2.003 Fall 1999 Homework Assignment 7

1. **The start-up transient of a CD player**. In a typical CD player the disc is mounted directly on the shaft of an electric motor. The rotor of the motor has a moment of inertia I_r and experiences a frictional retarding torque which can be modeled by a linear damper with damping coefficient B_m.

   (a) The mass of a compact disc is 0.028 kg, and its diameter is 0.120 meters. The diameter of the hole in the center is 0.015 meters. Calculate the moment of inertia I_l of the CD.

   (b) Assume that the electric motor, when it is powered, produces a constant torque T_m independently of the rotational speed ω_m. It is observed that, after an initial starting transient, the steady-state rotor speed ω_{ss} is the *same*, whether a CD is mounted or not. The start-up transients are, however, different. With no disc mounted, the rotor spindle-speed reaches 95% of ω_{ss} in 0.30 seconds, but when a disc is mounted it takes 2.0 seconds to reach that same speed. Assuming that this behavior can described by a simple linear model, estimate the value of the damping coefficient B_m,

      (i) when there is no disc mounted, and

      (ii) when a CD is mounted.

   (c) Estimate the moment of inertia of the motor rotor I_r.

   (d) Construct a state-determined representation of your linear model for the system with the CD mounted. Take T_m to be the input and ω_m to be the output.

2. **Speed control of CD player**. A common design for a speed controller uses simple proportional velocity feedback. The actual speed of the disc is sensed from information written onto the disc. The motor torque T_m is then generated according to the following control algorithm

$$T_m = G(r - \omega_{sensed})$$

where r is a constant reference speed, and G is a constant gain.

   (a) What are the dimensions of the gain parameter G? What would be the units for G in the SI system of units?

   (b) Consider that this controller is applied to the CD player modelled in Problem 1(d). Construct a state-determined representation of a linear model for the controlled system with r as the input and ω_m as the output.

   (c) Compare the inputs required to achieve a steady-state disc rotation speed of 4.0 revolutions per second for the uncontrolled open-loop system of Problem 1 and for the controlled closed-loop system of the present Problem:

(i) What is the magnitude of the constant, suddenly applied, motor torque T_m required to reach the desired steady-state speed in the uncontrolled system.

(ii) Obtain a formula (involving the gain parameter G) for the magnitude of the constant, suddenly applied, reference speed r required to reach the desired steady-state speed in the controlled system.

(d) Design the controller; *i.e.,* choose the value of G, such that the time to reach 95% of the steady-state speed for the controlled system is 1/5 of the corresponding time for the uncontrolled system.

(e) Write a MATLAB script which can integrate the equation (or equations) of Part (b) and plot the time history of the motor torque T_m during the starting transient of the controlled system. Compare the magnitude of the *maximum* motor torque in the controlled case with the magnitude of the constant torque found in (c{i}) for the uncontrolled case.

3. **Locked load response.** The sketch depicts a common mechanical drive configuration discussed in class. To identify parameters of the motor and coupler it is often useful to conduct a test of the locked-load dynamic response. The load is immobilized; a step change in motor torque is applied; and the resulting motion of the motor is observed.

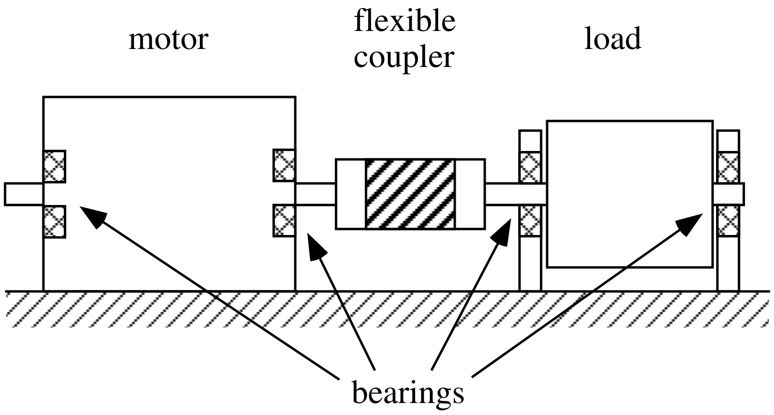

Figure 1: Mechanical Drive

(a) Assuming that all model elements have linear constitutive equations, develop a model to predict the angular position of the motor in response to an applied motor torque T_m. Express this model as a state-determined system.

(b) Calculate

(i) the damped natural period, and

(ii) the decrement ratio for the oscillatory response, when the model parameters have the following values:

$Irotor$ = 5e-5 kg-m^2
B_{motor} = 1e-4 N-m-s/r
$B_{coupler}$ = 2e-5 N-m-s/r
$K_{coupler}$ = 1.24e-2 N-m/r
T_m = 4e-3 N-m abruptly applied constant torque
$\omega(0) = 0$ initial angular velocity
$\theta(0) = 0$ initial displacement

(c) A proportional *velocity* feedback controller is added to the uncontrolled system described in (b) above. The motor angular speed is sensed and used in the control algorithm

$$T_m = G_1(r_1 - \omega_{sensed}) \tag{1}$$

where r_1 is a reference *speed* parameter, and G_1 is a gain parameter. In what units should G_1 be expressed? Analyze the closed-loop system and construct a state-determined system which describes its dynamic behavior. Calculate the controller gain, G_1, required to yield a critically-damped locked-load response.

(d) A proportional *position* feedback controller is added to the uncontrolled system described in (b) above. The motor angular position is sensed and used in the control algorithm

$$T_m = G_2(r_2 - \theta_{sensed}) \tag{2}$$

where r_2 is a reference *position* parameter, and G_2 is a gain parameter. In what units should G_2 be expressed? Analyze the closed-loop system and construct a state-determined system which describes its dynamic behavior. Derive algebraic expressions which show how the following closed-loop behavioral parameters depend on the feedback gain, G_2:

(i) The locked-load undamped natural frequency ω_O.

(ii) The locked-load damping ratio ζ.

(e) The usual implementation of the control algorithms (1) and (2) is with electronic signal processing and amplification. It is however theoretically possible to implement these algorithms mechanically by interpreting the control algorithms as constitutive equations for elements connected between the motor rotor and a reference rotor which is driven with the desired motion. Can you identify the type of mechanical element which is represented by:

(i) The algorithm (1)?

(ii) The algorithm (2)?

4. **Third-order dynamic system.** Consider the mechanical drive of Problem 3 with the load *free* to move. The model parameters have the following values:

I_{rotor} = 5e-5 kg-m^2
B_{motor} = 1e-4 N-m-s/r
$B_{coupler}$ = 2e-5 N-m-s/r
$K_{coupler}$ = 1.24e-2 N-m/r
I_{load} = 5e-5 kg-m^2
B_{load} = 1e-4 N-m-s/r

This is a system with fairly special symmetry. Note that $I_{motor} = I_{load} = I$, and that $B_{motor} = B_{load} = B$. For this special case, develop a model to predict the angular velocity of the load in response to an applied motor torque T_m.

(a) Use the variables:

$\Delta\theta$, relative angular displacement of motor with respect to load,

ω_m, motor angular velocity, and

ω_l, load angular velocity,

as the state variables for this model in a state-determined system. Construct the state equations for this model.

(b) Consider a change of variables to the following three variables:

$$\Delta\theta, \qquad \Delta\omega = \omega_m - \omega_l, \qquad \text{and} \qquad \omega_{avg} = \frac{\omega_m + \omega_l}{2}$$

Express the variables ω_m and ω_l in terms of the variables $\Delta\omega$, and ω_{avg}.

(c) Re-write the state-determined system using $\Delta\theta$, $\Delta\omega$, and ω_{avg} as state variables. Show that the dynamic behavior of the system may be viewed as a combination of a first-order system with state variable ω_{avg} and a second-order system with state variables $\Delta\theta$, and $\Delta\omega$. Note that both sub-systems are excited by the motor torque T_m but are otherwise independent of one another.

(d) Evaluate the first-order behavioral parameter: the decay time-constant τ. Estimate the steady-state speed ω_{ss} when T_m = 6.0e-3 N-m.

(e) Evaluate the second-order behavioral parameters: the undamped natural frequency ω_o, and the damping ratio ζ.