18.06 Professor Edelman Quiz 1 March 2, 2018

Your PRINTED name is: ____________

Please circle your recitation:

(1)	T 10	26-328	D. Kubrak
(2)	T 11	26-328	D. Kubrak
(3)	T 12	4-159	P.B. Alvarez
(7)	T 12	4-153	E. Belmont
(4)	T 1	4-149	P.B. Alvarez
(5)	T 2	4-149	E. Belmont
(6)	T 3	4-261	J. Wang

Grading
1
2
3
Total:

Important Instructions: We will be using Gradescope which requires your solutions appear in the boxes provided. Please place your solutions in the boxes if possible.

If you need extra pages, please write continued in the box, and on the extra pages clearly label with problem number and letter.

1 (30 pts.)

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 8 & 2 \\ 5 & 12 & 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

(a) (15 pts.) Perform elimination on $[A\ b]$ to determine the condition that b is in the column space of A.

1. (a) Continued if needed.

(b) (5 pts.) What is the rank of A?

(c) (10 pts.) Find a nonzero vector in the nullspace of A^T (this space is known as the left nullspace.)

2 (30 pts.) (6 points each part) Consider the 5x5 matrices

$$E_1 = \begin{pmatrix} 1 & . & . & . & . \\ a & 1 & . & . & . \\ b & . & 1 & . & . \\ c & . & . & 1 & . \\ d & . & . & . & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & . & . & . & . \\ . & 1 & . & . & . \\ . & . & 1 & . & . \\ . & . & . & 1 & . \\ . & . & . & x & 1 \end{pmatrix}, \text{and } E = E_1 E_2.$$

Here the dot ("**.**") denotes 0.

(a) Solve $E_1 x = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ for x.

(b) Solve $E_1^T x = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ for x.

(c) Compute E.

(d) Compute E^{-1}. Check your answer.

(e) Compute $(E_1)^{10} = E_1 \times E_1 \times E_1 \times E_1 \times E_1 \times E_1 \times E_1 \times E_1 \times E_1 \times E_1$.

3 (40 pts.) Answer the following with TRUE/FALSE and explain briefly and convincingly. (1 point for the correct answer, and 3 points for a correct justification. No points for the wrong answer no matter how creative the explanation.)

(a) Is it TRUE/FALSE that the matrix $M = vv^T$ is always symmetric, when v is a vector of length n?

True. Note that v defaults to a column vector as is the convention in this class. Explanation: $(vv^T)^T = (vv^T)$ by the rules of transpose. Any matrix equal to its transpose is symmetric.

Alternatively one can check that $M_{ij} = v_i v_j = v_j v_i$.

Points lost for doing only one example or only $n = 2$ or $n = 3$. Points lost if notations written such as v_i^T which indicated lack of understanding.

(b) Is it TRUE/FALSE that the matrix $M = vv^T$ might possibly have rank 0, or rank 1, but could never have rank 2?

(c) Is it TRUE/FALSE that it is possible to compute $y = ABx$ with approximately a constant multiple of n^2 operations, where A and B are general $n \times n$ matrices, and x is a general n vector?

(d) Is it TRUE/FALSE that in the above computation of y, if x is known to be the first column of the identity matrix, then it is generally possible to reduce the number of operations to a constant multiple of n?

The ikj formulation of square matrix multiply has the following key lines:

```
C = zeros(n,n)
for i=1:n, k=1:n, j=1:n
    C[i,j] += A[i,k] * B[k,j]
end
```

where first $i = 1$ and $k = 1$ and then j goes from 1 to n.

Next $i = 1$ and $k = 2$ and then j goes from 1 to n, etc.

(e) Is it TRUE/FALSE that the calculation of the (1,1) entry of C is completed before the calculation of the (1,2) entry is started?

(f) Is it TRUE/FALSE that the ikj formulation is a ROW based view of matrix multiply?

(g) Is it TRUE/FALSE that a 3x3 rank 2 matrix can have an RREF whose first column is all zeros?

(h) Is it TRUE/FALSE that a 3x3 rank 2 matrix can have an RREF whose second column is all zeros?

(i) Let x be a given nonzero vector of length 4. Consider the set of matrices A for which x is in the nullspace of A. Is it TRUE/FALSE that this set of matrices form a vector space?

(j) Let b be a given nonzero vector of length 4. Consider the set of matrices A for which b is in the column space of A. Is it TRUE/FALSE that this set of matrices form a vector space?

Extra Page. Please write problem number and letter if needed.

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