2.003 Fall 1999 Homework Assignment 9

1. Consider uniaxial vibration of an engine block that weighs 200 pounds and is supported on mounts that have an effective stiffness of 18,000 pounds per inch. and an effective linear damping coefficient of 2 pounds per inch per second, and is subjected to an oscillating force of the form

$$f(t) = f_a \sin \Omega t$$

where the amplitude f_a equals 2 pounds. Derive an equation of motion for the displacement $y(t)$, of the engine block from its equilibrium position.

(a) Write an expression for the complex amplitude A of the steady-state displacement response.

(b) Obtain:

(i) the low-frequency asymptote for the complex amplitude (a);

(ii) the high-frequency asymptote for the complex amplitude (a);

(iii) the break frequency Ω_{break} where the magnitudes of (i) and (ii) are equal.

(c) Evaluate the input frequency Ω_{peak} at which the engine displacement amplitude has the greatest magnitude.

(d) Evaluate the ratio $|A(\Omega_{peak})/A(0)|$ where $A(0)$ is the limit of $A(\Omega)$ as $\Omega \to 0$.

(e) Use MATLAB to make Bode plots for:

(i) the magnitude of the ratio $A(\Omega)/A(0)$;

(ii) the phase of the complex amplitude $A(\Omega)$.

Use a logarithmic scale for frequency Ω which extends at least one decade below, and one decade above, the break frequency. Express the magnitude scale in decibels; *i.e.,* plot $10 \log_{10}(\text{magnitude})^2$.

2. Reconsider Problem 1 for the same engine block and mounts, but instead of the input force with constant amplitude $f_a = 2$ pounds, consider the amplitude of the input force $f(t) = f_a sin\Omega t$ to increase with the square of the frequency, as it does when the force is a reaction to the force required to accelerate a mass $m = w/g$ which oscillates at frequency Ω with displacement amplitude r. Take $f_a = (w/g)r\Omega^2$, where $(w/g)r = (1.93/386)(1) = 0.005$ pound-sec^2.

(a) Write an expression for the complex amplitude A of the steady-state displacement response.

(b) Obtain:

   (i) the low-frequency asymptote for the complex amplitude (a);

   (ii) the high-frequency asymptote for the complex amplitude (a);

   (iii) the break frequency Ω_{break} where the magnitudes of (i) and (ii) are equal.

(c) Evaluate the input frequency Ω_{peak} at which the engine displacement amplitude has the greatest magnitude.

(d) Evaluate the ratio $|A(\Omega_{peak})/A(\infty)|$ where $A(\infty)$ is the limit of $A(\Omega)$ as $\Omega \to \infty$.

(e) Use MATLAB to make Bode plots for:

   (i) the magnitude of the ratio $A(\Omega)/A(\infty)$;

   (ii) the phase of the complex amplitude $A(\Omega)$.

Use a logarithmic scale for frequency Ω which extends at least one decade below, and one decade above, the break frequency. Express the magnitude scale in decibels; *i.e.,* plot $10 \log_{10}(\text{magnitude})^2$.

3. Reconsider Problem 1, but now with emphasis on the start-up transient. Cast the equations of motion in the standard form for state-determined systems:

$$\begin{aligned} \tfrac{d}{dt}\mathbf{x} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{aligned}$$

with the force $f(t)$ as input and the displacement $y(t)$ as the output. Consider that the engine is at rest in its equilibrium position until $t = 0$. Take the force $f(t)$ to be an abruptly initiated sinusoidal force of the form

$$f(t) = \left\{ \begin{array}{cc} 0, & t < 0 \\ f_a \sin \Omega t, & t > 0 \end{array} \right\}$$

with $f_a = 2$ pounds.

(a) Write a MATLAB script to integrate the state equations from the given initial condition. The script should have the capability of accepting an arbitrary value of the forcing frequency Ω.

(b) The MATLAB solution for $\mathbf{x}$ is a two-column matrix consisting of the functions $y(t)$ and $v(t) = dy/dt(t)$ evaluated at the many intermediate t-values listed in the one-column $\mathbf{t}$-matrix. Write some addional script to make MATLAB construct three one-column matices of the following power quantities evaluated at the same set of intermediate t-values:

(i) $P_{in}(t) = f(t) * v(t)$, the power transmitted to the vibratory system by the input force $f(t)$.

(ii) $P_{diss}(t) = f_{fric}(t) * v(t) = bv^2(t)$, the power dissipated by the damping element.

(iii) $P_{stored}(t) = P_{in}(t) - P_{diss}(t)$, the power transmitted to the storage elements where it is continually shifted back and forth from kinetic energy of the engine block to potential energy of the suspension springs.

(c) Have MATLAB make the following four plots:

Engine block displacement $y(t)$ vs. t;

Power input $P_{in}(t)$ vs. t;

Power dissipated $P_{diss}(t)$ vs. t;

Power stored $P_{stored}(t)$ vs. t;

for each of three input frequencies:

(i) $\Omega = 1/3\Omega_{break}$;

(ii) $\Omega = \Omega_{break}$;

(iii) $\Omega = 3\Omega_{break}$.

In order to see both the transient response and the steady state response in the plots, the time span in the plots should be at least ten times longer than the system decay time-constant.

4. This is a different kind of vibration problem for the light aircraft engine we have been considering. Previously, we considered uniaxial translation of the engine block due to the inertia loading from accelerating pistons. We now consider oscillations in the rotational speed of the propellor due to torsional vibration of the short elastic coupler shaft connecting the propellor to the crankshaft. The source of the oscillation is the fluctuating speed generated by the reciprocating engine.

Periodic firing of the cylinders in an internal combustion engine causes its rotational speed to vary periodically. One stroke of a piston is one move from top dead center to bottom dead center (or from bottom to top). In a four-stroke engine, three strokes are used to clear out the products of combustion from the previoius firing, let in fresh air and fuel, and compress the mixture prior to firing. It is only in the fourth stroke that the explosion occurs and a very large force on the piston exerts torque around the axis of the crankshaft. In a four-cylinder engine the resulting torque on the crankshaft is smoothed out considerably by arranging it so that one cylinder fires on every stroke. The remaining fluctuation in torque, when applied to the inertia of the crankshaft, results in a fluctuation in the output speed of the engine, which varies in an approximately sinusoidal manner at the firing frequency, which is half the rotational frequency of the crankshaft. It is this periodic engine speed fluctuation which excites the torsional vibration.

Consider the case of a four-dylinder 150 horsepower engine which operates between 500 and 2700 rpm. The moment of inertia of the two-bladed propellor can be estimated to be the same as that of a uniform solid rod of aluminum, six feet long and two inches in diameter (the density of aluminum is 2.72 grams/cc). It is observed that the steady-state oscillations of propellor speed at the firing frequency reach a peak amplitude when the engine runs at 2200 rpm. Furthermore the magnitude of the oscillation at 2200 rpm is four times larger than the magnitude at 500 rpm.

(a) Develop a model to describe the steady-state fluctuations in propellor speed (output) in response to the fluctuations in engine speed (input). To keep the analysis simple, assume that the amplitude of the engine-speed fluctuations delivered to the coupler shaft are independent of the engine speed, so that in the steady state the angular positiion θ_{eng} and the angular speed ω_{eng} of the engine can be assumed to take the form

$$\begin{aligned}\theta_{eng} &= \Omega t + \epsilon \sin \frac{\Omega}{2} t \\ \omega_{eng} &= \Omega(1 + \frac{\epsilon}{2} \cos \frac{\Omega}{2} t)\end{aligned}$$

(b) Use your model to estimate the torsional stiffness K of the elastic coupler shaft.