2.003 Fall 1999 Homework Assignment 5

1. Reconsider the 500-pound engine with shock-absrbing packaging in a shipping crate that was treated in Problem 4 of Homework Assignment 4. Take the packing material at each end to have an effective stiffness k of 100 pounds/inch, and an effective damping parameter b of 150 pounds/inch/second. Now suppose the crate is placed on a flat-bed truck so that the end-to-end orientation of the crate is parallel to the direction of motion of the truck. The crate is secured to the truck so that there is no relative motion of the crate with respect to the truck. However, because of the shock-absorbing packaging, the engine may move with respect to the crate when the truck accelerates and decelerates.

(a) Formulate a mathematical model to describe the longitudinal motion of the engine *with respect to the crate* in response to longitudinal motion of truck.

(b) Derive a single ordinary differential equation relating the (input) absolute velocity of the truck $v_{truck}(t)$ to the the (output) relative velocity of the engine to the crate $v_{engine}(t)$.

(c) Derive a state-determined system in which $v_{truck}(t)$ is the input and $v_{engine}(t)$ is the output.

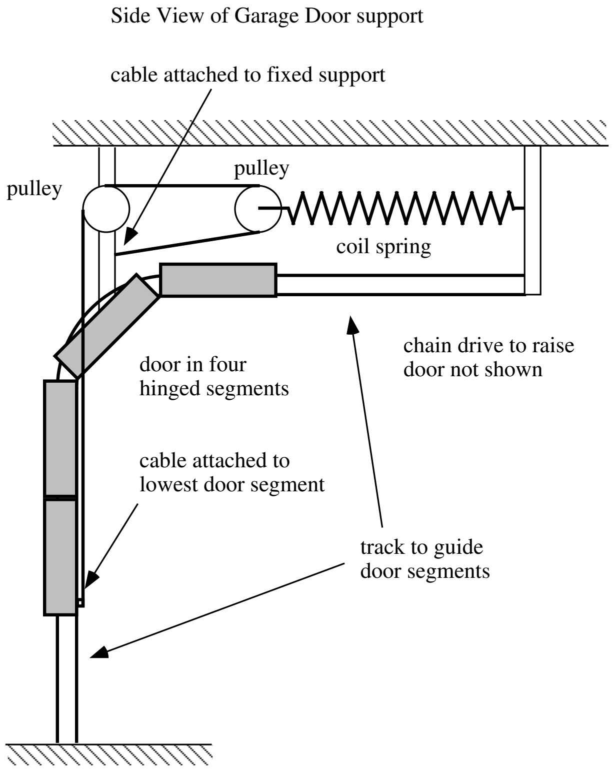

Figure 1: Garage-Door-Support System

2. Figure 1 shows a side view of one side of the support mechanism used in many garage-door openers. On each side of the garage door a portion of the weight of the door is supported by a long coil spring attached to the door through a cable and pulley system. It was observed, after the mechanism was recently serviced and freshly lubricated, that the suspended door had a tendency to oscillate up-and-down when disturbed.
Figure 2 shows a simplified schematic of one-half of the door support system. Half of the total door inertia is coupled to one of the overhead springs by the pulley system. Take the total weight of the door to be 200 pounds and the stiffness of each spring to be 5 pounds/inch.

(a) Formulate a model to analyze the oscillations of the door.

(b) Estimate the frequency, in Hz, of the oscillations.

(c) List the main assumptions underlying your model.

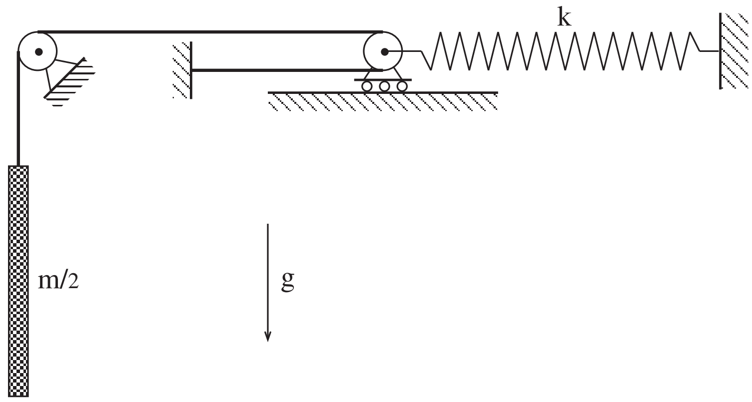

Figure 2: Simplified Schematic of Garage-Door-Support System

3. The steel plate on springs demonstrated in class has been modeled by the differential equation

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = f(t)$$

Remodel this as a state-determined system with the force $f(t)$ as the input and the vertical position and velocity of the plate as the outputs.

(a) The effective stiffness of the springs is known to be 3000 N/m. Choose the mass parameter m and the damping parameter b such that the steady state deflection of the plate under its own weight is $\Delta = 7$ millimeters and the damping ratio is $\zeta = 0.5$.

(b) Write a MATLAB script to integrate the state equations for the case where, at $t = 0$, an abruptly applied force equal to the weight of the plate is applied in the *upward* direction.

(c) Write a MATLAB script to produce plots of the first two seconds of transient response in (b). Produce plots of the following forms:

  (i) position vs. time

  (ii) velocity vs. time

  (iii) velocity vs. position