2.003 Fall 1999 Homework Assignment 3

1. Consider the vertical oscillations of a 5-pound steel plate supported on springs, as demonstrated in class. Begin by formulating a simple linear model for the motion of the plate in response to an applied force $f(t)$. Your job is to use the MATLAB scripts MassSprgDmpr1, 2 & 3 based on closed form analytical solutions, of the type developed in class, to obtain twelve plots of the time history of response for twelve different sets of model parameters. In every case the plate is at rest in its equilibrium position, when, at $t = 0$ a constant force of 5 pounds is suddenly applied in the direction of positive displacement. In each case you must select the model stiffness and damping parameters to achieve the specified behavioral parameter values.

(a) In each of the first four plots the undamped natural frequency is 5 Hz (cycles/second), and the damping ratios have the following values: (i) $\zeta = 0.1$; (ii) $\zeta = 0.5$; (iii) $\zeta = 1.0$; (iv) $\zeta = 1.5$. You must choose the model parameters to insert in the MATLAB scripts to get the time histories of response corresponding to these combinations of ω_o and ζ. The script 'MassSprgDmpr1.m' must be used for Cases (i) and (ii). The script 'MassSprgDmpr2.m' must be used for Case (iii), and the script 'MassSprgDmpr3.m' must be used for Case (iv). For Cases (i), (ii), and (iii) take the total duration of the time history to be equal to 5 times the decay time constant. In Case (iv) take the total duration of the time history to be 5 times the *longest* time constant.

(b) In each of the next four plots, the *damped* natural frequency is 5 Hz, and the damping ratios have the following values: (i) $\zeta = 0.1$; (ii) $\zeta = 0.3$; (iii) $\zeta = 0.5$; (iv) $\zeta = 0.7$. Again choose model parameters to get the time histories of response corresponding to these combinations of ω_d and ζ. Here all cases can be run with 'MassSprgDmpr1.m'. In each case take the total duration of the time history to be equal to 5 times the decay time constant.

(c) In the final four plots the decay time constant is fixed at $\tau = 0.10$ seconds, and the damping ratios have the following values: (i) $\zeta = 0.3$; (ii) $\zeta = 0.5$; (iii) $\zeta = 0.7$; (iv) $\zeta = 0.9$. Choose model parameters to get the time histories of response corresponding to these combinations of decay time constant and ζ. In each case take the total duration of the time history to be equal to 5 times the decay time constant.

2. Make a careful sketch of the complex plane showing the location of the eigenvalues, λ corresponding to the twelve parameter sets used to produce the time histories in Problem 1.

3. Reconsider the spring-supported plate of Problem 1, but now take the motion to be caused by a 2-pound rubber ball which strikes the plate and bounces away.

Assume that the plate is at rest in its equilibrium position, when it is struck by the ball at $t = 0$. Assume the ball is falling vertically with a velocity of 10 feet/second and rebounds with an upward velocity of 3 feet/second, and that it does not impact the plate again.

(a) Determine the initial velocity of the plate, immediately after the impacxt.

(b) Select the model stiffness and damping parameters so that the damped natural frequency is 5 Hz, and the damping ratio ζ is 0.3.

(c) Insert the model parameters in 'MassSprgDmpr1.m' and plot the displaement response of the plate for an interval equal to 5 times the decay time constant.

(d) What is the relationship between the time history in 2(c) and the time history in the plot for b(ii) in Problem 1?

4. At a tailgate party, it was observed that when Uncle Massive, a sprightly 250-pounder, hopped up on the back of his pick-up truck, the truck suspension bounced for more than 5 clearly defined cycles (it has been some time since Uncle Massive has replaced his shock-absorbers) at a frequency of 2 Hz. When the vibration stopped it was noted that the back of the truck was one inch lower than it was before Uncle Massive sat down.

(a) Estimate the effective mass of the back end of the truck.

(b) Briefly explain any assumptions you have made in arriving at your estimate..