2.003 Fall 1999 Homework Assignment 4

1. A rigid plate supported on four identical springs, like the system demonstrated in class, has unknown model parameters, m, k, and b. A dynamic test is performed in which a constant force of 1.0 Newton is suddenly applied at $t = 0$. The measured displacement response is displayed in Fig.1

Your job is to use the data revealed in this Figure to estimate:

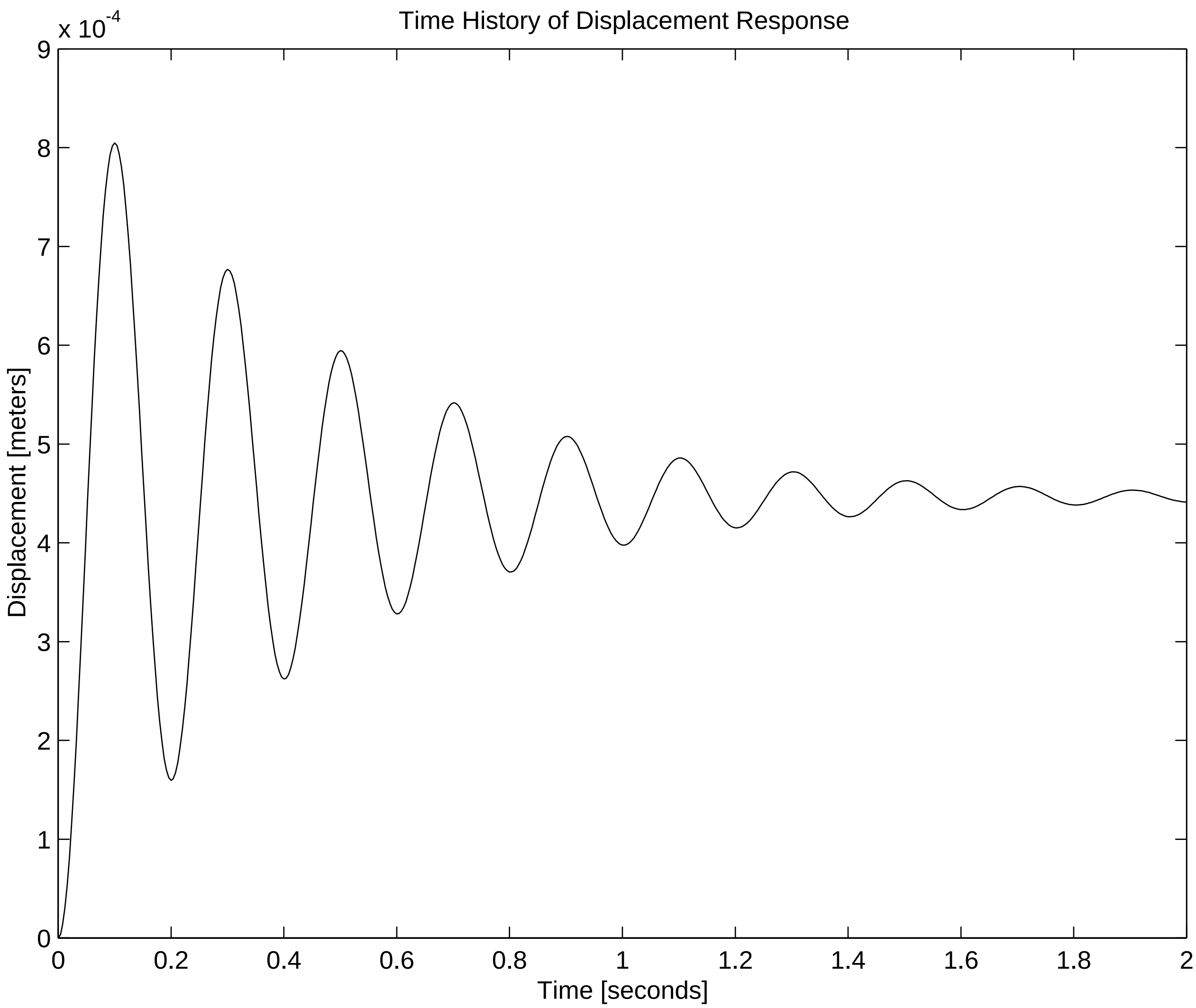

Figure 1: Response to 1.0 Newton Step-Force

(a) The effective stiffness k of the four springs;

(b) The effective mass m of the plate;

(c) The effective damping coefficient b of the system.

2. Each, of two identical plate-on-springs assemblies, like those demonstrated in class, is accurately represented by a model with $m = 2.0$ kg, $k = 2000$ N/m, and $b = 20$ N/m/s. Consider the system obtained by placing the two individual units face-to-face, with the first plate resting on a firm table with its springs extending upward, and the second plate reversed, so that its springs extend downwards and are connected to the springs of the first plate. Your job is to derive a differential equation for free vertical motion of the second plate, and:

(a) Evaluate the undamped natural frequency ω_o for the model of the combined system;

(b) Evaluate the damping ratio ζ for the model of the combined system;

(c) Evaluate the decay time constant τ for the model of the combined system.

3. A bungee jumper weighs 150 pounds. Bungee cords attached to her ankles have a slack length of 100 feet. She dives off a high tower, the elastic cords extend and instantaneously arrest her motion at a lowest point A (well above the ground). The cords then retract and she bounces through several cycles before finally coming to rest at a point B, where her ankles are 120 feet below the upper attachment point of the bungee cord.

(a) Estimate the location of the low-point A.

(b) During the portion of her jump from the initial take-off to the point A, estimate her maximum *downward* acceleration?

(c) During the portion of her jump from the initial take-off to the point A, estimate her maximum *upward* acceleration.

(d) Briefly explain the assumptions made in obtaining the previous estimates .

Seeking an ever-greater thrill the jumper doubles the slack length of the bungee cords (the new lowest point A' of her jump is still comfortably above the ground). For this second jump

(e) Estimate the location of the low-point A'.

(f) Estimate the location of B' where she finally comes to rest.

(g) During the portion of her jump from the initial take-off to the point A', estimate her maximum *downward* acceleration?

(h) During the portion of her jump from the initial take-off to the point A', estimate her maximum *upward* acceleration?

(i) Make a careful sketch of the time histories of vertical position during the initial downward phase (from initial take-off to point A , or A') for the two jumps. Use the same time and position axes for both curves.

4. Sensitive machinery and instruments are typically packed with shock-absorbing material for shipping. Fig.2 shows a plan view sketch of a 500-pound engine in a

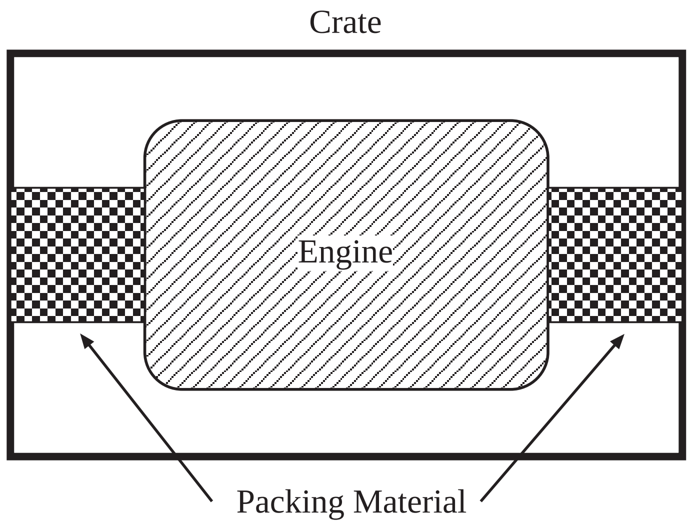

Figure 2: Engine in Shipping Crate

shipping crate with packing material at each end. In reality packing material is provided on all sides, but to keep things simple we only consider the end-to-end horizontal translation of the engine with respect to the crate. Assume that the packing material at each end can be adequately modeled as a linear elastic element with stiffness k combined with a linear friction element with damping parameter b.

(a) Derive a differential equation for the end-to-end motion of the engine with respect to a stationary crate.

(b) Specify the values of k and b required to provide a damped natural frquency of 1 Hz and a damping ratio of 0.707.