2.003 Fall 1999 Homework Assignment 2

1. In class a simple model of a Pinewood Derby racecar was analyzed under the assumption that its motion was opposed by linear viscous friction with a constitutive relation

$$f_{viscous} = bv$$

(a) Formulate a revised model in which the viscous friction element is replaced by a model of air resistance (sometimes called *form drag*) with a constitutive equation

$$f_{drag} = AC_d v|v| = AC_d v^2 \mathrm{sgn}(v)$$

where A is the frontal area of the car and C_d is a drag coefficient that depends on the car's geometry; *i.e.,* on its *form.*

(b) What are the dimensions, in SI units, of the drag coefficient C_d in the above constitutive relation?

(c) Derive a mathematical expression for the time-history of the speed of the car, on a horizontal track, if it starts at $t = 0$ with initial velocity v_o.

2. Formulate a differential equation for the velocity of a racecar in a model in which the car of mass m descends a very long inclined track with angle (alpha), under the influence of gravity and a form drag, which has the constitutive equation described in Problem 1.

(a) Show that this model predicts a terminal velocity v_{ss} on an infinitely long track. Derive a formula for v_{ss} in terms of the parameters m, (alpha), g, A, and C_d.

(b) If, for the same parameter values considered in Assignment 1 ($m = 0.1418$ kg, (alpha) $= 10.81$ degrees), the terminal velocity is the same as it was for viscous friction ($v_{ss} = 8.60$ m/s), what is (are) the parameter value(s) in the form-drag model?

(c) Study the MATLAB scripts 'car2.m' and 'car_form.m' which can be used to explore the model described in this Problem. Determine the time T it takes for the car starting at rest to accelerate to 99.9% of the terminal velocity.

(d) Compare the result of 2(c) with the corresponding result in 4(b) of Assignment 1. Which friction model has a quicker approach to the terminal velocity? Give a physical and/or graphical explanation for this result.

3. In class a model of the vertical motion of a steel plate mounted on four springs was analyzed. Consider the limiting case where all friction is absent. Derive an equation for the displacement history of the plate when it starts at $t = 0$ from its equilibrium

position with the initial velocity v_o.

4. Consider the experiment demonstrated in class, in which a textbook is dropped on the spring-supported plate. Again, for simplicity, consider the limiting case where all friction is absent. Take the mass of the textbook to be one half the mass of the plate, and assume that during the motion the textbook remains in contact with the plate. There are two cases to be considered. In Case I there is a "zero height drop" in which the book is released from rest when just in contact with the plate. In Case II the drop is from a non-zero height such that the book strikes the motionless plate with a downward velocity of v_o. In both cases the motion begins at $t = 0$, at the instant when the book contacts the plate.

(a) What is the ratio of the frequency of oscillation in Case I to the frequency of free oscillation of the unloaded plate?

(b) What is the ratio of the frequency of oscillation in Case II to the frequency of free oscillation of the unloaded plate?

(c) Locate the equilibrium position of the (book + plate) system with respect to the equilibrium position of the unloaded plate .

(d) Determine the initial conditions for y_k and v_k for Case I.

(e) Determine the initial conditions for y_k and v_k for Case II.

(f) For each case write a mathematical expression for the vertical motion of the book and plate for $t > 0$.