2.003 Fall 1999 Homework Assignment 6

1. Reconsider the problem of the hovering ballon which was discussed in Practice Session 4. A string dangling from a helium-filled balloon has it's free end resting on the floor as shown in Fig.1. As a result the balloon hovers at a fixed height off the floor and when it is deflected a little from that height it oscillates up and down for a while, eventually returning to the same height. Develop the simplest mathematical model competent to describe the vertical motion of the balloon. You may take all the elements of your model to be linear. Show that your model is competent to describe the observed hovering behavior.

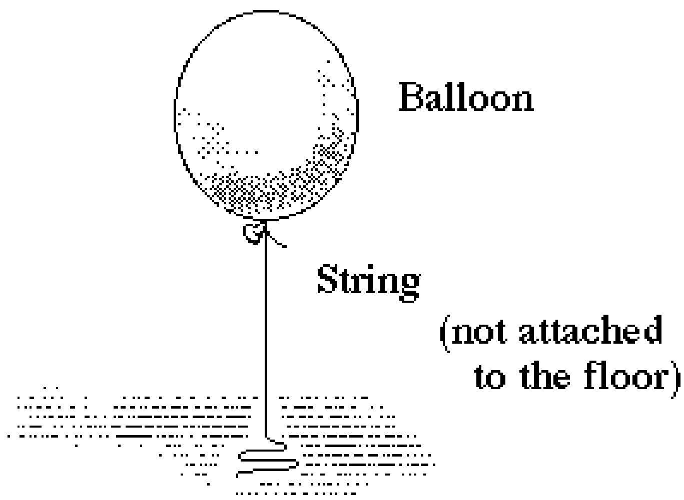

Figure 1: Hovering Balloon

(a) Show that your model has a steady state.

(b) Show that displacement from the steady state evokes a restoring force. What is the equivalent stiffness parameter?

(c) Show that your model could predict oscillation about the steady state.

(d) Show that your model predicts that the motion will eventually approach the steady state.

The balloon is released from rest at a small distance a above its steady-state height of 3.0 feet. After the first half-cycle of oscillation, which takes 1.5 seconds, the balloon is at a point $0.2a$ below the steady-state height, with an instantaneous velocity of zero. The string is known to weigh 0.5 ounces per foot. Use these data to estimate the following *behavioral* parameters:

(e) the damping ratio ζ;

(f) the undamped natural frequency ω_o;

and the following *model element* parameters:

(g) the effective mass m;

(h) the effective damping coefficient b;

(i) the effective stiffness k.

2. Reconsider the problem of blocked springs in an automobile suspension which was discussed in Practice Session 3. Figure 2 depicts the main components of the suspension for one wheel of an automobile. To change the ride and handling qualities, automobile enthusiasts sometimes insert "blocks" between some of the coils of the spring to prevent that part of it from deflecting. Consider the case where "blocks" are added to immobilize exactly *half* of the coils of each spring. Assume that:

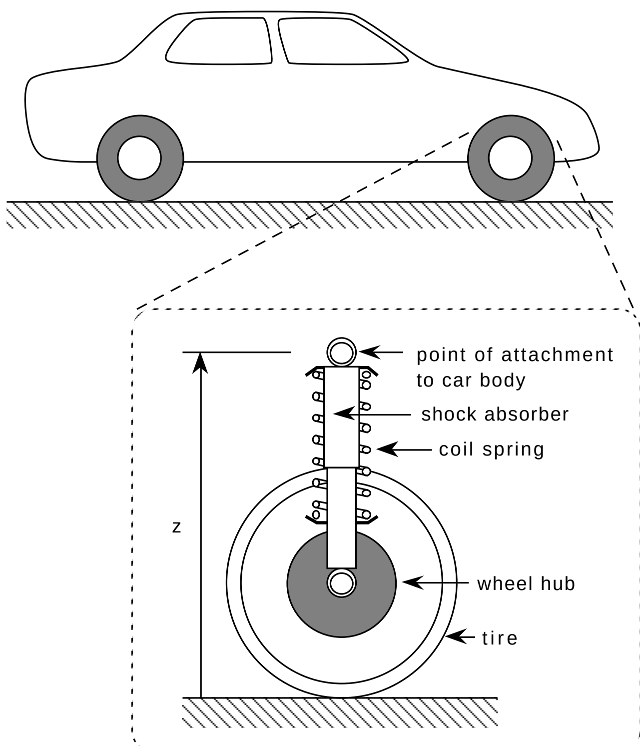

Figure 2: Automobile Suspension

All four wheels are identical and have identical suspensions.

The car moves vertically as a rigid body.

The tire deflections are negligibly small compared to the spring deflections.

The shock absorbers exhibit linear viscous behavior.

(In practice these assumptions are not especially accurate but they will keep the analysis simple and provide insight to the behavior of the suspension.)
It is known that the vehicle weighs 2,500 pounds, and that before the blocks were added the suspension was critically damped ($\zeta = 1$). After the blocks were added, the ride height was changed by 2.5 inches. Use these data to estimate the following parameters. Work in SI units.

(a) The suspension stiffness before, and after, the blocks were added.

(b) The undamped natural frequency of oscillation before, and after, the blocks were added.

(c) The suspension damping coefficient before the blocks were added.

(d) The suspension damping ratio after the blocks were added.

Write state equations and output equations to compute the response to an abruptly applied vertical load of 1,000 pounds. Adapt the MATLAB scripts of Homework Asasignment 5 to provide plots of the vertical displacement of the vehicle from its resting height vs. time, for (i) the suspension without blocks, and for (ii) the suspension with blocks added.

3. Reconsider the problem of automobile crashworthiness discussed in Practice Session 5. A vehicle weighing 1 ton is driven into a fixed concrete barrier at 10 mph. The vehicle's fender, which strikes the barrier first, is of the type that can deform under this load and return to its original shape (undamaged) when it is unloaded. If the fender were unable to dissipate energy, its maximum deflection would be 6 inches.

(a) Estimate the effective stiffness of the fender.

(b) Estimate the peak deceleration of the vehicle in SI units.

To absorb collision energy, linear dampers are added to the fender. Write state equations and output equations to predict the dynamics of the vehicle-fender system while the fender is in contact with the barrier. The outputs should include:

(i) The deflection of the fender.

(ii) The deceleration of the vehicle.

(iii) The total force exerted on the barrier.

Adapt the MATLAB script of Humework Assignment 5 to integrate these state equations to find the response of these outputs starting from the moment the fender first contacts the barrier. Make plots of the time histories of these three outputs for the following values of the damping ratio ζ:

(c) $\zeta = 0.25$

(d) $\zeta = 0.50$

(e) $\zeta = 0.75$

(f) $\zeta = 1.00$

(g) In which, if any, of the cases (c) through (f) does the fender remain in contact with the barrier after the impact is over?

(h) Can the peak deceleration of a vehicle with a fender with damping ever be greater than the peak deceleration of a vehicle with an undamped fender? Give a brief physical explanation for your answer.

(i) In which of the cases (c) through (f) is the peak deceleration the greatest?

(j) Estimate the value of the damping ratio ζ which would *minimize* the peak deceleration.