18.06 Professor Edelman Quiz 2 April 6, 2018

Your PRINTED name is: ____________

Please circle your recitation:

(1)	T 10	26-328	D. Kubrak
(2)	T 11	26-328	D. Kubrak
(3)	T 12	4-159	P.B. Alvarez
(7)	T 12	4-153	E. Belmont
(4)	T 1	4-149	P.B. Alvarez
(5)	T 2	4-149	E. Belmont
(6)	T 3	4-261	J. Wang

Grading	
1	
2	
3	
4	
Total:	

Note: We are not planning to use gradescope for this exam.

Your Initials: ______________

1 (25 pts.) Find the QR decomposition (Q is 4×2, R is 2×2 upper triangular) of

$A = \begin{pmatrix} 1 & a \\ 1 & b \\ 1 & c \\ 1 & d \end{pmatrix}$ in terms of $\mu = \frac{a+b+c+d}{4}$, the mean of the second column and the elements of A.

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2 (20 pts.) An experimenter has data in the form of pairs (x_i, y_i) for $i = 1, \ldots, n$ where the x_i are distinct and positive. Given the matrix

$$A = \begin{pmatrix} \sin(x_1) & e^{x_1} & \sqrt{x_1} \\ \sin(x_2) & e^{x_2} & \sqrt{x_2} \\ \vdots & \vdots & \vdots \\ \sin(x_n) & e^{x_n} & \sqrt{x_n} \end{pmatrix},$$

suggest a method for computing the best fit function of the form $f(x) = C\sin(x) + De^x + E\sqrt{x}$ through the n points. In what precise sense is your answer a best fit?

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3 (20 pts.)

A form of the singular value decomposition of a rank r, $m \times n$ matrix A is $U\Sigma V^T$ where Σ is square r by r with positive diagonal entries, U is $m \times r$ and V is $n \times r$. Write down projection matrices for the four fundamental subspaces of A, in terms of one of U, Σ, or V in each expression. Be sure to clearly identify which fundamental subspace of A goes with which projection matrix.

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4 (35 pts.)

Let $d(A)$ be a scalar function of 3×2 matrices A with the following properties:

α) If you interchange the two columns of A, $d(A)$ flips sign.

β) $d(A)$ is linear in each of the columns of A.

γ) $d(A)$ is non-zero for at least one 3×2 A.

a. (5 pts.) What is $d(2A)$ in terms of $d(A)$?

b. (10 pts.) Give an example $d(A)$ that satisfies the three requirements of this question.

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c. (10 pts.) We recall that the determinant of square matrices is linear in each column and each row of the square matrix. Can property β be extended to rows and columns of 3×2 matrices A to create a $d(A)$ with the three requirements of this question? If yes, give an example, if not, why not?

d. (10 pts.) If we discard property γ to allow the "zero" function, the set of all functions $d(A)$ satisfying α and β form a three dimensional vector space. Describe explicitly this vector space of functions in terms of the elements of A.

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