MASSACHUSETTS INSTITUTE OF TECHNOLOGY

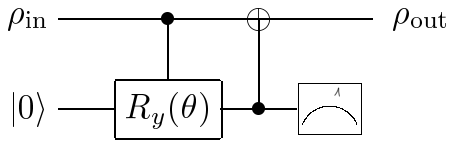
MIT 2.111/8.411/6.898/18.435
Quantum Information Science I

November 22, 2010

Problem Set #10
(due in class, THURSDAY 02-Dec-10)

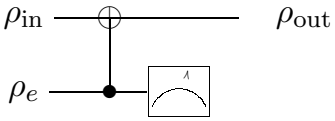
P1: (**Circuit models of quantum operations**) Let $\rho_{\rm out} = \sum_k E_k \rho_{\rm in} E_k^\dagger$.

(a) Give the operation elements E_k describing the mapping for this circuit:



What physical process does this describe?

(b) Give the E_k for this circuit, assuming $\rho_e = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$:



What physical process does this describe?

P2: (**Quantum noise and codes**) Single qubit quantum operations $\mathcal{E}(\rho)$ model quantum noise which is corrected by quantum error correction codes.

(a) Construct operation elements for $\mathcal{E}$ such that upon input of any state ρ replaces it with the completely randomized state $I/2$. It is amazing that even such noise models as this may be corrected by codes such as the Shor code!

(b) The action of the bit flip channel can be described by the quantum operation $\mathcal{E}(\rho) = (1-p)\rho + pX\rho X$. Show that this may be given an alternate operator-sum representation, as $\mathcal{E}(\rho) = (1-2p)\rho + 2pP_+\rho P_+ + 2pP_-\rho P_-$ where P_+ and P_- are projectors onto the $+1$ and -1 eigenstates of X, $(|0\rangle + |1\rangle)/\sqrt{2}$ and $(|0\rangle - |1\rangle)/\sqrt{2}$, respectively. This latter representation can be understood as a model in which the qubit is left alone with probability $1-2p$, and is 'measured' by the environment in the $|+\rangle, |-\rangle$ basis with probability $2p$.

P3: (**Shor's 9 qubit code**) The Shor code is able to protect against phase flip and bit flip errors on any qubit.

(a) Show that the syndrome measurement for detecting phase flip errors in the Shor code corresponds to measuring the observables $X_1X_2X_3X_4X_5X_6$ and $X_4X_5X_6X_7X_8X_9$.

(b) Show that recovery from a phase flip on any of the first three qubits may be accomplished by applying the operator $Z_1Z_2Z_3$.

P4: (**Recent quantum communication results**) . Find a recent paper in the literature about a recent (post-2008) theoteical or experimental advance in quantum communication, involving distributed qubits. Write a short ($<$ 500 word) summary of it, on the QIS wiki. See instructions on the course homepage, `http://web.mit.edu/2.111/`