{o2}}{p_{o1}} = \frac{p_2}{p_1} \left(\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right)^{\gamma/(\gamma-1)} \quad (1)$$

where both p_2/p_1 and M_2 are functions of the upstream Mach number M_1 , as derived previously. The figures show the p_{o2}/p_{o1} ratio, with the second figure showing an expanded scale near $M_1 \simeq 1$.



The fractional shock total-pressure loss $1 - p_{o2}/p_{o1}$ is seen to be small for M_1 close to unity, but increases rapidly for higher Mach numbers. Minimizing this loss is of great practical importance, since it cuts directly into the performance of supersonic air-breathing engines.

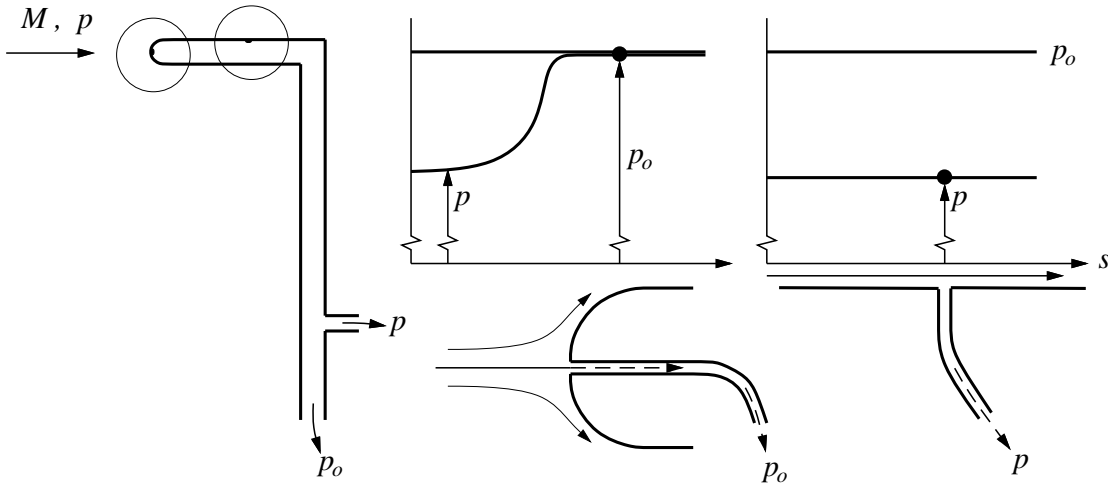
Compressible-Flow Pitot Tube

Subsonic pitot tube

A pitot tube in subsonic flow measures the local total pressure p_o . Together with a measurement of the static pressure p , the Mach number can be computed from the p_o/p ratio relation.

$$\frac{p_o}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

$$M^2 = \frac{2}{\gamma-1} \left[\left(\frac{p_o}{p}\right)^{(\gamma-1)/\gamma} - 1 \right] \quad (2)$$



The pitot-static combination therefore constitutes a *Mach meter*. With M^2 known, we can then also determine the dynamic pressure.

$$\frac{1}{2}\rho V^2 = \frac{\gamma}{2}pM^2 = \frac{\gamma}{\gamma-1}p \left[\left(\frac{p_o}{p}\right)^{(\gamma-1)/\gamma} - 1 \right] \quad (3)$$

The velocity can be determined from

$$V^2 = a^2M^2 = \frac{a_o^2 M^2}{1 + \frac{\gamma-1}{2}M^2}$$

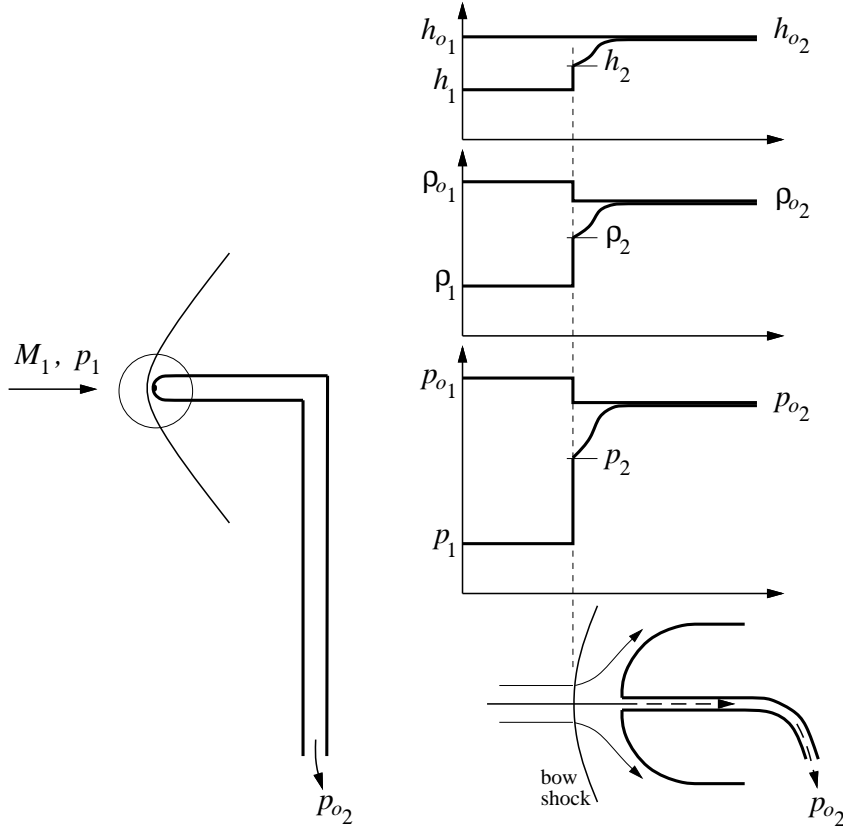
but this requires knowing either the static speed of sound a , or the stagnation speed of sound a_o . The latter can be obtained by measuring the stagnation temperature at the tip of the pitot probe.

Supersonic pitot tube

A pitot probe in a supersonic stream will have a *bow shock* ahead of it. This complicates the flow measurement, since the bow shock will cause a drop in the total pressure, from p_{o1} to p_{o2} , the latter being sensed by the pitot port. It's useful to note that the shock will also cause a drop in ρ_o , but h_o will not change.

The pressures and Mach number immediately behind the shock are related by

$$\frac{p_{o2}}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$



In addition, we also have M_2 and p_2/p_1 as functions of M_1 from the earlier normal-shock analysis. Combining these produces the relation between the p_{o2} measured by the pitot probe, the static p_1 , and the required flow Mach number M_1 . After some manipulation, the result is the *Rayleigh Pitot tube formula*.

$$\frac{p_{o2}}{p_1} = \frac{p_{o2} p_2}{p_2 p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)} \right)^{\gamma/(\gamma-1)} \frac{1 - \gamma + 2\gamma M_1^2}{\gamma+1} \quad (4)$$

The figure shows p_{o2}/p_1 versus M_1 , compared with the isentropic ratio p_{o1}/p_1 .

$$\frac{p_{o1}}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\gamma/(\gamma-1)} \quad (5)$$

Only the latter is plotted for $M_1 < 1$, where there is no bow shock, and so equation (4) does not apply. The effect of the pitot bow shock's total pressure loss, indicated by the difference $p_{o1} - p_{o2}$, becomes substantial at larger Mach numbers.

Ideally, we would like to have the pitot formula (4) give M_1 as an explicit function of the pitot/static pressure ratio p_{o2}/p_1 . However, this is not possible due to its complexity, so a numerical solution is required. The function is also readily available in table form.

