2.003 HW #10

#1 **DYNAMIC BRAKING IN AN ELECTRIC MOTOR**

The mechanical work required to rotate the shaft of a direct-current permanent magnet motor is greater when the electrical terminals are short-circuited (i.e., connected together with nothing in between) than when they are open-circuited (i.e., disconnected). This phenomenon is known as “dynamic braking” and is often used in applications to bring a spinning motor quickly to rest.

The shaft of a motor with the specifications shown below is initially rotating at 1,000 rpm.

(a) Develop the simplest linear model competent to describe the time-course of the shaft speed as it slows down in two cases:
  (i) with the electrical terminals disconnected (open-circuited)
  (ii) with the electrical terminals connected together (short-circuited)

(b) Plot the time-course of shaft speed in each case.

(c) Calculate the effective (mechanical) viscous damping coefficient due to electrical power dissipation

Excerpt from specification sheet of a direct-current permanent-magnet motor

MOTOR CONSTANTS: INTRINSIC (AT 25 DEG C)	**SYMBOL**	**UNITS**	
TORQUE CONSTANT	KT	OZ IN/AMP	5.03
BACK EMF CONSTANT	KE	VOLTS/KRPM	3.72
TERMINAL RESISTANCE	RT	OHMS	1.400
ARMATURE RESISTANCE	RA	OHMS	1.120
VISCOUS DAMPING CONSTANT	KD	OZ IN/KRPM	0.59
MOMENT OF INERTIA	JM	OZ IN SEC-SEC	0.0028
ARMATURE INDUCTANCE	L	MICRO HENRY	<100.0
TEMPERATURE COEFFICIENT OF KE	C	%/DEG C RISE	-0.02

#2

A 30,000 μF (micro-Farad) electrical capacitor is connected to the electrical terminals of a DC permanent-magnet motor with the specifications shown in problem #1. Starting from rest, a torque of 4 N-m is abruptly applied to the shaft.

(a) Develop the simplest linear model competent to describe the time-course of the shaft speed in response to this torque.

(b) Plot the time-course of shaft speed in response to this torque.

(c) In problem #1 you saw that the electrical resistor has the same effect as an equivalent rotational damper.
  (i) Which rotational element (inertia, torsional spring or rotational damper) would have the same effect as the capacitor?
  (ii) Write an expression relating the parameter of that element (i.e., inertia, I, stiffness, K, or damping, B) to the parameters of the capacitor and motor.

#3

A Boston-based group of venture-capital investors have been approached by Shamus O’Veroptimistic, a Celtic Tiger entrepreneur who proposes to launch a new company. Its products are a line of portable generators to provide electrical power for night-lights for bicycles, skateboards, rollerblades, etc. Your task is to determine if this line of products is technically sound. That is, would they work in practice? (never mind whether they would sell)

The underlying technology is a lightweight AC (alternating current) generator. Permanent magnets attached to a shaft rotate inside a set of wire coils arranged so that electro-magnetic induction generates a voltage that sends electrical power to a light. Rotation of the shaft with the magnets is produced by coupling it to a wheel on the bicycle (or skateboard, rollerblade, etc.)

In one design to be used with a bicycle, the AC generator is mounted on the bicycle frame. Its shaft is driven by a 0.5 inch diameter wheel that rolls without slipping on the sidewall of the rear bicycle wheel, which is 30 inches in diameter.

If the generator shaft rotates at constant speed, ω, the voltage varies sinusoidally with time as follows:

$$e(t) = A\,\omega\,\sin(\omega t)$$

where A = 0.12 volts-sec/radian.

The wire coils of the AC generator have an inductance of 140 millihenries and a resistance of 10 ohms.

When driven by a constant 60 volts, the light bulb consumes 40 watts.

(a) Develop a differential equation relating generator voltage to the voltage across the light bulb.

(b) Derive an expression relating the light-bulb voltage amplitude to bicycle speed in mph.

(c) For a range of speeds from 2 to 20 mph, plot light-bulb power amplitude as a function of bicycle speed. Use log scales (as in a Bode magnitude plot).

#4
In another of his designs, O'Veroptimistic the entrepreneur proposes to boost the output of the light bulb by adding a 5 μF (microfarad) electrical capacitor to the circuit in series between the generator and light bulb. He claims this will cause the circuit to resonate, yielding substantially higher voltages and hence more output light power. Using the parameters of problem #3,

(a) Develop state and output equations relating generator voltage to
  (i) capacitor voltage
  (ii) light-bulb voltage

(b) For a range of speeds from 2 to 20 mph, make Bode magnitude plots of
  (i) capacitor voltage
  (ii) light-bulb voltage

(c) Will this design work as planned? Explain briefly